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EXPLANATORY HANDBOOK TO IRC:112-2011 CODE OF PRACTICE FOR CONCRETE ROAD BRIDGES

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EXPLANATORY HANDBOOK TO IRC:112-2011 CODE OF PRACTICE FOR CONCRETE ROAD BRIDGES

INTRODUCTION

The principal aim of this Explanatory Handbook is to provide the user with guidance on the interpretation and use of IRC:112 and to present the worked examples. The examples cover topics that are in line with the codal clauses and that will be encountered in a typical bridge designs.

The work of preparing the document was entrusted to Consultants, M/s. B&S Engineering Consultant Pvt. Ltd. In Joint Venture Aassociation with M/s. Spectrum Techno Consultant (P) Ltd. The draft prepared by the Consultants was discussed by the B-4 Committee at several meetings and finalised on 15th March 2014. The present composition of the B-4 Committee is as follows:

Accordingly, the draft document was approved by the Bridges Specifications & Standards Committee in its meeting held on 8th August 2014. The document was considered by IRC Council in its 203rd meeting held on 19th and 20th August in New Delhi and approved.

This publication is not to be regarded as an IRC Code for any contractual or legal considerations. The publication is meant only to serve as a guide to the designers. It contains non-contradictory and complementary information which does not purport to include other necessary provisions of a contract.

IRC Thank to all the Committee members of the B-4 committee for their support in bringing out this handbook. Special thanks are due to the following persons for their immense contribution, perseverance, patience and for their selfless work in authoring and/or reviewing the various chapters of this explanatory handbook:

Shri Alok Bhowmick	Shri S.G. Joglekar
Dr. A.K. Mittal	Shri T. Viswanathan
Dr. A.K. Mullick	Shri V.N. Heggade
Shri D.A. Bhide	Shri U.K. Rajeshirke

CHAPTER 1

BACKGROUND AND OVERVIEW OF THE CODE

This Explanatory Handbook is a complementary document which is non-contradictory in its contents to the limit state code IRC:112-2011 ‘Code of Practice for Concrete Road Bridges’. The document gives:

a) Background to Code clauses, wherever considered as necessary

b) Explanation and commentary on some specific clauses/sub-clauses. All clauses of the Code are not covered

c) Worked-out examples highlighting application of specific clauses

IRC:112-2011 replaces two earlier codes IRC: 21 for ‘Plain and Reinforced Concrete Bridges’ and IRC:18 for ‘Post-Tensioned Concrete Bridges’. Some of the design procedures and analytical model from these Codes are permitted for use till further notice of withdrawal of the same, details of which are given in the Normative **Annexure A-4** of the Code.

IRC:112 is a unified Code covering various types of concrete bridges, using plain concrete, reinforced concrete, and prestressed concrete, constructed by cast-in-place, precast and composite construction methods.

For some special applications, like suspension bridges and cable stayed bridges, provisions of this Code can be applied to the extent applicable, combined with the use of specialist literature for these types of bridges. IRC:112-2011 is in line with the new generation of rationalised international concrete codes using semi-probabilistic Limit State approach to arrive at the desired targets of safety, serviceability, durability and economy in a consistent and reliable way. It incorporates up-to-date knowledge of the behaviour of concrete, steel and composite structures. It takes into account the present state of the art of bridge construction, using modern construction technology, together with current conventional practices in India. It also incorporates new developments in the field of bridge engineering that have taken place internationally. As such the Code includes many new details within its body, which were not covered in earlier codes.

It is realised that the Code is to be used by the present generation of practising engineers, who will have to familiarise themselves with the new and more advanced methods of the analysis and design used with the Limit State approach. This Explanatory Handbook is intended to help the users using such methods.

2 LAYOUT OF THIS GUIDE

This Explanatory Handbook has 19 Chapters covering **Section 4 to Section 18, Annexure A-1 & Annexure B-1** of the Code. The content sheet gives the correlation between the handbook chapter and the corresponding section of the Code, sequentially. The

1st chapter of the handbook (i.e. this chapter) provides a brief background information and broad overview of the Code to enable the users of IRC:112 to understand the origin and objectives of its provisions.

The handbook generally follows the chapter numbers as per the handbook on the left hand side and corresponding relevant section numbers of the Code on the right hand side of the page so that guidance can be sought on the Code on a section by section basis. Figure numbers generally follow the relevant section number of the Code.

3 NEED FOR NEW GENERATION OF CODES

The unprecedented and rapid growth of concrete construction, both in developed and developing countries, is the driving force behind the search for stronger, better and cheaper materials, improvements in analysis and design methods, improved technology and use of mechanised, fast track construction methods. The revised Code therefore has to take this in to account. The significant developments that necessitate the new generation of Codes can be grouped in three categories as follows:

A) Scientific Developments

- Unprecedented growth of knowledge about concrete and its structural behaviour in the last 20 years
- Development of new structural forms
- Research on durability of concrete
- Growing awareness about sustainable construction
- Research and experience of seismic response of structures
- New powerful methods of computer based analysis and design

B) Technological Developments

- Development of stronger reinforcing steels, prestressing steels and concretes of high strength and high performance
- Applications of fast track construction techniques and large scale mechanisation of construction
- Use of large sized pre-cast segments and heavy lifting capabilities, which allow rapid construction of longer, taller and bigger structures in all kinds of difficult environments

C) Development of Semi-Probabilistic, Limit-State Design Philosophy

The ‘Semi-Probabilistic, Limit State Design Philosophy’, which allows application of uniform and rational safety norms to all types of structural elements, provides the basis for many international Codes

Since the development efforts are continuing at accelerated pace all over the world, concrete technology has truly become an international activity, especially so for the bridges. It is reasonable to expect that this trend will continue.

To meet the changing needs of society, new developments in materials, new technologies and additional or new concerns are bound to emerge in different parts of the world. The Codes of new generation need to accommodate these developments rapidly. It therefore becomes imperative that the Code adopts a rational design philosophy with transparent aims and appropriate strategies.

4 CONTENTS OF IRC:112-2011

4.1 New Features of the Code

A) Section 5 : Basis of Design

The desired aims and objectives are listed in '**Section 4 General**' of the Code. **Section 5** will help users to understand the basis and significance of various provisions of the Code.

The **Section 5** includes brief descriptions of the multiple strategies adopted by the Code using concepts of limit state philosophy, reliability, limit states considered for design, types of actions and their combinations, analytical modelling, material properties, service life, design life (normally 100 years), methods to achieve durability and design based on full scale testing.

B) Materials and their Properties

These are covered in **Sections 6 & 18 and Annexure A-2**.

a) **Section 6: Material Properties and their Design Values**

- This section covers the main materials, viz. reinforcing and prestressing steels, and concretes of various grades. Grades of steel reinforcement are extended upto and including Fe 600. The concrete grades are extended from earlier M 60 upto and including M 90.
- The simplified design values of properties, which are sufficiently accurate for normal applications, are specified. Bilinear stress-strain diagrams are specified for reinforcing and prestressing steels.

b) **Annexure A-2: Additional Information and Data about Properties of Concrete and Steel**

This Annexure gives more accurate values and laws covering material properties, which are required for extrapolation of solutions beyond the normal range and for use in innovative or new applications. For a normal user of this Code, awareness of these properties will help to understand the situations in which the design (and construction) should be based on more exact values of properties as given in the Annexure.

c) **Section 18**: Materials, Quality Control and Workmanship

This Section provides the material properties of manufactured items, which are controlled by BIS or other International Codes. These are to be used for the procurement, testing and quality assurance purposes.

C) Section 7 : Analysis

The Code covers many types of bridges with different geometry, which are exposed to different types of actions and combinations thereof, each of which represents a different design situation. In order to assess the response of the structure in different situations, distinct types of analysis are required. Linear elastic analysis are most commonly used and are generally adequate. The developments in powerful Finite Element techniques have allowed analysis of complex structural forms and loading conditions. The resultant stress fields from the analysis can be directly converted to detailing of reinforcing steel.

The advent of computerised analysis and availability of advanced software have put very powerful analytical tools in the hands of designers and raised the standards of analysis far above those of the past. These developments called for the new Section on 'Analysis'. This Section also includes a number of simplifications which have been found to be adequate from past practices and experience.

The new trend set up by fib Model Code 2010 (MC 2010) is for the Codes to indicate different levels of analyses, from simplified methods to be used in normal applications to more and more complex methods needed when the load effects and behaviour of the structures (not considered in the normal design situations) become significant and important for proper understanding, design and construction.

D) Detailing of Reinforcement, Prestressing Tendons, and Limiting Concrete Dimensions

Three Sections cover these in greater details than hitherto

- Sections 15 Detailing General Requirements
- Section 16 Detailing Requirements of Structural Members
- Section 17 Ductile Detailing for Seismic Resistance

Large numbers of figures explain the requirements

E) Informative Annexures

This is a new concept adopted to bring some of the pertinent technical information to the attention of users. The Annexures do not form part of the requirements of the Code. However, by using the information or methods given therein, the recommendations of the Code can be implemented effectively. These also provide additional or supplementary information for creating more awareness and better understanding of the Code among the users.

Three numbers of **Annexures B-1, B-2 and B-3** are included in the Code.

F) Repeating Technical Information/Requirements in Format of Tables as well as Equations

The tables are given for ready reference and ease of hand calculations. The equations repeat the same in formats suitable for computerising the calculations. The tabulated values may not exactly match with those derived by equations as the same are rounded off in certain cases.

G) Optional Use of Working Load/Allowable Stress Method

To ensure continuity and smooth transition from old Codes to methods of new Code, use of working load/allowable stresses method is also permitted for some time. This is done through provisions in **Annexure A-4**. The scope and details of this Annexure are on similar lines as those of IRC:18 and IRC:21. The exceptions are specifically included in the Annexure.

Almost all operative sections are brought upto date with relevant international standards and practices. The Sections and the salient features of the same are as below:

4.2 Other Improvements

A) Section 3: Definitions and Notations

In view of relatively new terminology needed for describing the limit state methods and extensive use of mathematical equations and notations, an exhaustive coverage is included as ready reference.

B) Section 4: General

a) This Section, after describing the applicability to all structural elements using normal weight concrete, further allows use of its relevant parts for other concretes, (e.g. light weight concretes and hybrid structures) based on special knowledge, specialist literature and/or experimental data at the discretion and responsibility of the owner/designer.

b) The underlying assumptions also bring out the important aspect of quality assurance and routine maintenance.

C) Section 8: Ultimate Limit State of Linear Elements for Bending and Axial Forces

For all linear members (including beams, columns, ties, struts etc.) carrying axial forces arising from external loads or prestressing effects of bonded or unbonded tendons, and resisting simultaneously the bending moment, if any, arising from any source, the distribution of strains at any section is taken as linear. In other words, plane section before action of forces remains plane after the action of forces, right upto the failure state.

Under this single assumption, which is reasonably valid for most of the loadings upto failure stage, the ultimate strength of all types of linear members is calculated, using stress-strain relationships given in the Code. Either the simplified diagrams or more accurate relationship can be used.

D) Section 9: Ultimate Limit State of Two and Three Dimensional Elements for Out-of-Plane and in-Plane Loading Effects

The generalised or classical solutions for such elements subjected to combined in-plane and out-of-plane loading conditions are quite complex. This Section gives simplified approaches for the design of slabs and webs of box sections.

E) Section 10: Design for Shear, Punching Shear and Torsion

The design verification of shear is carried out at Ultimate Limit State (ULS) only. The design of members requiring shear reinforcement is based on truss model. Shear design of members not requiring shear reinforcement is with empirical formulas, evolved based on results of extensive experimentation.

The design for shear of both reinforced and prestressed members is based on the same model. This is a deviation from the past. The rules of torsional resistance have also been changed from the past practice. Due to introduction of the new methods, detailed explanation is included in the Section itself.

F) Section 11: Ultimate Limit State of Induced Deformation

The slender bridge sub-structures such as piers of variable cross-sections, with or without piles to support them, could not be checked for buckling of overall height by methods given in earlier codes. Cumbersome calculations based on advanced elastic methods were required. The present Code has rather simplified the work by introducing the criteria of permisssible increase of stresses due to second order deformations, which do not require further detailed ULS checks.

For concrete members of uniform cross-section, slenderness is defined not only in terms of l_e/i, (l_e/r in the old notation) but is based on a factor λ lim, defined in the Code, which is a more accurate estimator. The general method of calculating the effective length depending upon the stiffness of end restraints given in Euro Code is followed. However, to simplify the calculations for normally met conditions of piers in bridges, simplified and well established values are provided in the tabular form, based on BS 5400. For calculating the ultimate strength of slender members, if required, a generalised method is included.

G) Section 12: Serviceability Limit State

The serviceability checks are restricted to check of stress levels in concrete, check of crack widths and check of deflections. Deemed control of crack widths by certain detailing parameters of reinforcement without calculation is permitted. Other serviceability states, such as vibration, are not covered in the Code. For these, specialist literature may be referred.

H) Section 13: Prestressing

This Section is in line with international practices. For anchorages and couplers, the requirements of acceptance through testing as per the methods based on the CEB/FIP recommendations are introduced in the Code.

I) Section 14: Durability

The durability requirements specified in this Section are consistent with the requirement of 100 years design life. The criteria of aggressiveness are based on the general environment in which the bridges are located in the country.

CHAPTER 2

SECTION 4 : GENERAL

2.1 Scope Cl. 1.4.1

Unlike earlier Codes, IRC:112-2011 strictly defines its scope and applicability. The Code deals with the structural use of plain cement concrete, reinforced concrete and prestressed concrete in highway structures using normal weight concrete. The Code also permits partial use of its recommendations for other types of concretes having different properties and different applications in which concrete is one of the components, e.g. hybrid structures.

Though the provisions of this Code are strictly applicable to highway bridges (& culverts) only, the choice of making use of the appropriately valid provisions of the Code for other highway and appurtenant structures is left to the wisdom of qualified and experienced personnel involved in such designs.

The Code recognizes that the limit state methods are not yet established in India for design of bridges and declares that it “strives to establish a common procedure for road bridges including foot bridges in India.” It is inferred, though not stated, that the recommendations are based on international practices, which are examined and modified in light of the Indian experience of using working load/allowable stress methods.

A ‘hybrid system’ is a system in which load is resisted by combination of two or more component materials in such a way that each component supplements its capacity with the capacity of the other component. Reinforced concrete and rolled or fabricated structural steel can be effectively used to make hybrid structure. Structural steel tubes with concrete in-fill are another example of hybrid structure. The consistency of internal strains at the contact surfaces, arising from bond is not an essential condition at ULS, although overall deformations have to be consistent.

There are several bridge elements (like precast segmental bridges, voided slab bridges, continuous bridges and pretensioned girder type bridges), which deserves more detailed coverage in the Code. It is expected that the Special Publications of IRC on these elements (i.e. IRC:SP: 64,65,66 & 71) will be amended soon to provide for design as per Limit State Methods in line with IRC:112.

2.2 Underlying Assumptions Cl. 43

The underlying assumptions stated in this Section are mostly the assumptions which the designers make about the standards of construction and site management which they expect to be followed on the project they are designing. The validity and satisfactory performance of the designed structure depends on satisfactory execution about these aspects. These assumptions emphasize the role of all the agencies involved in construction viz. Owner, Contractor, Supervision Consultant, Design Consultant, in the right perspective for fulfilling the design intents, especially the intended service life of 100 years. It is useful to discuss some of the aspects in detail, to cover the unstated but obviously related issues.

The Code becomes applicable with certain basic conditions. The intention behind stating these conditions is not to use them as a disclaimer, but to bring to the attention of users the fact that following the Code faithfully in the design process alone will not result in the satisfactory long term performance of bridges. The role of other agencies in realising the intents of the Code, in long service life of the structure is equally important. It is imperative that these assumptions in the design Code must be converted into practical, effective and contractually enforceable “specified requirements” for each construction process.

The explanations and clarifications on underlying assumptions are as under:

Assumption (1): The choice of structural system and the design of the structure are made by appropriately qualified and experienced personnel.

Bridges provide vital communication links, sometimes the only link, in the highway system. They dominate our landscape and play a vital role in our visual environment. The importance of good conceptual design for any bridge therefore cannot be over emphasised. The ability to produce a ‘conceptual design’ is one which is acquired through experience over a period of time and is nurtured by the successes and failures of past projects. A great effort is called for from professional engineers in understanding these requirements of a bridge project so as to fix the optimum structural scheme. The present and future traffic needs, knowledge of the hydraulics and flood-history of the river, geotechnical conditions, behaviour and experience of other bridges on the same river and in comparable environmental exposure conditions, available construction technology, time needed for the construction, cost of materials and labour and sustainability concerns are some of the issues involved in identifying suitable alternatives at the conceptual design stage.

In short, the conceptual design as well as proper application of the Code can be satisfactorily done only by appropriately qualified and experienced personnel, working individually or collectively as a team.

Assumption (2): Execution is carried out by personnel having the appropriate qualification, skill and experience.

This assumption is self evident, but it is the case of ‘easier said than done’. In practice, at all stages of detailed design and execution some of the personnel are in the stage of gaining ‘hands on’ experience. They need to be properly trained, guided and supervised in order to fulfil this condition. How to achieve this and the other assumptions numbered (3), (4) and (5) in **Clause 4.2** of the Code is the subject matter of the ‘Quality System’ to be set up for controlling the entire activities of project preparation, design, construction and maintenance. It should be realised that in expounding all the strategies and their successful and reliable application, the Code implicitly depends upon human skills. Management of the involved personnel is the subject matter of the ‘Quality Systems’. The IRC has published Special Publication SP: 47-1998 ‘Guidelines for Quality Systems for Road Bridges’ which should be referred for setting up and operating management of all these activities. Mandatory process control and verification requirements should be specified in the tender documents and enforced during design and construction stages.

Assumption (3): Adequate supervision and quality control are provided during all stages of design and construction.

This is needed even if the Assumption (2) is otherwise satisfied. It is based on the sound principle that systems and human beings are fallible and the resulting errors from non-application of efforts, un-intended oversights or downright mistakes can be controlled by introducing at least one more level of supervision of the activity. In important projects, more than two levels of quality controls are used. Reference is made to IRC:SP:47 for details of Quality Systems.

Assumption (4): The construction materials and products are provided and used as specified by relevant national standards.

This is another self-evident statement. However it will be a sobering thought to keep in mind that the national standards specify only the minimum acceptable requirements. "Doing better than the minimum" will normally improve the quality of the end product.

Assumption (5): The intended levels of properties of materials adopted in the design are available.

This is obvious for ensuring the validity and adequacy of the design using the Code.

Assumption (6): The structure will be used as intended and is maintained adequately.

This stipulation has come out of the aim of achieving the specified design life using methods stated in the Code. The provisions of the Code in themselves are not adequate to do so. Timely and proper maintenance and repair of the structures are needed. IRC has published a number of guidelines, listed below, which need to be implemented by the owner, by appointing the experts or its own in-house staff for this activity.

a) IRC:SP:18 Manual for Highway Bridge Maintenance Inspection

b) IRC:SP:35 Guidelines for Inspection and Maintenance of Bridges

c) IRC:SP:37 Guidelines for Load Carrying Capacity of Bridges

d) IRC:SP:40 Guidelines on Techniques for Strengthening and Rehabilitation of Bridges

e) IRC:SP:51 Guidelines for Load Testing of Bridges

f) IRC:SP:52 Bridge Inspector's Reference Manual

g) IRC:SP:60 An Approach Document for Assessment of Remaining Life of Concrete Bridges

h) IRC:SP:80 Guidelines for Corrosion Prevention, Monitoring and Remedial Measures for Concrete Bridge Structures

CHAPTER 3

SECTION 5: BASIS OF DESIGN

3 GENERAL

This Section describes the overall basis of the recommendations of the Code. It indicates the design philosophy, aims of design, methods of design and other strategies adopted by the Code to achieve the stated and unstated aims of design. Various basic choices and strategies adopted in the Code are described under appropriate headings. Collectively, they provide assurance of achieving the aim of designing functional, safe and durable bridges.

On the basis of the approach outlined in this section, the recommendations can be used with full understanding of their context, applicability and limitations. Where the design needs to be supplemented by information available from specialist literature or other international codes, it needs to be critically evaluated for its applicability and consistency before combining the same with the approach outlined in this Section. Format of the Code will allow the future modifications and revisions of the Code itself, to include new knowledge and technological developments to be presented in a way which is consistent with the overall philosophy and basis of design of the present Code.

The Code is based on clear and scientifically well founded theories and models. Though the Code presumes that these scientific concepts and methods stated in the Code are generally known to practicing engineers, this Section provides some explanations to the fundamental concepts adopted in the Code.

3.1 Aims of Design — Cl. 5.1

3.1.2 *Reliability Aspects and Codal Approach* — Cl. 5.1.2

The unprecedented development of computational capabilities, the increasingly available databases on variability of materials and loads, the development of new sensor technologies, the use of new materials, the new level of maturity of probabilistic methods and many advances in the field of structural mechanics have paved the way for a more prominent role of Structural Reliability methods as rational tools for development of design codes. Current international design codes and standards (e.g. ACI 318, and Euro Codes) are based on level 1 methods that employ only one "characteristic" value of each uncertain parameter and its variability. (These are also known as 'Semi-probabilistic' methods).

Target reliability level is often expressed in terms of reliability index β, which is mathematically related to the probability of the event of which reliability is investigated. The relationship of probability (of failure in reference period of one year) and index β is expressed in a tabular form bellow:

β Values Related to the to the Failure Probability P_f

P_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-6}
β	1.28	2.32	3.09	3.72	4.75

For more detailed explanation refer to the literature on reliability and the fib Model Code 2010.

The Euro Code defines target β = 4.8 for a one-year return period, which is equivalent to 3.8 for a 50-year return period. This is for regular structures. Other values are given in these codes, if the consequences of collapse of the structure are especially severe or insignificant.

The probability and reliability based evaluation of the risk of achieving or not achieving a certain aim and keeping the risk within 'acceptable limits' by choosing appropriate partial factors for loads and strengths is the fundamental method adopted by the Code. This Code has not defined the values of 'acceptable limits' used in the Code. However from the literature regarding the basis of Euro Codes the targeted 'β' values can be judged by comparison. In cases where 'β' values cannot be assessed with any degree of certainty, the prevailing design practices are re-examined and adopted with modifications, and retrofitting the requirement in the probability format of the 'partial factor method' as is done for other requirements (provisions) for sake of consistency. The Code does not use the direct evaluation of risk using methods of mathematical probability. It uses semi-probabilistic methods following the design format based upon statistical concepts of characteristic values of loads and material properties, and multipliers to modify them, which are termed as partial factors. The Code strives to achieve the desirable degree of reliability by approximate methods, using a combination of factors as outlined in this clause of the Code.

3.1.3 *Safety, Serviceability, Durability and Economy* Cl. 5.1.3

The Code aims to achieve safety, serviceability, durability and economy in the design and construction of bridges by stipulating certain set of requirements about the materials, structural models, methods of analysis, design approach and detailing apart from the controlled quality of construction for realizing the design aims.

The acceptable limits of safety and serviceability, measured in terms of probability with wide international acceptance, are as follows:

a) The level of the probability of structural failure under action of the working loads (i.e. safety) is kept less than 10^{-6} (one in a million) and less than 10^{-4} (one in 10,000) of exceeding the specified performance levels at service loads (i.e. serviceability), in period of one year. The Codal method of doing so, namely the use of partial factors on loads and material properties, reasonably assures that the targeted levels of probability are met. This assessment does not cover risks arising out of human error or accidents of non-structural nature. Based on these basic risk levels, the risks of failure within the design life are approximately given by the annual risk multiplied by the design life in years. (This is true enough for low values of annual probabilities, although not mathematically accurate).

b) The aim of achieving durability is based on the past experience of the behavior of structures located in various climatic environments in India. The international experience and current practices of achieving durability are taken into account. These methods are covered in **Section 14** and further discussed in detail in **Chapter 12** of this handbook.

   For the methods of calculation of the concrete cover to the reinforcement based on the rate of penetration of the attacking agents or the types of deteriorating mechanisms for targeting a minimum stipulated service life, the designers can refer to special literature such as fib bulletins.

c) Achieving economy is addressed indirectly in the Code by allowing maximum exploitation of the materials used and accepting risk levels appropriate for various situations as indicated in **Clause 5.1.3, (1) and (2)**.

3.2 Limit State Philosophy of Design **Cl. 5.2**

The basic approaches of Limit State Methods are stated in **Section 5.2 (1) to (6)**. The explanations for 5.2 (1) to (3) are as below. Others are self explanatory.

1) A structure designed to serve its function is subjected to various direct external actions or indirect actions resulting from environmental and geo-technical phenomenon during its service life, which defines its loading history. It experiences different physical situations having exposed to different combinations of actions, termed as 'Design situations'.

   Limit state philosophy of design refers to a condition of a structure beyond which it no longer fulfills the relevant design criteria. The condition may refer to a degree of loading or other actions on the structure, while the criteria refer to structural integrity, fitness for use, durability or other design requirements. A structure designed by limit state method is proportioned to sustain all actions likely to occur during its design life, and to remain fit for use, with an appropriate level of reliability, for each limit state.

2) In the limit state approach for designs, ultimate strength including strength controlled by induced deformations (ULS) and serviceability limit state (SLS) are mainly considered. Semi-probabilistic methods are used to verify that the limits are not exceeded. The serviceability limit states presently include checks to control overstress in concrete, crack-widths and limiting the deflection of the structure. The deflection limits specified are aimed to achieve indirectly the rigidity and robustness and to avoid any visual discomfort, rather than to achieve any functional need of the road traffic.

   The Code indicates that for some structures the vibration control may be an important consideration (e.g. for foot bridges and foot paths of road bridges) although it is not considered in the Code. Limit state of fatigue has also not been included.

3) Considering the statistical nature of variation of loads and material properties, time dependent changes in the same, uncertainties and limitations of structural models and methods of analysis, quality of construction and finally the deterioration and maintenance, a margin called factor of safety against risk of failure in meeting the performance has to be provided in the design. However over design has to be kept within limits for sake economy, considering initial cost as well as life-cycle cost.

   Use of partial factors, which are different for the same load in verification of different limit states, is made together with appropriate material factors describing the minimum strength properties of the materials to achieve the targeted level of reliability (safety). Appropriate experience based methods are used to achieve the same where statistical methods have not developed sufficiently.

3.3 Limit States **Cl. 5.3**

In the context of performance-based Limit State Design, performance criteria for serviceability and structural safety are specified for two basic groups of limit states.

a) **Ultimate Limit State Criteria:** Ultimate limit states are limit states associated with the loss of structure by collapse or unacceptable damage to the structure, thereby leading to loss of life, disruption of operation and/or damage to the environment. Thus, the Ultimate Limit States are related to design principles with respect to the performance-based Limit States for structural safety. They may relate to limit state of equilibrium and/or limit state of strength.

b) **Serviceability Limit State Criteria:** Serviceability Limit States correspond to the states beyond which specified performance Requirements for a structure or a structural component are no longer met. Various limit states to be satisfies under serviceability are:

i) Limit State of Crack Control

ii) Limit State of Deformation

iii) Limit State of Vibration

iv) Limit State of Fatigue

Fatigue verification can be carried out only when details of fatigue vehicles are made available. Fatigue vehicles are not yet defined in IRC codes. This is still under consideration of IRC. However, pending the finalization of fatigue vehicles, the fatigue verification can be avoided provided following conditions are met with:

i) For reinforced concrete structures when the stress in the tensile reinforcement is maintained less than 300 MPa under Rare Combination of Serviceability Limit State as against 0.8 fy specified in **Clause No. 12.2.2** of the Code.

ii) For prestressed concrete structures under the frequent combination of actions and prestressing force, only compressive stresses occur at the extreme concrete fibers, under frequent load combination of Serviceability Limit State.

Vibration verification for bridges is specifically required but is deemed complied by limiting deformations specified in the Code in all cases except for special types of bridges and footbridges or components of footways, as indicated in the Code.

3.4 Actions and their Combinations

Comprehensive description of this is included in **Chapter 17, Annexure A1**, 'Combination of Actions for Bridge Design'. The values of actions and partial factors to be used in different combinations for verification of design by Limit State Method are available in **Annexure B of IRC:6**.

It may be noted that limit state approach is to be followed for structural design of bridge components only. Until the Foundation Code, IRC:78 is modified to include material safety factors and resistance factors for the soil parameters, un-factored loads are to be used for checking of base pressure under foundation, stability check for foundation and for checking of maximum load on pile foundation. **Table 3.4** of IRC:6-2010 therefore shall be used only for the structural design till such time.

3.5 Representative Values of the Properties of Materials — Cl. 5.5

The material properties are, in principle, based on the statistical distribution of the values, using characteristic properties such as mean value or upper or lower fractiles. However, in actual practice this is possible for limited cases. For tensile strength of steels, the manufacture is controlled by a minimum specified value as defined by BIS Standards and this nominal minimum strength is assumed to represent the characteristic strength of 5 percent fractile, which assumption is on the conservative side for assessment of structural strength.

The compressive strength of concrete is based on the statistical parameters. The other properties needed for the design are derived from co-relation equations with the compressive strength, which have been established in laboratories. These properties are not directly verified at site by testing, although the Code does not prohibit such verification.

The available correlations are generally based on tests conducted on cylinders of 150 mm diameter and 300 mm length. In Indian scenario 150 mm cubes are used for acceptance tests. The Code has followed a factor of 0.8 over cube strength for arriving at the cylinder strength. This is an acceptable approximation and will not have significant impact from design considerations. Some inherent small discrepancy may be observed on this account. This is more pronounced, but on conservative side, for concretes of grade M60 and higher.

Where higher levels of accuracy is desired, the Code recommends use of more accurate properties (**Clause 5.5.3**), obtained from one of the two ways. One way is to use established expression of the property derived after incorporating the larger number of factors which have effect on the property than the one recommended by the Code (Refer **Annexure A-2**) for the design in **Section 6**. The other method is to use the experimentally established values, which are arrived at by using proper statistical methods, and sufficient number of test samples to enable estimates to have 95 percent level of reliability.

3.6 Analytical Methods to Evaluate Behavior of Structure — Cl. 5.6

The use of both Global and Local analysis is required by the Code using the appropriate methods. Emphasis is for use of an appropriate model/method for applicable actions, materials and desired level of accuracy. For details refer **Chapter 5 (Section 7)**.

3.7 Design Based on Full Scale Testing — Cl. 5.7

For some elements, design based on the experimentally established use of materials, structural configurations and detailing is accepted by the Code. Salient features of such testing and problems faced in the interpretation of results from such tests are:

- Testing carried out on full scale structural element (prototype) and not on a scale model.

- The methods of analysis required to explain or predict the actually observed behavior are far too complex for use in design office. The failure is taken as reaching the ultimate load capacity, or deformations which are large enough to make the element or structure unsuitable for use.
- No mention has been made about the factor of safety to be used on the load capacity or deflections thus obtained, nor of the number of tests required to establish the design, or the statistical methods to be used, as has been done in case of material properties established experimentally in **Clause 5.5.3**.
- The choice of the acceptance criteria is left to the mutual agreement between the testing agency and user, except in the case of acceptance testing of prestressing anchorages and devices for which the methods of testing as well the acceptance criteria are defined by the applicable national standards/ or fib publication.

3.8 Durability Aspects — Cl. 5.8

The overall approach of the Code for achieving the desired durability is discussed in **Clause 5.1.3**. For further explanatory discussion refer to **Chapter 12 (Section 14)**.

Design service life is specified in the Code for normal structures, temporary structures and special structures (**Table 5.1**). The operational way of designing for durability is to define durability as a design service life requirement. In this way the non-factual and rather subjective concept of “durability” is transformed into a factual requirement of the “number of years” during which the structure shall perform satisfactorily without unforeseen high costs for maintenance. (exact definition or criteria of ‘satisfactory behavior’ are not given, but are to be understood as qualitative descriptions) The designed service life can be achieved by using two principles one, deem-to-satisfy rules and other, the performance-based parameters.

The Code follows deem-to-satisfy rules, which are based on specifying a certain concrete composition, minimum cement content, minimum concrete cover, controlling material properties (especially by restricting harmful ingredients), time dependent properties of concrete appropriate to the design life, appropriate return periods for actions of environmental origin, specifying intended use along with maintenance requirements which is assumed to result in achievement of the specified service life.

The performance based design for durability and service life is usually based on requirements of performance of the structure. Performance based design for service life of structure is quite complex. It not only requires in-depth knowledge and data base of the parameters determining the ageing and deterioration of concrete structures and constituent materials, but also the quality of workmanship needs to be factored in from data base. The performance level of deterioration also needs to be well defined in this case. This design method is not covered in the present Code.

CHAPTER 4

SECTION 6 : MATERIAL PROPERTIES & THEIR DESIGN VALUES

4.1 General Cl. 6.1

The Code, has dealt with materials with two aspects. The first is the "Manufacturing Specifications" and actual properties of manufactured materials and other is the "Design properties" or the "Design Models". Design properties are simplified descriptions of mechanical properties of different materials used in the process of design of bridges.

As far as the manufacturing specifications of materials are concerned, there is no change in new IRC:112 as compared to the earlier codes (IRC:18 & 21). For the specifications of the materials used viz. reinforcement, prestressing steel & cement, reference is made to the relevant Indian Standards which are listed in **Section 18 and Annexure A-2** of the Code. However, substantial modifications are made in the design models of the material which are based on the large amount of the data gathered in past few years. This vast pool of knowledge available today is now incorporated in this Code which will help designers to get more rational designs.

Section 6.0 describes these simplified models considered as adequate for design of bridges. More elaborate models are included in **Annexure A-2** of the Code.

4.2 Untensioned Steel Reinforcement Cl. 6.2

4.2.1 *Specifications and Grades* Cl. 6.2.1

Over past few years, better varieties of reinforcing steels are being used in structures in other parts of the world. Also, higher grades of steel which have more ductility are manufactured in our country and are now covered in the latest version of IS:1786. To get benefit of these developments, these improved verities are included the Code. Use of reinforcing steel of grades up to Fe 600 is introduced. This change will help in reducing amount of steel used in RCC structures. Code has also introduced the galvanized and stainless steel that have improved corrosion resistance. This provides more alternatives to designer to achieve longer service life of the bridges, particularly in aggressive environments.

4.2.2 *Strength, Stress-Strain Diagrams, Modulus of Elasticity and Ductility* Cl. 6.2.2

To have consistent approach with the limit state philosophy, the term characteristic strength, f_{yk} is introduced for steel in the Code. It is the same as the "yield stress" as defined in IS:1786 which is,

f_{yk} = yield strength in case of mild steel or hot rolled/heat treated HYSD bars.

= 0.2 percent proof strength in case of cold worked HYSD bars.

Typical stress-strain diagrams for mild steel and HYSD (both Hot rolled/heat treated and cold worked) are shown in **Fig. 6.1** of the Code.

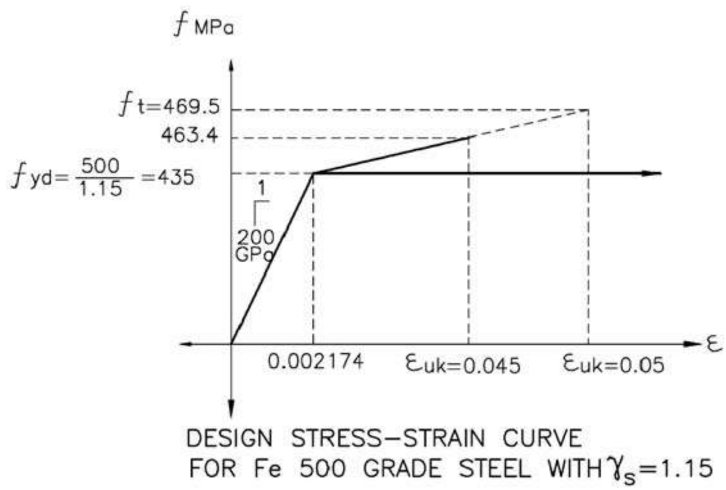


Fig. 6.1

For design purpose, code has suggested two alternate models, bilinear or simplified bilinear stress-strain curves. These, with partial material safety factor γ_s, are shown in **Fig. 6.2** of the Code.

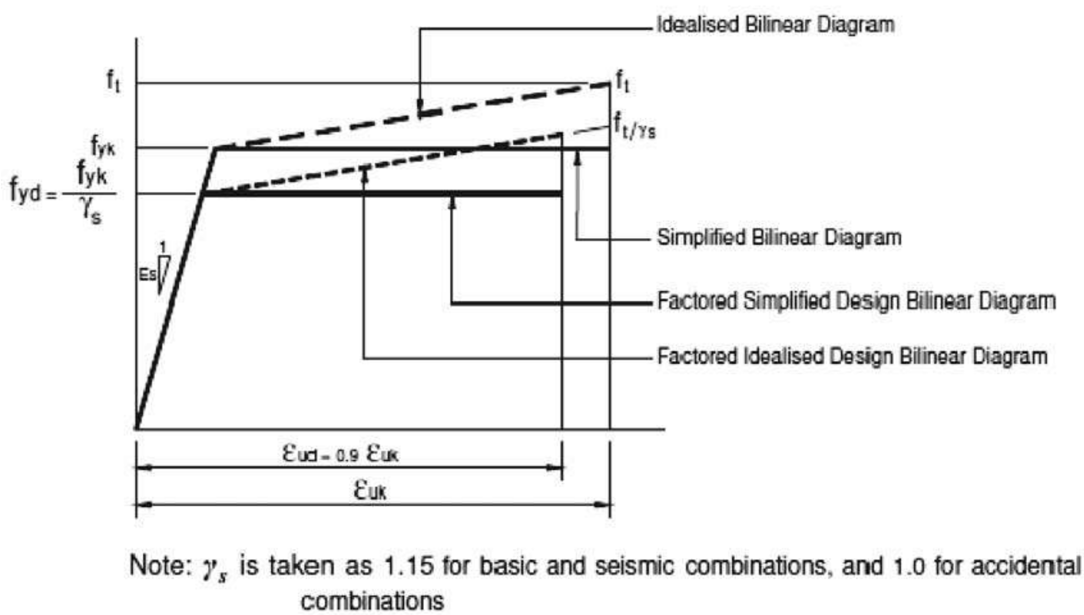


Fig. 6.2 Bilinear Stress-Strain Diagram of Reinforcing Steel for Design

To explain the use of this diagram, a typical curve is generated for grade of Fe 500 with γ_s = 1.15 in **Fig. 6.1**. The elastic modulus for this grade is typically 200 GPa. Here, yield stress f_{yk} = 500 MPa, hence, $f_{yd} = f_{yk}/\gamma_s$ = 500/1.15 = 435 MPa. The value of strain at this point is 435 MPa/200 GPa = 0.002174. For design purpose, the tensile strength, f_t shall be considered as minimum value given in **Table 18.1** of the code (reproduced from IS:1786) which is 108 percent of f_{yk} i.e. 540 MPa. Note that the minimum value of 545 MPa is specified in IS:1786 for grade Fe 500 as a manufacturing requirement. Thus ultimate stress will be f_t/γ_s = 469.5 MPa. In absence of data from manufacturer, the value of characteristic strain, ε_{uk} can be assumed as 5 percent as given in **Table 18.1** and IS:1786 for uniform elongation.

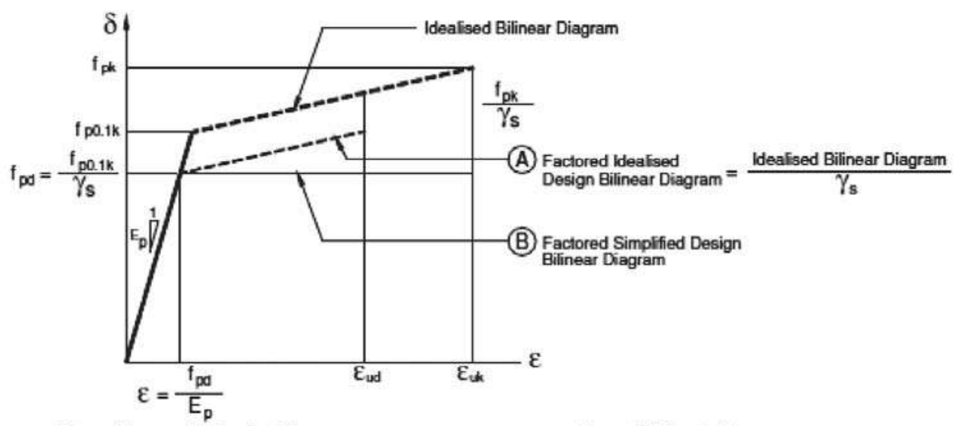
Hence, the strain limit for sloping arm of the curve shall be 0.9 ε_{uk} = 4.5 percent. Then the ultimate stress for design purpose will be 463.4 MPa, arrived at by liner interpolation.

Similar diagrams for other grades of steel can be constructed. It shall be noted that there is no limit on strain if the horizontal branch of the curve is used, as limiting ultimate stress remains as f_{yd}.

4.3 Prestressing Steel **Cl. 6.3**

4.3.1 *Stress-Strain Properties for Design* Cl. 6.3.5

For purpose of analysis & design, code has allowed to use the representative stress-strain curve as shown in **Fig. 6.3** of the Code for wire, strands and bars. For design purpose simplified two stress-strain diagrams, first the bilinear and other simplified bilinear as in **Fig. 6.4** of the code are suggested. These are reproduced below:



E_p = Slope of Elastic Phase = 200GPa/195 GPa irrespective of γ_s values

f_{pk} = Ultimate Stress

ε_{ud} = Strain at Design Ultimate Stress

Note: γ_s is taken as 1.15 for basic and seismic combinations, and 1.0 for accidental combinations

Fig. 6.3 Bi-Linear Stress-Strain Diagram of Prestressing Steel for Design

These diagrams shall be constructed as explained for un-tensioned steel as well. Note that like reinforcing steel, there is no limit on strain for horizontal branch of simplified bilinear design diagram.

In **Fig. 6.4** of the Code, the yield point is defined at 0.1 percent proof stress ($f_{p0.1k}$). This value can be taken as 0.87 times of f_{pk}. The term f_{pk} is the characteristic tensile strength of prestressing steel, which is same as f_p, corresponding to breaking load as given in relevant IS code. As per IS:14268, for strand of dia 15.2 mm, f_{pk} = 260.2 kN (cross section:140 mm^2) = 1862 MPa. Hence, $f_{p0.1k}$ = 0.87 * 1862 = 1620 MPa and f_{pd} = 1620/1.15 = 1409 MPa. Then corresponding strain ε = 1409MPa/195 GPa = 0.007224. In absence of accurate data on characteristic strain of prestressing steel, the value of ε_u can be taken as 0.02. Thus, the stress strain diagram for 15.2 mm, 7 ply, Class II, stress relived strand to be used for design is shown in **Fig. 6.4**.

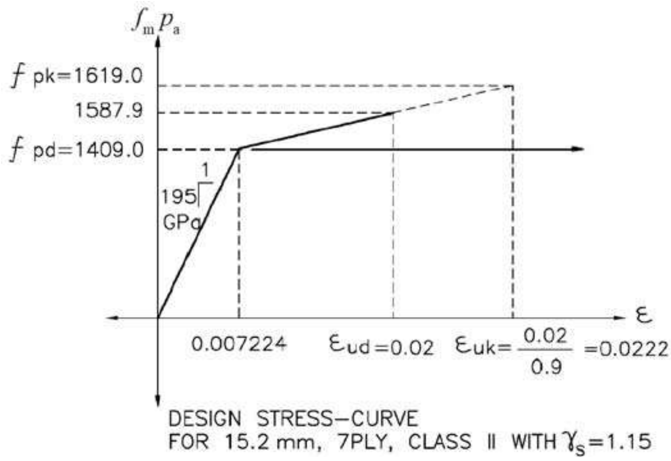


Fig. 6.4

4.3.2 *Relaxation Loss for Design* Cl. 6.3.6

The method of calculations of loss in force due to relaxation of prestressing steel recommended in the Code. This Code has introduced the expression for estimation of the loss due to relaxation at any time after tensioning upto 30 years in **Annexure A-2** as well. It has also introduced the method for estimating the effect of temperature on relaxation of steel. This is particularly useful for steam cured elements. The relationship between relaxation losses and time upto 1000 hours is in **Table 6.3** of the Code and it is markedly different from the corresponding **Table of IRC:18 (i.e. Table 4B)**.

4.4 Concrete Cl. 6.4

In previous codes (i.e. IRC:18 & 21), design properties of concrete were provided upto grade M60. In this code, these properties such as stress strain relationship, models for predication of creep & shrinkage strains, multi axial state of stress and the variation of these properties with respect to time, etc. are given for the grades upto M90. It is now possible to use concrete upto grade M90 for the design of bridges.

4.4.1 *Grade Designation* Cl. 6.4.1

Three groups of concrete are specified in **Table 6.4**, Ordinary concrete, Standard concrete and High performance concrete, based on method of proportioning the concrete and use of mineral admixtures (to improve any specific performance parameters).

Grades which are produced using nominal mix proportions by weight of its main ingredients are termed as Ordinary Concrete. M15 and M20 are covered under this group. Grades M15 to M50 are covered under Standard Concrete, which is produced using chemical admixtures to achieve certain properties in fresh condition and when mineral admixtures are used to achieve certain parameters of the concrete like reduced porosity, the concrete is termed as High Performance Concrete.

As standard practice, the grades of the concrete are defined in term of the characteristic strength (5 percent fractile) at 28 days on cube size of 150 mm. However, the Code has also allowed using the term “characteristic strength” at other than 28 days strength for concrete, which is produced using high content of ingredients that slow down the setting process of concrete in its initial days, but gain the full target strength at say 56, 72 or 84 days. Hence, now it is possible to take advantage of realistic strength of concrete, for design at service stage. However, in such cases, it is necessary to check the design for the loads which may act during initial period of low strength.

The Eurocode has adopted concrete strength described in terms of cylinder (as also in the US and many sources in international literature). The Indian Codes, as well as IRC:112 use cube strength. The two are assumed, for simplicity, to be connected by co-relation of $f_{ck.cyl} = 0.8\ f_{ck.cube}$ and the relation between $f_{cm.cyl} = f_{ck.cyl} + 8$ MPa, translates to $f_{cm.cube} = f_{ck.cube} + 10$ MPa. This has been used in IRC:112. This difference should be kept in mind while referring to the international sources.

4.4.2 *Design Properties of Concrete* Cl. 6.4.2

4.4.2.1 General Cl. 6.4.2.1

Design properties of concrete, such as f_{cm}, f_{ctm}, $f_{ctk,0.05}$, $f_{ctk,0.95}$, E_{cm}, are specified directly in **Table 6.5** of the Code for grades of concrete M15 to M90 and their correlation equations with f_{ck} are included in **Annexure A-2**. The tabulated values are rounded off and may not exactly match with the computed values arrived at by use of the correlations.

Depending on the purpose of analysis, it is necessary to use appropriate probabilistic value of these properties, i.e. either their mean value or 5 percent fractile or 95 percent fractile values. For example, for a section design, concrete strength shall be taken as lower 5 percent fractile i.e. f_{ck}, whereas, the mean value of the modules of elasticity (E_{cm}) shall be used for calculating the deflection of the members. This is because a small local patch of acceptable but having low strength concrete (upto 5 percent fractile of concrete) in the member decides the ultimate strength carrying capacity of the entire member whereas the value of E_c at every section of the member, and not a small weak zone, influences the deflection of the member.

4.4.2.2 Compressive strength and strength development with time Cl. 6.4.2.2

This clause gives the expression for development of compressive strength with time. It can be noted that for normal Portland cement, 34%, 60%, 78% 90% and 120% strength is expected on 1st, 3rd, 7th, 14th day and one year after casting of concrete. This data is useful for taking number of decisions during construction such as, the time for applying initial prestress, striking off formwork etc. It should be noted that the gain in strength after 28 days is not allowed to be used for design of new structures by **Clause 6.4.2.2 (4)**, except in situations as given in Note No. 3 below **Table 6.4**.

Concrete compressive strength also depends on the duration during which it is subjected to a constant stress. A sustained stress in the range of working stress may lead to a slight increase in compressive strength. However, high sustained stresses accelerate the process of micro-cracking and may eventually lead to failure. As mentioned in **Clause 6.4.2.2 (2)**, this effect of reduction of strength due to sustained loading is considered in factor 0.67 in ultimate strength calculations.

The **Clause 6.4.2.2 (3)** provides the principles for acceptance of concrete strength obtained at site with limited number of cubes. Though the title of this clause is `the verification of early age strength by testing`, the principles given here are equally applicable for the results of the cube tests for other than the early strength i.e. say at 28 days.

4.4.2.3 Tensile strength and its development with time Cl. 6.4.2.3

Tensile strength of concrete is one of the important parameters in reinforced and prestressed concrete structure. It is expressed either as direct or axial tensile strength (which is difficult to measure in laboratory), flexural tensile strength or split cylindrical tensile strength. The mean, 5 percent fractile & 95 percent fractile values of direct tensile strength for different grades of concrete are given in **Table 6.5** of the code and the co-relations between the three are given in Equations 6.4 & 6.5. As per the code, the tensile strength of concrete is required to be used for calculation of shear resistance of section (Eq. 10.4), for deciding the minimum reinforcement (Eq. 12.1), for calculations of crack width (Eq. 12.6), to control of shear cracks within webs (Eq. 12.14) and to find out anchorage length of pre-tensioned tendons (Eq. 15.7). It is influenced significantly by the fracture mechanical properties of the concrete which in turn is function of type, size and shape of the aggregates used. Hence for important projects it is necessary to verify the tensile strength of the concrete using split cylinder or flexural beam test.

4.4.2.4 Multi-axial state of stresses Cl. 6.4.2.4

The compressive strength of concrete under multi-axial state of stresses is higher than uniaxial compressive strength and is generally presented in form of failure surface. CEB-FIB Model Code 2010[1] may be referred for further details. The higher strength is not for design of main elements but for local zones only.

This effect is considered in specifying the higher strength & higher critical strains of concrete confined by adequately closed links or cross ties which reach the plastic condition due to lateral extension of the concrete at ULS.

The expressions are available in **Annexure A-2**. In these equations σ_2 can be estimated as explained in following example:

For circular pier of diameter 1.5 m, the closed links if Φ 16 of Fe 500 are provided at spacing of 200 mm c/c with clear cover of 50 mm, σ_2, i.e. the radial pressure exerted by the links at ULS

$$\sigma_2 = 2 \text{ x } A_s*(f_y \gamma_s) \text{ (dia. of link *spacing)}$$
$$= 2 \text{ x } 201 * (500/1.15)/(1500-2 \text{ x } 50-16) *200)$$
$$= 0.6317 \text{ MPa}$$

4.4.2.5 Stress-Strain relationship and modulus of elasticity Cl. 6.4.2.5

For unconfined concrete, the Code has allowed the use of parabolic-rectangular (**Fig. 6.5**) as well as simplified equivalent stress blocks, such as rectangle or bi-linear for the design purposes (**Annexure-A-2**). Here it should be noted that the shape of parabolic rectangular stress-strain diagram is not the same for all grades of the concrete (as given in IS:456) but varies with the value of exponent ‘n’, which is different for different grades of concrete.

The Code suggests different values of Young's modulus E depending on purpose of analysis. For example, for static & quasi-static loads acting for short duration, and dynamic loads such as Earthquake & Wind loads, secant modulus of elasticity, E_{cm} (**Fig. A2-1** of the Code) is recommended. For analyzing impact/shock loading, dynamic modulus of elasticity to be used which can be taken as 1.25 times of E_{cm}. For analysis for seasonal variation of temperature 0.5 E_{cm} shall be used. For temperature gradient analysis, E_{cm} shall be used.

4.4.2.6 Shrinkage

Total shrinkage strain ε_{cs}, consists of two parts; drying shrinkage strain, ε_{cd} and autogenous shrinkage, ε_{ca}. Drying shrinkage is time dependent strain, primarily caused by loss of water when ordinary hardened concrete is exposed to air with relative humidity of less than 100 percent. Incidentally the term 'relative humidity' referred to in the Code means the average annual value of relative humidity of the geographical area under consideration. For a particular humidity and temperature, the total drying shrinkage of concrete is most influenced by the total amount of water present in the concrete at the time of mixing and to a lesser extent, by the cement content. Autogenous shrinkage, also known as basic shrinkage, self-desiccation shrinkage or chemical shrinkage is associated with the ongoing hydration reaction of the cement. It occurs irrespective of the ambient medium due to chemical volume changes and internal drying.

The process of drying shrinkage is slow due to the slow diffusion process of loss of moisture from concrete. Within a short period of time, the surface region of the concrete section reaches the state of moisture equilibrium with the surrounding environment but due to the slow diffusion process of moisture loss, the relative humidity in the pore system of the concrete region away from the surface remains high. Thus the moisture distribution in the concrete section is non-uniform, increases from surface towards the center and leads to the development of internal stresses, tensile in the regions near surface and compressive in the interior regions. This often leads to the surface cracks. In case of autogenous shrinkage, similar stresses do not develop as this deformation develops nearly uniform throughout the section of the member.

Since the shrinkage stresses develop gradually with time they are substantially reduced by the creep. Also these are influenced by mechanical properties, especially the modulus of elasticity of the aggregates. For high performance concrete drying shrinkage is substantially reduced as the capillary porosity is low and restricts the loss of water.

Prediction of shrinkage:

The final values of autogenous and drying shrinkage strains for different grades of the concrete are tabulated in **Tables 6.6 & 6.8** of the code and to predict these strains at any time after the casting of concrete, the multiplying coefficient β_{as} and β_{ds} are as per Equations 6.13 and 6.15. are given.

The values of final autogenous shrinkage, $\varepsilon_{ca}(\infty)$ as in **Table 6.6** are obtained from following equation:

$$\varepsilon_{ca}(\infty) = 2.5\,(0.8\,f_{ck} - 10)\,10^{-6}$$

Similarly, the final values of drying shrinkage ε_{cd} (∞) as given in **Table 6.8** are derived from the basic Equation A2-26 of **Clause A2.6 of Annexure-2** of the Code, which is reproduced below:

The basis drying shrinkage strain $\varepsilon_{cd.0}$ is calculated from:

$$\varepsilon_{cd.0} = 0.85\left[(220 + 110.\alpha_{ds1}).\exp\left(-\alpha_{ds2}.\frac{f_{cm}}{f_{cmo}}\right)\right].10^{-6}.\beta_{RH}$$

$$\beta_{RH} = 1.55\left[1 - \frac{RH}{RH_O}\right]^3$$

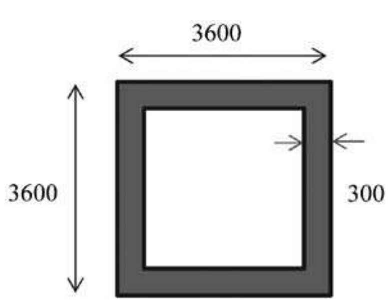
where,

f_{cm}		is the mean compressive strength (MPa)
f_{cmo}	=	125 MPa
α_{ds1}	=	is a coefficient which depends on the type of coment
	=	3 for slow setting cement
	=	4 for Normal cerrent
	=	6 for cement Class R
α_{ds2}		is a coefficient which depends on the type of cement
	=	0.13 for cement Class S
	=	0.12 for cement Class N
	=	0.11 for rapid hardening cement.
RH		is the ambient relative humidity (%)
RHo	=	100%

Note: exp(*) has the same meaning as e* where e is the Napierian base

In **Table 6.7** and Equation 6.15 of the Code, h_o represents the notional size of the member in mm, expressed as $h_o = 2\,A_c/u$, where A_c is the cross-sectional area (mm^2), u is the perimeter of the member in contact with the atmosphere (mm). h_o is approximately the distance travelled by water molecule from the center point of the cross section to the surface of the concrete. The concept will be clear from the following example:

Consider the following box section: Assuming that both inner and outer surfaces are exposed to atmosphere, water loss from both the surfaces, i.e. the maximum distance the water molecule can travel, i.e. from center of the wall to the outer surface of the concrete. The concept is also true for other regularly used concrete sections like Solid Square, solid rectangle, solid circular or hollow circular section. It shall be noted that autogenous shrinkage is not dependent on the RH or member size.



$A_c = 3600^2 - 3000^2$
$= 3960000$ mm^2
$U = 4*(3600 + 3000)$
$= 26400$ mm
$h_o = 2\,A_c/u$
$= 150$ mm

In the super-structure of the bridge various components have different effective thicknesses. This will lead to differential strains and stresses in the components, unless this difference is significant, it is not necessary to calculate the effect separately, relying on the shrinkage steel to control the adverse effects. If significant, special analysis and provision of steel is necessary.

4.4.2.7 Creep Cl. 6.4.2.7

The time dependent strain due to constant stresses is referred as creep. The clause explicitly relates to creep of concrete.

The concrete may be considered as an aging linear visco-elastic material. If the stress in concrete does not exceed 0.36 f_{ck} (t) the creep remains proportional to the stress. Also, it shall be noted that the creep is partially reversible. The ratio of creep strain and elastic strain is called the creep coefficient Φ. Similar to shrinkage, the Code has specified final creep coefficient for design i.e. at end of 70 years in **Table 6.9**. These values are tabulated for two values of RH i.e. at 50 percent and 80 percent for notional sizes of 50 mm, 150 & 600 mm and at different ages at the time of loading. For other notional sizes and RH, the basic equations are available in **Annexure A-2** i.e. Equations A2-14 to A2-25.

An example for calculation of shrinkage and creep strains at various stages of loading for a PSC T Girder is given below:

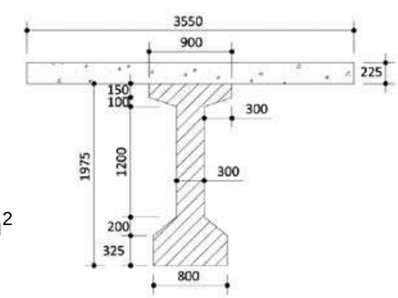
Worked Example:

Calculate shrinkage and creep strain at different stages for a typical PSC T Girder type Super-structure of Span 25.0 m (c/c bearing) composite with RCC Deck Slab.

1 CALCULATION OF CREEP & SHRINKAGE STRAINS

1.1 Material Properties

Concrete Grade for PSC Girder	M 45
Concrete Grade for Deck Slab	M 40
Grade of Steel	Fe 500



TYPICAL SECTION OF PSC GIRDER COMPOSITE WITH RCC SLAB

1.2 Section Properties

- Precast Section of Girder

Overall Depth, D	1.975 m
Area, A	0.9250 m^2
CG of Section from top, y_t	1.050 m
CG of Section from bottom, y_b	0.925 m
Moment of Inertia, I_{xx}	0.4134 m^4
Section Modulus, Z_t	0.3938 m^3
Section Modulus, Z_b	0.4468 m^3

- Composite Section

Overall Depth, D	2.200 m
Area, A	1.701 m^2
CG of Section from top, y_{ts}	0.744 m
CG girder top, y_{tg}	0.519 m
CG of Section from bottom, y_b	1.456 m

Moment of Inertia, I_{xx}	0.9868 m^4
Section Modulus, Z_{ts}	1.3256 m^3
Section Modulus, Z_{tg}	1.8998 m^3
Section Modulus, Z_b	0.6780 m^3
Perimeter of Composite Section, exposed to drying	8.800 m

1.3 Sequence of Casting & Stressing

1^{st} stage stressing	14 days (after casting of girder)
Deck Slab casting	28 days
Casting/Laying of SIDL	90 days

1.4 Variation of Shrinkage Strains with time

1.4.1 Autogeneous Shrinkage

Total Autogenous. Shrinkage Strain,

$\varepsilon_{ca} \times 10^6$ 65 (Refer **Table 6.6**)

Shrinkage strain variation with time: (Refer Eqs. 6.12 & 6.13)

DAYS	β_{as}	Res. Auto Sh. Strain $(1 - \beta_{as})*\varepsilon_{ca}$
14	0.527	$30.76 \times 10^{(-6)}$
28	0.653	$22.56 \times 10^{(-6)}$
90	0.850	$9.75 \times 10^{(-6)}$

Auto. Shrinkage bet. 0-14 days	$34.26 \times 10^{(-6)}$
Auto. Shrinkage bet. 14-28 days	$8.20 \times 10^{(-6)}$
Auto. Shrinkage bet. 28-90 days	$12.81 \times 10^{(-6)}$
Auto. Shrinkage bet. 90- Infinity	$9.75 \times 10^{(-6)}$

1.4.2 Drying Shrinkage

Notional size of cross section, h_o	0.387 m (Refer **Clause 6.4.2.6 (4)**
Coefficient, k_h	0.75 (Refer **Table 6.7**)
Humidity Considered	70%
Unrestrained Drying Sh. Strain, $\varepsilon_{cd} \times 10^6$	315.7 (Refer **Table 6.8**)

Age of Concrete at the end of curing, ts 5 days (Refer Eq 6.15)

DAYS	$\beta_{ds}(t,ts)$	$\varepsilon_{cd}(t) \times 10^6$	Res. Drying Sh Strain $(\varepsilon_{cd}(\infty)-\varepsilon_{cd}(t)) \times 10^6$
14	0.02874	6.8052	229.94
28	0.07031	16.6467	220.10
90	0.21845	51.7177	185.03
∞	1	236.7500	0.00

Drying Sh. between 0-14days	$6.81 \times 10^{(-6)}$
Drying Sh. between 14-28 days	$9.841 \times 10^{(-6)}$
Drying Sh. between 28-90 days	$35.071 \times 10^{(-6)}$
Drying Sh. between 90-∞ days	$185.032 \times 10^{(-6)}$

TOTAL SHRINKAGE STRAIN	$\varepsilon_{ca}(t)$	$\varepsilon_{cd}(t)$	Total Shrinkage Strain, ε_{cs}
Total Shrinkage between 0-14days	8.20	6.81	$18.04 \times 10^{(-6)}$
Total Shrinkage between 14-28 days	8.20	9.84	$18.04 \times 10^{(-6)}$
Total Shrinkage between 28-90 days	12.81	35.07	$47.88 \times 10^{(-6)}$
Total Shrinkage between 90-∞ days	9.75	185.03	$194.78 \times 10^{(-6)}$

1.5 Variation of Creep strains with time

Creep Coeff. Calculated as per Annexure A2:

σ_1 = 0.852

σ_2 = 0.955

σ_3 = 0.892

$\beta(f_{cm})$ = 2.532

φ_{RH} = 1.290

$\beta(t_o)$ for 14 days loading = 0.557

β_H = 828.08

$\varphi_o = \varphi_{RH} \times \beta(f_{cm}) \times \beta(t_o)$ = 1.820

DAYS	$\beta_c(t,t_o)$	$\varphi(t,t_o)$
28	0.293	0.533
90	0.476	0.866
∞	**1.000**	1.820

Modulus of Elasticity, E_c	34000 MPa
Elastic Strain per 10 Mpa stress	2.94E-04
Total Creep Strain	5.35E-04 per 10 MPa
Creep Strain between 0 to 14 days	1.57E-04
Creep Strain between 14 to 28 days	1.57E-04
Creep Strain between 28 to 90 days	9.81E-05
Creep Strain between 90 days to ∞	2.81E-04 per 10 MPa

References:

1. CEB-FIP Model Code 2010.

2. fib bulletin 51-Structural Concrete: Textbook on Behavior, Design and Performance, Volume 1, Second Edition 2009.

CHAPTER 5

SECTION 7 : ANALYSIS

5.1 General Provisions

5.1.1 *Response of Structure to Loads* Cl. 7.1.1

This section covers various simple as well as advanced methods of analysis for RCC and Prestressed members. Behaviour of special load transferring devices (like STU's, Bearings and Seismic Arresters) and their influence on global analysis is also covered in this section. These provisions are based on the current national and international practices.

Structural analyses are performed at three levels; Global analysis of overall structure, member analysis of parts of the structure and local analysis of a part of the member or of structure. These are carried out together in one analysis or separately for individual members Separate local analyses are often necessary in situations where global analyses performed do not cover all relevant structural effects.

Structural analysis is a fundamental tool and basic to make design decisions. The Engineer chooses the idealisation (or model) to represent the structure. The basic requirement in modelling is that the model should correctly represent the expected behaviour of the structure and its constituent elements. Since concrete is a heterogeneous material, its properties are dependent on member size, amount of reinforcement or prestressing of the constituent members as well as the age of concrete. Simulations of the material heterogeneity in a computationally efficient and simple manner are the challenges, to be met by the Structural Engineer.

Generally, analysis outputs will provide the designer with the internal forces or stress resultants, namely bending moments, shear forces and axial forces and may provide displacements. Methods which provide internal stresses and strains, such as finite element methods, are not precluded, although care is needed in the interpretation of results in such cases.

This Section assumes that the designer has adequate knowledge of the analytical methods. In order to have better understanding of the clauses, it is useful to have general background of the classical as well as the modern methods of analysis. These methods have developed rapidly in last 50 years for calculating linear and non linear static and dynamic effects. More explanations of these analysis, their historical development, background and applicability are included in the **Chapter 19** of this Explanatory Handbook.

5.1.2 *Methods of Analysis*

These clauses give a general statement of the types of structural analyses that may be used for bridges. The use of various types of analyses in appropriate situations is also indicated. For details of these analysis refer detailed discussion given in **Section 19** of this Explanatory Handbook.

5.1.3 *Modelling of Foundations* Cl. 7.1.2, 7.1.3

As of now the design of the foundations is based on the classical methods of soil mechanics. Although many are based on the estimation of Ultimate Capacity of the type of foundation and use of an overall safety factor against failure or excessive settlement, the format of Limit State is not followed. For details of the design recommendations reference is made to IRC:78. Where required, reference is made to the standard text books and literature.

5.1.4 *Redistribution of Moments*

Concrete members are subjected to creep shrinkage and other time dependent effects which lead to redistribution of internal stresses in the members. The code provides for including these effects in the design without carrying a detailed analysis of these effects in this clause. This is based on the experience. For situations not covered under the conditions of redistribution, detailed analysis may be required.

5.1.5 *Non-Linear Analyses and Plastic Analyses*

For detailed guidance of the use of these methods refer **Chapter 19** of these Guidelines.

5.5 Analysis for SLS and ULS **Cl. 7.2, 7.3**

a) A first order analysis may be useful for short to medium span bridges. A second order analysis should always be encouraged for bridges with long spans and/or bridges with tall piers.

b) A linear elastic analysis is sufficient for strength based design. Redistribution of moments can be done as per the code to account for material non-linearity, without complicating the analysis method. When redistribution of moments is done, the design shear forces and design reactions shall be taken as the higher of the values 'with' or 'without' redistribution. Linear Elastic Method of Analysis is applicable for both SLS & ULS checks. For SLS checks, the section properties for various members shall be based on un-cracked sections. Fully cracked section analysis may be used for ULS checks to reduce effects of imposed deformations. Modulus of Elasticity of concrete for SLS checks and deformation calculations shall be appropriate to the nature of load. For sustained loads, effective modulus of elasticity, '$E_{c,eff}$' as per Eq. 12.15 of the code shall be used to account for creep of concrete under sustained loads. For instantaneous loads, secant modulus, 'E_{cm}' shall be used.

c) Plastic analysis may be resorted to for checking of the structure under accidental load combination/seismic load combination. In such cases, a preferred design failure mechanism and its attendant hinge locations shall be determined. Plastic method of analysis is applicable for ULS checks, provided adequate ductility is provided at sections/locations where successive hinge/yield lines form and the method adequately model the global effects in combination with the local effects.

d) Strut & Tie Model may be adopted for design of structural elements like Blister Blocks, Anchorage zones, Corbels, Nibs, Diaphragm of Box Girder, Pile Caps etc. (i.e. structures with areas of non-linear strains, where Hooke's law is not applicable).

e) Non-Linear analysis is usually resorted to (though very rarely) for verifying the sufficiency of the provided strength to accommodate the expected in-elastic deformations under seismic loads. For the non linear analysis, the bridge model shall include non-linear properties of the material being used.

5.3 Combined Global & Local Effects — Cl. 7.5

In addition to global analysis, local analyses may also be necessary, particularly when the assumption of linear strain distribution does not apply. Examples of this include:

- Prestressing anchorage zones & Blister & Deviator Blocks
- Members with significant changes in cross section or geometry
- Articulations or Half-joints
- Deep Beams & Brackets
- Areas of discontinuity like opening in member
- Concentrated load locations

Local effects need to be combined with the global effects, wherever necessary. For detailed discussion of combining Global and Local analysis or performing sub-structure analysis refer **Chapter 19** of the Explanatory Handbook.

5.4 Structures & Structural Frames — Cl. 7.6

5.4.1 *General*

The elements of structure are classified by consideration of their nature and function as beams, columns, slabs, walls, plates, arches, shells .etc.

A beam is a member for which the span is not less than 3 times the overall section depth, primarily without axial forces. Otherwise it should be considered as deep beam.

A slab is a member for which the minimum panel dimension is not less than 5 times the overall slab thickness, primarily loads perpendicular to its plane. For beams and slabs, the effective span shall be taken as per **Clause 7.6.1.1** of the Code.

A column is a member for which the ratio of largest lateral dimension to the lesser lateral dimension does not exceed 4, primarily with axial forces. Column with ratio exceeding 4 shall be considered as a wall. A column shall be considered 'short', if the ratio 'l_e/i' in each plane of buckling is such that the failure takes place without involving secondary effects.

5.4.2 Clause 7.6.1.2 of the code gives simplified expressions for the effective width of deck slab and bottom slab which can be taken in the analysis of the overall section at the mid span as well as at the support span for simply supported beams, although in reality, the participating width is varying and is not independent of the load and load position. For the continuous spans the situation is more complex. An FEM elastic analysis of the full span subjected to a defined position and magnitude will show the participating width. However, in practice this concept of effective width simplifies the calculation tremendously, and is found to yield satisfactory Designs.

5.5 Composite Concrete Construction Cl. 7.7

5.5.1 *General*

Pre-cast concrete girders composite with cast-in-situ deck slab is quite commonly used for bridges. In such composite members, where only RC and PSC is used as materials, the concrete is placed in two or more separate stages, generally leading to different time dependent properties of the concretes cast at different time. The analysis needs to account for these properties during various stages of construction and service Transformed sectional properties need to be considered for the analyses. Also, relative (i.e. differential) shrinkage of the deck slab with respect to the precast girder, duly reduced on account of creep of concrete, needs to be taken into account in case of composite decks as per **Clause 7.7.2.1** of the code.

At the serviceability stage the stresses in the concrete and steel are in the elastic or low level of plasticity. The differential shrinkage and creep between deck slab and girder are locked-in and needs to be taken into account.

For ULS condition checks both the concretes of girder and deck slab reach very high levels of strains well in to the plastic range, where the initial differential strains do not change the stress levels due to relatively small differences at the elastic stage This is the logic behind sub **Clause 7.1.1(2)**.

5.6 Structural Effects of Time Dependent Properties of Concrete Cl. 7.8

The effects of creep & shrinkage in reinforced & prestressed concrete members can cause significant changes in not only the deformations but also the internal stresses. Creep substantially modifies the initial stress and strain patterns, increasing the load induced deformations, relaxing the stresses due to artificially introduced and naturally imposed strains and progressively activating the restraints to deformations.

Consequently, there is a need to consider the stress redistribution and its effect in design in order to make a proper assessment of the load carrying capacity of the structural member in service.

There is also a need to take special care for design of slender or thin sections, where second order deformations may have significant influence from considerations of buckling instability.

5.7 Prestressed Members & Structures Cl. 7.9

This clause covers specific rules for prestressed concrete members & structures, dealing specifically with pre-tensioned as well as post tensioned structures. The rules for design and detailing aspects such as prestressing forces, prestress losses, the treatment of prestress in section design and global analysis etc. are included.

The Code has introduced the concept of prestress as an action. For ultimate limit state checks, effects of prestressing are considered as resistance, duly taking into account the pre-strain.

The maximum force applied to a tendon is specified in **Clause 7.9.2**. The limit is set at 90 percent of 0.1 percent proof stress. Since proof stress is $0.87f_p$ as per **Fig. 6.3** of the code, maximum force works out to 78.3 percent of the breaking strength. This limit can be exceeded only in exceptional situations, up to 95 percent of 0.1 percent proof stress which works out to 82.6 percent of breaking strength, only if greater accuracy of measurement (i.e. within ±5 percent) is assured at site. The maximum prestressing force in tendon after instantaneous losses is limited to 75 percent of breaking load or 85 percent of 0.1 percent proof stress which works out to 72.3 percent of breaking strength.

Short and long term losses in prestressing steel for pre-tensioned elements as well as post tensioned elements are defined in **Clause 7.9.3** of the Code, which are as under :

Short Term Losses:

- Elastic Shortening
- Anchorage Draw-in including deformation of the anchorage itself, if any
- Friction

Long Term Losses:

- Concrete Shrinkage
- Concrete Creep
- Steel Relaxation

These losses are explained in the Code in detail. The values for seating at the anchorage on account of wedge draw-in & deformation of the Anchorages itself are to be defined by the prestressing system supplier.

Regarding 'Losses Due to Friction and wobble' given in 7.9.3.2 (2) of the Code, it is worth remembering that 'μ' , the coefficient of friction between the tendon and its duct is a basic property between two materials and the 'k' is a coefficient for wobble effect (representing angular displacement per unit length of duct multiplied by the length is the function of 'μ' and the average effective deviation per unit length of the tendon/duct assembly. The later is the function of tendon fixing accuracy, diameter of the duct compared to the tendon assembly, its weight, support distances, stiffening by curvature, quality controls & workmanship etc. It varies from site to site, The values given in the table are based on experience and are to be used in the design. In actual use these values should be ascertained at site by using two jacks, one as active and other as passive and thus measuring the actual effect of friction and wobble together.

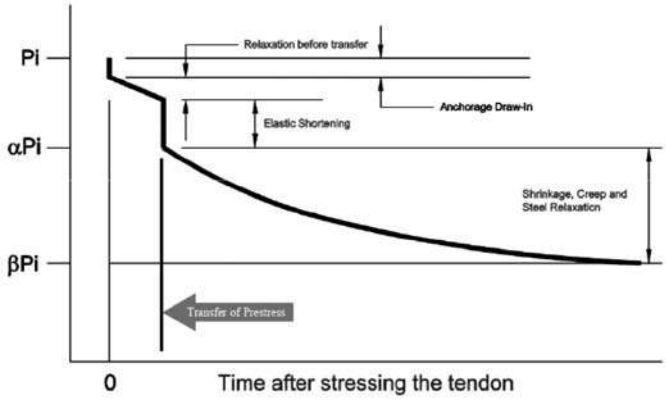


Fig. 7.1 Prestressing Losses for Pre-tensioning

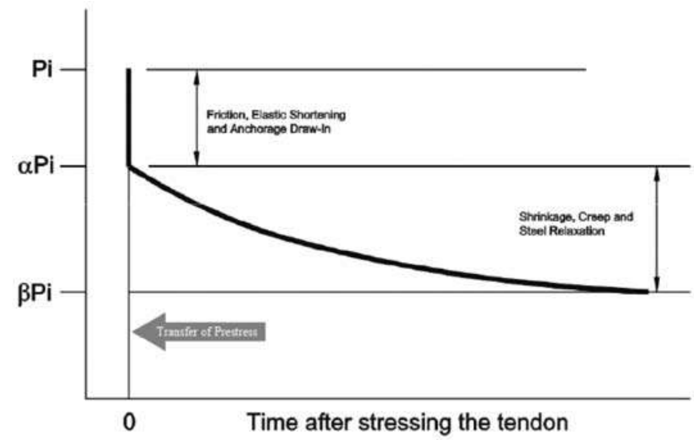


Fig. 7.2 Prestressing Losses for Post-Tensioning

Fig. 7.1 gives the diagrammatic representation of the losses in pre-tensioning while **Fig. 7.2** gives the similar diagram for post tensioning.

5.8 Design and Detailing for Curved Tendons in thin Sections Cl. 7.10

Prestressing tendons embedded in curved thin sections (slabs or shells) of single or double curvature exerts inward, in plane and/or out of plane pressures. This pressure has tendency to punch the tendon out of the section under effect of local punching. For derivation of the internal tensile stresses set-up in the section due to curvature, refer **Chapter 19**. For curved tendons in the webs which curve in plan, the action is further complicated, Apart from the local punching out effects (punching shear) overall inward thrust is created on the webs, This is resisted by the web acting like a curved slab resisting out of plane loads and carrying them to the upper and lower slabs acting as supports.

The Code includes the method of checking for the punching effect, bending effects and corresponding shears to achieve safe design. This is based on the recommendations published by Prof. Breen in Proceedings of FIB Congress in Washington in 1994. **Fig. 7.2** of the Code is self-explanatory in this respect.

5.9 Special Load Transferring Devices Cl. 7.11

Vertical and horizontal load transfer from Superstructure to the foundation takes place with load transferring devices like bearings, dislodgement preventing stoppers and shock transmission units. Type and disposition of these loads transferring devices controls the magnitude and the manner in which the load is transferred to foundation. The deign needs to account for the reactions developed from these devices as well as restraints imposed, if any.

For bridges built in grade or cross fall, the bearings are normally set in horizontal position. This ensures that only vertical reactions are transferred to the bearing resulting from vertical loads. In case the bearings are placed parallel to the grade, the bearings are subjected to in-plane forces also in addition to out-of-plane forces, resulting from vertical loads. Permanent in-plane forces can lead to increased time dependent displacements of bearings due to creep, which is not desirable. Type of bearings and bearing arrangement provided in a bridge therefore has significant influence on the design forces for the substructure & foundation. Seismic forces transferred to the substructure are also greatly influenced by the bearing type and arrangement.

CHAPTER 6

SECTION 8 : ULTIMATE LIMIT STATE OF LINEAR ELEMENTS FOR BENDING AND AXIAL FORCES

6.1 This section covers Ultimate Limit State (ULS) design of linear elements which are subjected to the bending with or without axial force. The rules for ULS design for shear, torsion, punching, and membrane elements are in **Sections 9 & 10**. The methods given here are applicable for the members like piers, slabs, I girders, longitudinal design of box girders, etc. The additional checks for buckling of slender columns are in **Section 11**. Cl. 8.1

6.2 Stress-Strain Distribution at ULS Cl. 8.2

Unlike working stress method where the design checks are based on stress limit, in ultimate limit state the design checks are based on strain limits. The strain limits, viz. ε_{c1}, ε_{c2} & ε_{c3} and ε_{cu1}, ε_{cu2} & ε_{cu3} for various grades of concrete are specified in **Table 6.5** of the Code. The values ε_{c1} & ε_{cu1} are used in generalized stress-strain distribution of concrete which are shown in **Fig. A2-1** of **Annexure A-2**.The values ε_{c2} & ε_{cu2} are used in defining the parabolic -rectangle stress-strain diagram as shown in **Fig. 6.5** whereas the values ε_{c3} & ε_{cu3} are used in defining the bilinear stress-strain relationship as shown in **Fig. A2-3** and rectangular stress distribution shown in **Fig. A2-4** of the **Annexure A2** of the Code. ε_{c1}, ε_{c2} & ε_{c3} are the strain limits when the cross section is subjected to the pure compression (without bending), whereas, ε_{cu1}, ε_{cu2} & ε_{cu3} are limits when the section is subjected to bending and axial forces. The limiting strain ε_{c2c} & ε_{cu2c} for confined concrete are in **Clause A2-8** of **Annexure A-2** of the Code. From **Table 6.5** of the code, it can be seen that for concrete grade upto M60, $\varepsilon_{cu1} = \varepsilon_{cu2} = \varepsilon_{cu3}$. It can also be seen that higher concrete strength show more brittle behavior, reflected by shorter horizontal branch of the concrete stress-strain graph. **Fig. 8.1** below makes it clear.

The possible range of strain distribution is shown in **Fig. 8.1** of the Code, which is presented below for better understanding of the clause.

The possible strain diagrams in accordance with the stress-strain diagrams discussed above are shown in **Fig. 8.2** of the Code. These diagrams are based on Bernoulli's hypothesis. The strain limits of the stress-strain diagram for concrete and steel results in five different zones for the design of cross-section. With the assumption of the perfect bond, the strain diagrams in the **Fig. 8.2** govern not only the concrete compressive stresses but also the stresses in reinforcement (reinforcing steel or prestressing steel including pre-strain $\varepsilon_{p(0)}$ at any place in the cross-section). The compressive strain of reinforcing steel caused by creep and shrinkage of concrete are normally negligible in ultimate limit state and hence not considered.

In zone 1, the whole section is subjected to tensile strain whereas in zone 5, it is subjected to compressive strain. In both the zones, the location of neutral axis is outside the section. The location of neutral axis changes from top fibre to bottom fibre of section in zones 2, 3 & 4. Similarly, the location of point 'C' (which is characterized by the compressive strain in concrete $\varepsilon_c = \varepsilon_{c2}$ also shifts from top to bottom in zones 2 to 4 following the movement of neutral axis.

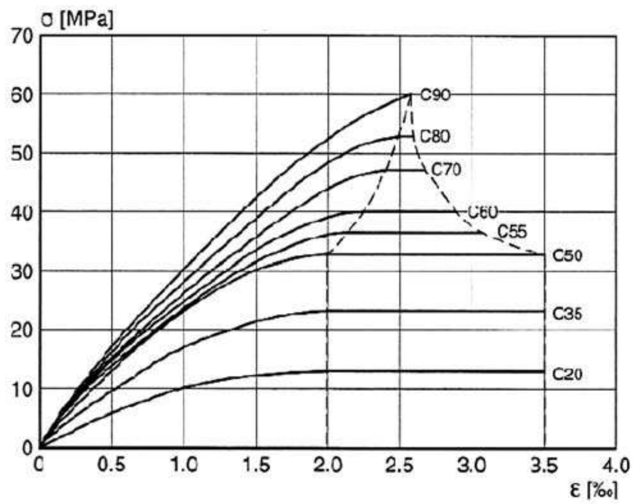


Fig. 8.1 Stress-Strain Relationship for Various Concrete Grades

Source : Presentation in Symposium on Euro Code : Feb 2008 by J C Walraven
[Note: Concrete grades are based on Cylinder Strength]

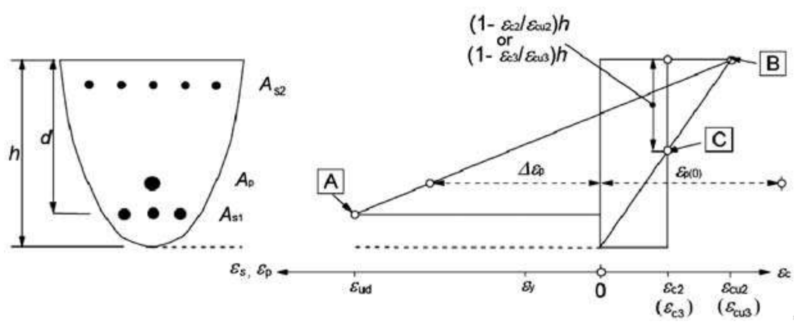


Fig. 8.1 Possible Strain Distributions in the Ultimate Limit State

Procedure for DESIGN CHECKS

(A) Procedure for Computer Program

For a given cross-section, the ultimate axial load carrying capacity can be checked for uniform strain of ε_{c2} if the applied force is pure compressive and for 0.9 ε_s if the applied axial force is pure tensile. The ultimate axial force carrying capacity of the section for given bending moment shall always be between these two limits. The position of neutral axis, x for which the ultimate force carrying capacity of the section matches with the factored axial force acting on the section is obtained by solving the following equilibrium condition.

$$f(x) = Pus\ (x) + Puc\ (x) + N = 0 \qquad \text{Eq.C 8.1}$$

Where N is the factored axial force applied and Pus (x) and Puc (x) are the contributions from steel and concrete respectively in the ultimate resistance of the section and can be express as

$$Pus\ (x) = \Sigma\sigma_s(x)\ A_s$$

and $$Puc\ (x) = \int\sigma_d A_c$$

The notation σ_s cover the steel stresses in both prestressed and non-prestressed steel.

After getting the location of neutral axis at equilibrium, the ultimate bending moment carrying capacity of the section about neutral axis is worked out and compared with the factored bending moment acting on the section.

Finding out the roots of **Eq. C8.1** involve solution of non-linear equation. The equation is solved using numerical method such as Regula-Falsi method. In this method, the initial assumptions of two values of x, one each on either side of solution are required. It is necessary to satisfy condition, i.e. $f(x_i) * f(x_{l+l}) < 0$ at each of iterations.

(B) The Direct Solution of Rectangular Sections

In case of direct solution, magnitude of compressive force (C_u) and its location is obtained from concrete stress block in compression. The ultimate failure of concrete is caused by crushing of concrete which requires the limiting strain (ε_{Cu2}) on extreme compressed fiber of concrete. Following two cases are discussed here:

1) Neutral axis within the section
2) Neutral axis outside the section

1) Neutral axis within the section

a) **Parabolic – Rectangular stress block**

When neutral axis is within the section, strain in extreme compressed fiber is limited to ε_{Cu2} and corresponding stress f_{cd}. For rectangular section of width b and neutral axis depth x the resultant force C_u is expressed by

$$C_u = \beta_1 f_{cd} bx$$

And its position, measured from extreme compressed edge is defined by b_{2x}. The expressions for β_1 and β_2, as function of strain $\varepsilon_{c,}$ are:

$$\beta_1(\varepsilon_{cu2}) = \frac{\int_0^{\varepsilon_{cu2}} \sigma_c d\varepsilon}{f_{cd}\varepsilon_{cu2}}$$

$$\beta_2(\varepsilon_{cu2}) = 1 - \frac{\int_0^{\varepsilon_{cu2}} \sigma_c(\varepsilon) d\varepsilon}{\varepsilon_{cu2} \int_0^{\varepsilon_{cu2}} \sigma_c(\varepsilon) d\varepsilon}$$

The numerical values of β_1 and β_2 are shown as the function of cube strength (f_{ck} in **Table C 8.1**.

Table C 8.1 Values of β1 and β2

f_{ck}(N/mm²)	upto 60	70	75	90	100	115
β_1	0.80952	0.74194	0.69496	0.63719	0.59936	0.58333
β_2	0.41597	0.39191	0.37723	0.36201	0.35482	0.35294

b) **Rectangular stress block**

Instead of parabolic – rectangular stress block, more simplified rectangular stress block can be used for evaluating compressive force and its line of action. With reference of **Appendix A-2.9** of the Code, the expressions for β_1 and β_2 are simplified to,

$$\beta_1 = \lambda\, \eta$$
$$\beta_2 = \lambda/2$$

The values are shown in **Table C 8.2**.

Table C8.2 Values of for Rectangular Stress Block

f_{ck}(N/mm^2)	upto 60	70	75	90	100	115
β1	0.8	0.76781	0.73625	0.675	0.61625	0.56
β2	0.4	0.39375	0.3875	0.375	0.3625	0.35

2) Neutral axis outside the section

a) **Parabolic** – Rectangular stress block

The adoption of the assumptions in **Clause 8.2.1** of the Code leads to the range of possible strain diagrams at ultimate limit states subjected to different forces. Numerous conditions of neutral axis outside the section arise between two cases of strain distribution, one with uniform ε_{c2}, over section for uniform ε_{cu2} compression and other is ε_{cu2} for extreme compressed edge and 0 at neutral axis. For this condition strain diagram is defined by assuming that compressive strain ε_{c2} is at level (1-ε_{c2}/ ε_{cu2}) h, with notation

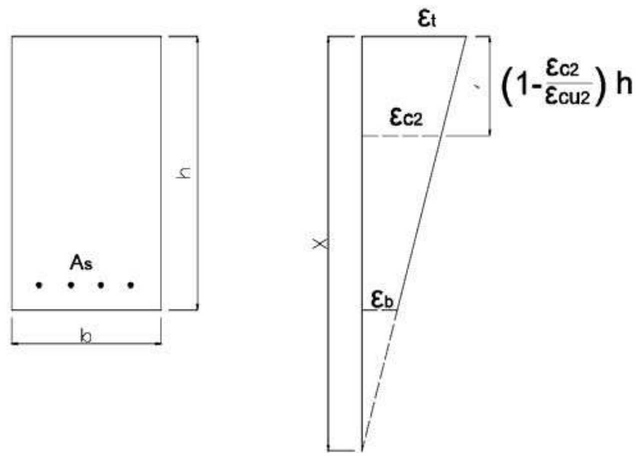


Fig. C8.2 Rectangular Section with Neutral Axis Outside the Section

$$\xi = \frac{x}{h}$$

The strains at top and bottom ε_1 and ε_b respectively, are given by formulae,

$$\varepsilon_t = \frac{\xi\, \varepsilon_{c2}}{\frac{\varepsilon_{c2}}{\varepsilon_{cu2}} + \xi - 1}$$

$$\varepsilon_b = \left(1 - \frac{1}{\xi}\right) \varepsilon_t$$

Indicating b_{1t} and b_{2t} resultant and its position for a length x and by β_{2b} and β_{2b} are the similar quantities for (x-h) part, the resultant β_e and its position β_4 compared to the most compressed edge and relation with depth h are given by:

$$\beta_3 = \zeta\, \beta_{1t} - (\xi - 1)\beta_{1b}$$
$$\beta_4 = \frac{\zeta\, \beta_{1t}\beta_{2t} - (\xi - 1)\beta_{1b}((\xi - 1)\beta_{2b} + 1)}{\beta_3}$$

The β_3 and β_4 values for a different x/h ratio are given in **Table C 8.3**.

b) **Rectangular stress block**

Though in IRC:112 there are no guide lines for obtaining rectangular stress block for (x > h) case, it is possible to write formula that gives the equivalent depth h* in relation to x as, $h^* = \frac{x-\lambda h}{x-kh}\, h$ where k factor is determined by imposing that . h* λh when x = h. It results in

$$k = 2 - \frac{1}{\lambda}$$

Putting value of k in first expression gives,

$$h^* = \frac{x - \lambda h}{x - \left(2 - \frac{1}{\lambda}\right) h}\, h$$

Values of λ, η, k, are given as a function of f_{ck} in **Table C8.4**.

The values of β_3 and β_4 were calculated in analogy to that was developed for case 2a) and presented in **Table C8.5**.

Table C8.4 Values of λ, η and k,

f_{ck}(N/mm^2)	λ	η	k
≤ 60	0.80000	1.00000	0.75000
70.0	0.78750	0.97500	0.73016
75.0	0.77500	0.95000	0.70968
90	0.75000	0.90000	0.66667
100.0	0.72500	0.85000	0.62069
115.0	0.70000	0.80000	0.57143

Table C 8.3. Values of β3 and β4

Parabolic Rectangle Constitutive Law												
x/h	f_{ck} = 60 N/mm²		f_{ck} = 70 N/mm²		f_{ck} = 75 N/mm²		f_{ck} = 90 N/mm²		f_{ck} = 100 N/mm²		f_{ck} =115 N/mm²	
	β_3	β_4	β_3	β_4	β_3	β_4	β_3	β_4	β_3	β_4	β_3	β_4
1.0	0.80952	0.41597	0.74194	0.39191	0.69496	0.37723	0.63719	0.36201	0.59936	0.35482	0.58333	0.35294
1.2	0.89549	0.45832	0.83288	0.43765	0.78714	0.42436	0.72968	0.41022	0.69249	0.40355	0.6772	0.40186
1.4	0.93409	0.4748	0.88197	0.45841	0.84129	0.44724	0.78831	0.43492	0.75381	0.42907	0.73986	0.42761
1.6	0.95468	0.48304	0.91168	0.4699	0.87615	0.46046	0.82826	0.44975	0.79679	0.44461	0.78422	0.44335
1.8	0.96693	0.48779	0.93113	0.47702	0.90007	0.46895	0.85695	0.45954	0.82834	0.45499	0.81702	0.45389
2.0	0.97481	0.49077	0.9446	0.48178	0.9173	0.47478	0.87838	0.46644	0.85234	0.46237	0.84211	0.4614
2.5	0.9855	0.49475	0.96464	0.48861	0.9442	0.48347	0.91348	0.47705	0.89255	0.47385	0.88448	0.47311
5.0	0.99702	0.49893	0.9906	0.49705	0.98285	0.49512	0.96937	0.49234	0.95972	0.49089	0.95622	0.49057

Table C 8.5 Values of β3 and β4 for Rectangular Stress Block

Rectangle Constitutive Law												
x/h	f_{ck} = 60 N/mm²		f_{ck} = 70 N/mm²		f_{ck} = 75 N/mm²		f_{ck} = 90 N/mm²		f_{ck} = 100 N/mm²		f_{ck} = 115 N/mm²	
	β_3	β_4	β_3	β_4	β_3	β_4	β_3	β_4	β_3	β_4	β_3	β_4
1.0	0.80000	0.40000	0.76781	0.39375	0.73625	0.38750	0.67500	0.37500	0.61625	0.36250	0.56000	0.35000
1.2	0.88889	0.44444	0.85601	0.43898	0.82344	0.43339	0.75938	0.42188	0.69695	0.40997	0.63636	0.39773
1.4	0.92308	0.46154	0.89154	0.45720	0.86011	0.45269	0.79773	0.44318	0.73623	0.43308	0.67586	0.42241
1.6	0.94118	0.47059	0.91073	0.46704	0.88030	0.46332	0.81964	0.45536	0.75946	0.44674	0.70000	0.43750
1.8	0.95238	0.47619	0.92274	0.47320	0.89308	0.47004	0.83382	0.46324	0.77482	0.45577	0.71628	0.44767
2.0	0.96000	0.48000	0.93097	0.47742	0.90191	0.47469	0.84375	0.46875	0.78572	0.46219	0.72800	0.45500
2.5	0.97143	0.48571	0.94341	0.48380	0.91534	0.48176	0.85909	0.47727	0.80282	0.47255	0.74667	0.46667
5.0	0.98824	0.49412	0.96191	0.49329	0.93554	0.49239	0.88269	0.49038	0.82975	0.48809	0.74677	0.48548

Calculation of Strength of Rectangular Section

Determination of N_{rd} and M_{rd}, i.e. resisting capacity of the section.

To illustrate the principles, a sample case is considered with neutral axis within the section for rectangular section.

Consider a rectangular section with breath b, total depth h. Strain at extreme compressed fiber of concrete is. Strain distribution at ultimate limit state across the section and corresponding stresses are illustrated in **Fig. C 8.3**. A's is steel in compression and whereas A_s is steel in tension.

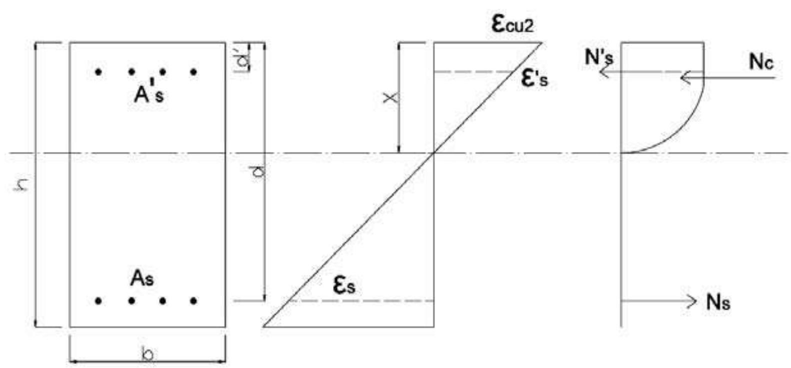


Fig. C 8.3 Rectangular Section at Ultimate Limit State

Two requirements are to be satisfied throughout the flexural analysis and design of reinforced concrete beams and columns:

1. **Stress and strain compatibility-** The stress at any point in a member must correspond to the strain at that point. A strain over the depth of the member is assumed to be linear.
2. **Equilibrium-** Internal forces must balance the external load effects, as illustrated in Eq. C8.1 and rewritten as:

For equilibrium,

$$N_{rd} = N_c + N'_s + N_s$$

where,

N_c is the resultant of compressive forces in concrete, N'_s is resultant force in compression reinforcement A_s' and N_s is resultant force in tension reinforcement A_s.

Each term can be expressed as,

$$N_c = \beta_1 f_{cd} bx$$
$$N'_s = f'_s A'_s$$
$$N_s = f_s A_s$$

Stress in reinforcement can be expressed as,

$$f'_s = \varepsilon'_s E_s \quad if\ \varepsilon'_s < \frac{f_{yd}}{E_s}$$

where,

$$\varepsilon'_s = \varepsilon_{cu2}\left(1 - \frac{d'}{x}\right)$$

$$f'_s = f_{yd} \quad if\ \varepsilon'_s \geq \frac{f_{yd}}{E_s}$$

Similarly, $f_s = \varepsilon_s E_s \quad if\ \varepsilon_s < \frac{f_{yd}}{E_s}$

where,

$$\varepsilon_s = \varepsilon_{cu2}\left(\frac{d}{x} - 1\right)$$

$$f_s = f_{yd} \quad if\ \varepsilon_s \geq \frac{f_{yd}}{E_s} \qquad (use - ve\ for\ steel\ in\ tension)$$

The nominal moment capacity, M_{rd}, for the assumed strain distribution is found by summing the moments of all the internal forces about the centroid of the column. The moments are summed about the centroid of the section, because this is the axis about which moments are computed in a conventional structural analysis.

$$M_{rd} = N_c\left(\frac{h}{2} - \beta_2 x\right) + f'_s A'_s\left(\frac{h}{2} - d'\right) + f_s A_s\left(\frac{h}{2} - d\right)$$

In case both the reinforcing bar yielded $(\sigma_s = \sigma'_s = f_{yd})$,

$$N_{rd} = \beta_1 f_{cd} bx + f_{yd} A'_s + f_{yd} A_s$$

$$M_{rd} = N_c\left(\frac{h}{2} - \beta_2 x\right) + f_{yd} A'_s\left(\frac{h}{2} - d'\right) + f_{yd} A_s\left(\frac{h}{2} - d\right)$$

β_1 and β_2 are factors given in **Tables C 8.1 and C 8.2**.

Design of Reinforcing Bars in Case of Bending Without Axial Force

Parabolic-rectangular stress block:

Consider a section with breadth b, depth h and effective depth d with a design bending moment M_{Ed}

In order to determine if section is sufficient using tensioned steel ($_{As}$) alone, the limiting bending moment $M_{ur,lim}$ is calculated and compared with design moment on section.

$$M_{ur,lim} = F_c z_{lim}$$

Where $F_c = \beta_1 f_{cd} b x_{lim}$ is resultant of compression stresses and $z_{lim} = (d - \beta_2 x_{lim})$

In order to ensure that the structure has ductile behavior, the strain in steel of tensioned reinforcement must be greater than that of strain corresponding to the limit of elasticity, which is $e_s \geq e_{yd} = f_{yd}/f_s$. This implies that neutral axis does not exceed depth x_{lim}, with e_{cu} as limiting strain in concrete,

$$x_{lim} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} d$$

a) If M_{Ed} is smaller than $M_{ur,lim}$, A_s is alone sufficient.

To find the actual neutral axis depth corresponding to M_{Ed},

$$\beta_1 f_{cd} bx(d - \beta_2 x) = M_{Ed}$$

This, by solving becomes,

$$x^2 - x\frac{d}{\beta_2} + \frac{M_{Ed}}{\beta_1\, \beta_2\, b\, f_{cd}} = 0$$

$$\therefore x = \frac{d}{2\beta_2} - \sqrt{\left(\frac{d}{2\beta_2}\right)^2 - \frac{M_{Ed}}{\beta_1\, \beta_2\, b\, f_{cd}}}$$

Finally, with z = (d – β_2 x),

$$A'_s = \frac{M_{Ed}}{f_{yd}\, z}$$

b) If If M_{Ed} is greater than $M_{ur,lim}$, A_s is alone is not sufficient, some compressive reinforcement A'_s in compression is needed. To calculate it,

$$A'_s = \frac{M_{Ed} - M_{ur,lim}}{f_{yd}(d - d')}$$

The tensioned reinforcement is

$$A_s = \frac{1}{f_{yd}}\left(\frac{M_{ur,lim}}{z_{lim}}\right) + A'_s$$

Rectangular Concrete Stress Block:

y = λx is the depth of compression zone and f_{cd} is designed compressive stress. Values of these factors are given in **Table C 8.4**.

Compressive force, $F_c = \eta\, f_{cd}\, y\, b$,

Liver arm $z = \left(d - \frac{y}{2}\right)$

$$M_{Ed} = F_c\, z = \eta\, f_{cd}\, y\, b\left(d - \frac{y}{2}\right)$$

Rearranging:

$$\left(\frac{y}{d}\right)^2 - 2\left(\frac{y}{d}\right) + \frac{2M_{Ed}}{bd^2(\eta\, f_{cd})} = 0$$

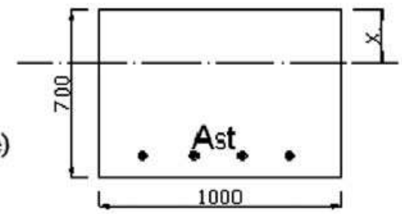
$$\frac{y}{d} = 1 - \sqrt{1 - \frac{2M_{Ed}}{bd^2(\eta\, f_{cd})}}$$

Once y is known area of steel can be calculated.

Note: To ensure the ductile failure, i.e. to make section under-reinforced, IS:456 has specified minimum strain in reinforcement as f_y/ (1.15 * Es) + 0.002 at failure. The Code has specified this limit as ε_{yd} which is equal to $f_{yk}/(\gamma_s * E_s)$; it results in higher depth of neutral axis for balanced section as compared to one calculated by IS:456.

Worked Example C8.1 : Reinforced Concrete Deck slab

Consider reinforced concrete deck slab of thickness 700 mm subjected to ultimate bending moment of 1200 kNm/m with grade of concrete M35 and Fe 500 steel, calculate amount of tension reinforcement required.



Assume clear cover = 50 mm, bar dia = 25 mm

d = 700-50-25/2 = 637.5 mm

For M35 concrete,

$\beta_1 = 0.80952,\ \beta_2 = 0.41597$

$f_{cd} = \frac{\alpha\, f_{ck}}{\gamma_c} = \frac{0.67x35}{1.5} = 15.633$ (Refer Fig. 6.5 of the Code)

To find value of neutral axis x,

$$x = \frac{d}{2\beta_2} - \sqrt{\left(\frac{d}{2\beta_2}\right)^2 - \frac{M_{Ed}}{\beta_1\,\beta_2\, b\, f_{cd}}}$$

$$= \frac{637.5}{2x0.41597} - \sqrt{\left(\frac{637.5}{2x0.41597}\right)^2 - \frac{1200x10^6}{0.809x0.415x1000x15.633}}$$

$$= 167.02\ mm$$

To check adequacy of section,

$$x_{lim} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} d \quad = \frac{0.0035}{0.0035 + 0.00217} d = 0.6168\, d$$

Where $\varepsilon_{yd} = \frac{500}{1.15x2x10^5} = 0.00217$

$$\therefore x_{lim} = 0.6168x637.5 = 393.21 > 167.02\ mm\ \ Hence\ OK$$

Liver arm,

z = 637.5 - 0.416 x 167.02

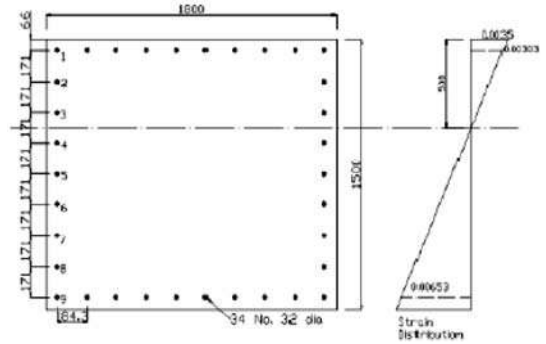
– 568.02 mm

$$\therefore A_s = \frac{M_{Ed}}{0.87\, f_y\, z} = \frac{1200x10^6}{0.87x500x568.02} = 4859\ mm^2$$

Provide 25 dia bar @ 100 c/c

Worked Example C8.2 : Reinforced concrete pier

Consider RCC pier of size 1800 x 1500 mm reinforced with 34 number of 32 dia bar as shown in Fig. Pier is loaded with 17000 kN axial load and bending moment of 14000 kNm about minor axis. Assuming grade of concrete as M60 and grade of steel as Fe 500 check the adequacy of the section.



As explained in C 8.2 first determine the location of NA so that the axial force resistance is equal to the factored applied force By trial and error, x_u obtained is 500 mm.

Yield strain for steel $\varepsilon_{yd} = \frac{f_{yd}}{E_s} = \frac{0.87x500}{2x10^5} = 0.002174$

For this location of x_u the strain and corresponding stresses at different level of steel are:

By d' = 66 mm and d = 1434 mm (to give clear cover of 50 mm).

$$\varepsilon_{s1}', = 0.0035\left(1 - \frac{66}{500}\right) = 0.00303 > \varepsilon_{yd}$$

$$f_{s1}' = \frac{500}{1.15} = 434.78\ MPa$$

$$\varepsilon_{s2}' = 0.0035\left(1 - \frac{237}{500}\right) = 0.00184 < \varepsilon_{yd}$$

$$f_{s2}' = 0.00184\ x\ 2\ x\ 10^5 = 368.2\ MPa$$

$$\varepsilon_{s3}' = 0.0035\left(1 - \frac{408}{500}\right) = 0.000644 < \varepsilon_{yd}$$

$$f_{s3}' = 0.000644\ x2\ x\ 10^5 = 128.8\ \text{MPa}$$

$$\varepsilon_{s6} = 0.0035\left(\frac{921}{500} - 1\right) = 0.00295 > \varepsilon_{yd}$$

$$f_{s6} = \frac{500}{1.15} = 434.78\ MPa\ \text{(tension)}$$

$$\varepsilon_{s7} = 0.0035\left(\frac{1092}{500} - 1\right) = 0.00414 > \varepsilon_{yd}$$

$$f_{s7} = \frac{500}{1.15} = 434.78\ MPa\ \text{(tension)}$$

$$\varepsilon_{s8} = 0.0035\left(\frac{1263}{500} - 1\right) = 0.0053 > \varepsilon_{yd}$$

$$f_{s8} = \frac{500}{1.15} = 434.78\ MPa\ \text{(tension)}$$

$$\varepsilon_{s9} = 0.0035\left(\frac{1434}{500} - 1\right) = 0.00653 > \varepsilon_{yd}$$

$$f_{s9} = \frac{500}{1.15} = 434.78\ MPa\ \text{(tension)}$$

Axial load carrying capacity given by,

$$N_{rd} = \beta_1 f_{cd} b x_u + \sum f_s' A_s' + \sum f_s A_s$$

$$where\ f_{cd} = \frac{\alpha f_{ck}}{\gamma_c} = \frac{0.67x60}{1.5} = 26.8$$

$$N_{rd} = 0.80952x26.8x1800x500 + (128.8x804.2x2 + 368.2x804.2x2 + 434.78x804.2x10) + 804.2\,x\,(-110.6x2 - 350x2 - 434.78x2 - 434.78x2 - 434.78x2 - 434.78x10)$$

$$= 17398.1\,KN \quad \approx 17000kN \qquad \therefore safe$$

Taking moment of all forces about C.G. of section,

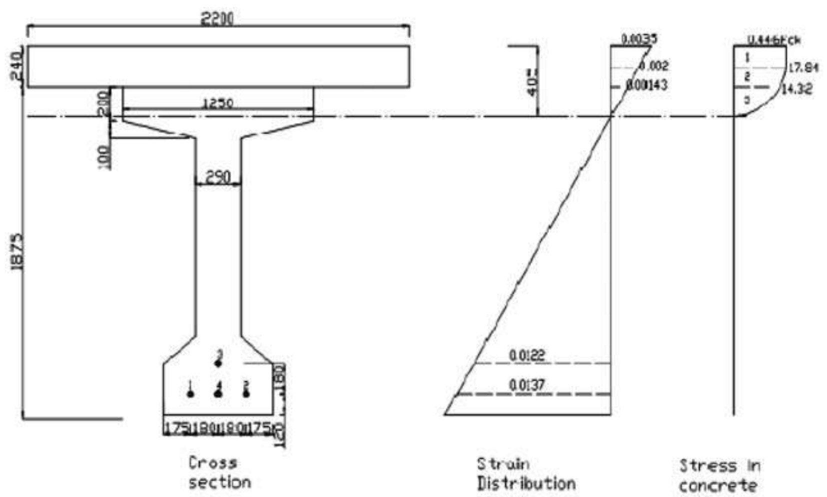
$$M_{rd} = \beta_1 f_{cd} b x_u \left(\frac{h}{2} - 0.42x_u\right) + \sum f_s' A_s' \left(\frac{h}{2} - d'\right) + \sum f_s A_s \left(\frac{h}{2} - d\right)$$

$$M_{rd} = 0.80952x26.8x1800x500x\left(\frac{1500}{2} - 0.42x500\right) + 804.2x(128.8x2x(750 - 408) + 368.2x2x(750 - 237) + 434.2x10x(750 - 66)) + 804.2x(-110.6x2x(750 - 579) - 350x2x(750 - 750) - 434.78x2x(750 - 921) - 434.78x2x(750 - 1092) - 434.78x2x(750 - 1263) - 434.78x10x(750 - 1434))$$

$$= 16398.6\,KNm \quad > 14000kNm \qquad \therefore safe$$

Worked Example C8.3 : Prestress concrete Girder

Consider prestress girder with 240 mm deep slab as in-situ monolithic construction shown in **Fig.** below. Calculate ultimate moment of resistant with following properties.



Area of group 1 cables (1,2 and 3) = 5625.9 mm^2 (1875.3 mm^2 each)

Area of group 2 cable (4) = 1579.2 mm^2

Total force in group 1 cable after slip = 7.213 MN

Total force in group 2 cable after slip = 2.014 MN

Total shrinkage+Relexation+creep Loss in group 1 cables = 1.732 MN

Total shrinkage+Relexation+creep Loss in group 2 cables = 0.31 MN

Final force in group 1 cables = 5.481 MN

Final force in Group 2 Cable = 1.704 MN

$$\text{Prestrain in Gr. 1 cables} = \frac{5.481\text{x}10^9}{5625.9\text{x}1.95\text{x}10^5}$$
$$= 0.005$$
$$\text{Prestrain in Gr. 2 cable} = \frac{1.704\text{x}10^9}{1579.2\text{x}1.95\text{x}10^5} = 0.0055$$

Ultimate stress in cable (f_{pk}) = 1861.2 MPa

stress @ 0.1% proof force = 0.87x1861.2 = 1619.2 MPa

$$f_{pd} = \frac{1619.2}{1.15} = 1408\ Mpa \qquad \varepsilon_{pd} = \frac{f_{pd}}{E_s} = \frac{1408}{1.95\text{x}10^5} = 0.00722$$

Considering neutral axis depth of 405 mm, obtained from trial and error method, the strain profile is shown in the **Figure**.

Therefore total strain at ULS in cables including Prestrain are:

$$\text{Total strain in cable 1,2} = 0.0137 + 0.005 = 0.0187 > \varepsilon_{pd}$$
$$\text{Total strain in cable 3} = 0.0122 + 0.005 = 0.0172 > \varepsilon_{pd}$$
$$\text{Total strain in cable 4} = 0.0137 + 0.0055 = 0.0192 > \varepsilon_{pd}$$

All strain are greater than ε_{pd} hence stresses in cables are = f_{pd} = 1408 MPa

Thus total force in steel (T_u) = 1408x(5625.9+1579.2) = 10144.78 kN

The neutral Axis is in top flange of beam therefore treat concrete in similar manner to the flanged beam calculations by splitting compression zone in to following three sections and taking account of different compressive strength,

1) Rectangular part of stress block in top slab (3/7 x 405 = 173.57 mm)
2) parabolic part of stress block in top slab (66.43 mm)
3) parabolic part of stress block in top flange of girder (165 mm)

Strain at bottom of slab = 0.0035 x (1-240/405) = 0.00143

$$\text{Corrosponding stress at bottom of slab } f_c = f_{cd}\left(1-\left(1-\frac{\varepsilon_c}{\varepsilon_c}\right)^2\right)$$
$$= 0.446\text{x}35\text{x}\left(1-\left(1-\frac{0.00143}{0.002}\right)^2\right)$$
$$= 14.32\ \text{Mpa}$$

Compressive force in area 1:

C1 = 0.446 x 35 x 173.57 x 2200

= 5960740 N

Compressive force in area 2:

C2 = 14.32 x 2200 x 66.43 + 2/3 x (15.61-14.32) x 2200 x 66.43
= 2092810 + 125685
= 2218495 N

Compressive force in area 3:

C3 = 2/3 x 14.32 x 1250 x 165
= 1969000 N

Thus Cu = 5960.7 + 2218.5 + 1969 = 10148.2 ≈ Tu, Therefore section balances and neutral axis is at correct level.

Taking moment of all forces about neutral axis level give,

$$\begin{aligned} M_{rd} = &\ 5960740\left(405-\frac{173.57}{2}\right)+2092810\left(405-173.57-\frac{66.43}{2}\right) \\ &+125685\left(405-173.57-\frac{3}{8}66.43\right)+1969000\left(\frac{5}{8}165\right) \\ &+1408(1875.3x2+1579.2)x(2115-405-120)+1408x1875.3x(2115 \\ &-405-300) \end{aligned}$$

$$= 2540.58x10^6 + 15654.93x10^6 \text{ Nmm}$$

$$= 18195.1 \text{ kNm.}$$

Worked Example C8.4 : Reinforced concrete Pier with Biaxial Bendi

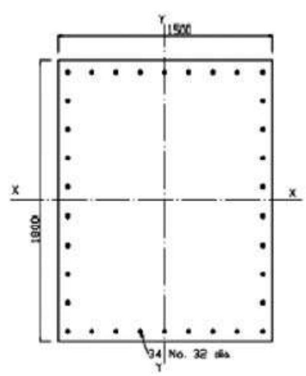
Consider RCC pier of size 1800 mm x 1500 mm reinforced with 34 number of 32 dia bar as shown in **Figure**. Pier is loaded with 17300 kN axial load, moment of 8000 kNm about minor axis. Assuming grade of concrete as M60 and grade of steel as Fe500, check the adequacy of section.

Total area of steel = 27344.4 mm^2

Pure compression load carring capacity,

$$\begin{aligned} N_{rd} &= 0.45 f_{ck} A_c + 0.87 f_y A_{sc} \\ &= 0.45x60x(1500x1800-27344.4)+0.75x500x27344.4 \\ &= 82415.\ 7 \text{ kN} \end{aligned}$$

$$\frac{N_{Ed}}{N_{rd}} = \frac{17300x10^3}{82415.7x10^3} = 0.21, \quad \text{by interpolation,} \quad \alpha = 1.092$$



Neutral axis depth for calculating ultimate moment of resistance (m_{rdx} and m_{rdy}) is obtained from trial and error method by shifting neutral axis locations to achieve given axial load carrying capacity as described in Example C8.2.

With the same procedure described in Example C8.2, values obtained is,

M_{rdx} = 19677.2 kNm
M_{rdy} = 16398.6 kNm

Check for interaction equation, for biaxial bending,

$$\left(\frac{M_{Edx}}{M_{rdx}}\right)^{\alpha} + \left(\frac{M_{Edy}}{M_{rdy}}\right)^{\alpha} \leq 1$$

$$\left(\frac{8000}{19677.2}\right)^{1.092} + \left(\frac{6000}{16398.6}\right)^{1.092} = 0.708 < 1 \quad Hence\ OK$$

6.3 Biaxial Bending — Cl. 8.3

The Code does not directly give any method for designing members subjected to direct load and bi-axial bending, other than working from first principles. However this is generally not a problem for the practicing engineers since there are several commercial softwares available internationally for solving such problems based on strain compatibility methods.

For those practicing engineers who do not have access to the computer software, a simplified approximate method is given in the Code for bi-symmetric sections, in which separate design in each principal direction is done disregarding the bi-axial bending, as a first step.

Graphical representation of the problem is shown in **Fig. C8.4** below as an example with rectangular column.

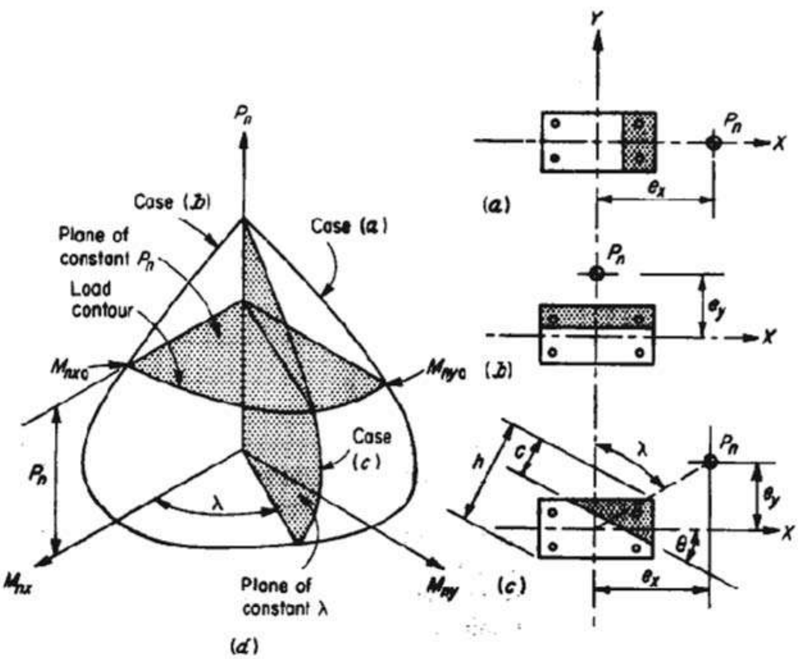


Fig. C8.4 Interaction Surface for Compression Plus Biaxial Bending; (a) Uniaxial Bending about Y-axis; (b) Uniaxial Bending about X-axis; (c) Biaxial Bending about X-axis and (d) Interaction Surface in Pear-Shaped Relationship

Horizontal sections of the interaction surface shown above at each value of axial load can be represented by Equation 8.3 of the Code. The application of this principal as given in the Code is self-explanatory.

CHAPTER 7

SECTION 9 : ULTIMATE LIMIT STATE OF TWO AND THREE DIMENSIONAL ELEMENTS FOR OUT OF PLANE AND IN PLANE LOADING EFFECTS

7.1 Scope — Ch. 9.1

This section deals with the design of two & three dimensional elements.

The procedure given is useful when the structure is analysed by Finite Element Method using membrane, plate or shell elements.

The membrane elements, which have only translation degrees of freedom have stress resultants in-plane only i.e., n_{Edx}, n_{Edy} & n_{Edxy} as shown in **Fig C9.1**. Membrane elements are used to idealise the shear walls etc. which are subjected to in plane forces and not out-of-plane forces.

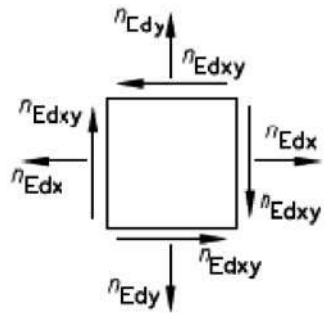


Fig. C9.1 Stress Resultants for Membrane Element

The plate bending elements on the other hand do not produce any in-plane forces and have three degrees of freedom per node i.e. one out-of plane translation and two rotations about two axes perpendicular to the out of plane (normal to surface) axis. It gives bending moments about x, y axis i.e. m_{Edx}, m_{Edy} & twisting moment m_{Edxy} and out of plane shear forces v_{Edx} and v_{Edy} as shown in **Fig. C9.2**.

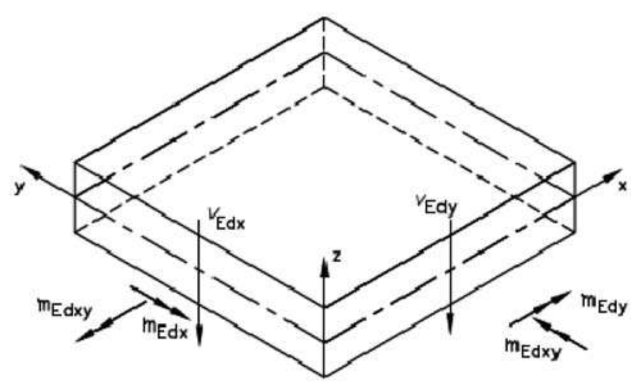


Fig. C.9.2 Force Resultants for Plate Bending Element

The shell element is combination of membrane element and plate bending element having six degrees of freedom at a node (three translations and three rotations) and gives all the eight force resultants i.e. n_{Edx}, n_{Edy}, n_{Edxy}, m_{Edx}, m_{Edy}, m_{Edxy}, v_{Edx} and v_{Edy}.

This section of the code gives the guidelines for design of these elements using Ultimate Limit State.

7.2 One Way and Two Way Slabs and Walls Cl. 9.2

One way two way slabs and the retaining walls are the typical examples which are subjected to the out-of-plane forces that generate the bending moments, out-of-plane shear forces and insignificant membrane forces. Hence "Plate bending" element is appropriate element for modeling of these structures. The Code prohibits use of the ultimate strength methods based on local yielding (e.g. yieldline method) in bridges, except for calculating resistance to accidental impact loads.

7.3 SUB-Elements of Box-Structures Cl. 9.3

1) The Code allows the design of the box type sections with separate designs for longitudinal effects and transverse effects.
2) For global analysis (longitudinal) of box-girders, a linear element i.e. beam element can be used, which gives overall longitudinal bending moment, shear forces and torsional forces for the section design.
3) For transverse analysis, a slice of 1.0 m length of the box girder at desired location can be modeled using beam element with pinned support at bottom-most nodes of webs, to represent the stiffness of webs. The design of these beam elements shall be done using ULS method for linear element as specified in **Section 8.0**.
4) In case of the box girder having complex geometry such as fish-belly- the girder can be modeled using shell elements. The design method for the same using sandwich model is available in **Annexure B-1**.

7.4 General Solution for Two Way Slabs, Walls and Shell Elements Cl. 9.4

7.4.1 *Simplified Design for Tensile Reinforcement for Orthogonal In-Plane Effects*

In this section the code has included a simplified design method for calculating tensile reinforcement for orthogonal in plane effects, $\sigma_{Edx} > \sigma_{Edy}$ and τ_{Edxy}.

In this method:

- The sign for compressive stresses needs to be taken as positive.
- X and Y and the direction along which the reinforcement is provided and s_{Edx} shall be greater than σ_{Edy}
- No reinforcement is required if both σ_{Edx} & σ_{Edy} are compressive (i.e. positive) and in-plane shear stress τ_{Edxy} is small. To determine whether τ_{Edxy} small or not, Code has included the criteria as $\sigma_{Edx}.\sigma_{Edy} > \tau_{Edxy}$.
- If τ_{Edxy} is significantly high or either σ_{Edx} or σ_{Edy} is tensile then the reinforcement needs to be provided in that direction.

Two worked examples demonstrate the procedure of the design.

7.4.2 *Simplified Design of Combined In-Plane Forces and Out of Plane Bending and Shear*

As per the approach suggested in this clause, the element is divided into three layers of plate, each having 1/3rd thickness; top & bottom resist the in plane forces & bending moments with resulting lever arm and the central one resists the out of plane shear forces. The detailed procedure is available in **Annexure B-1** and is explained with worked examples in the commentary of the said Annexure.

WORKED EXAMPLE **Cl. 9.1**

Following are the results at the center of membrane element used for analysis of diaphragm of bridge deck having thickness of 500 mm with Grade of concrete M40 and reinforcement Fe500

n_{Edx} = 3000 kN/m

n_{Edy} = 2000 kN/m

n_{Edxy} = 1500 kN/m

As per **Clause 6.4.2.8 (1)**, f_{cd} = 0.67 x 40/1.5 = 17.86 MPa

σ_{Edx} = 6.0 MPa

σ_{Edy} = 4.0 MPa

t_{Edxy} = 3.0 MPa

$\sigma_{Edx} \times \sigma_{Edy}$ = 24

t_{Edxy^2} = 9.0< 24

Since $\sigma_{Edx}.\sigma_{Edy} > \tau_{Edxy}^2$ and both σ_{Edx} & σ_{Edy} are compressive, no reinforcement in both direction is required (provide the minimum reinforcement as given in section).

Also both σ_{Edx} and σ_{Edy} are less than f_{cd}, the section is safe.

WORKED EXAMPLE **Cl. 9.2**

Keeping the other details same in example C-9.1, by changing the forces to following:

n_{Edx} = 2600kN/m

n_{Edy} = 2000kN/m

n_{Edxy} = 2500kN/m

σn_{Edx} = 5.2 MPa

σn_{Edy} = 4.0 MPa

t_{Edxy} = 5.0 MPa

Though both σ_{Edx} and σ_{Edy} are compressive, $\sigma_{Edx}.\sigma_{Edy}$ (i.e. 20.8) $<\tau_{Edxy}^2$ (i.e. 25), hence it is necessary to provide the reinforcement.

Since $\sigma_{Edx} > |\tau_{Edxy}|$

f'_{tdx} = 0

$$f'_{tdy} = \frac{\tau_{Edxy}^2}{\sigma_{Edx}} - \sigma_{Edy}$$

$$= \frac{25.0}{5.2} - 4.0$$

$$= 0.81 \text{MPa}$$

$$\sigma_{cd} = \sigma_{Edx} \left(1 + \left(\frac{tEdxy}{\sigma Edy}\right)^2\right)$$

$$= 5.2 \left(1 + \left(\frac{5}{5.2}\right)^2\right)$$

$$= 10.01 \text{ MPa}$$

$$v.f_{cd} = 0.6 \left(1 + \frac{40}{310}\right) \times 17.867$$

$$= 9.33677 \text{MPa}$$

$$< 10.01 \text{ MPa}$$

Here it is necessary to revise the thickness. After increasing from 500 mm to 600 mm, f'_{tdy} = 0.67 MPa and σ_{cd} = 8.34 MPa < 9.33677 MPa.

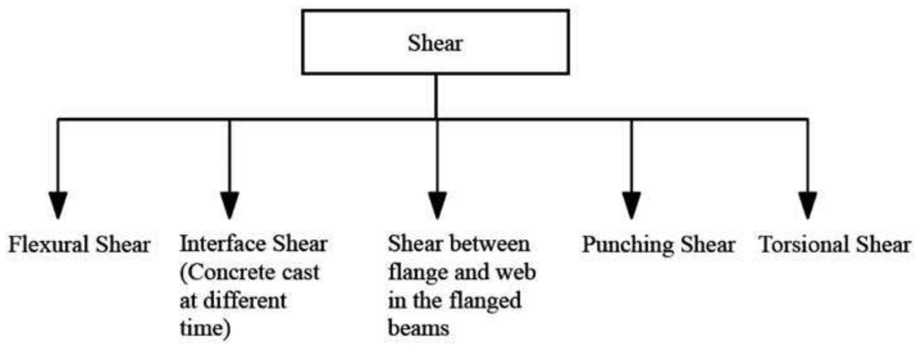
CHAPTER 8

SECTION 10 : DESIGN FOR SHEAR, PUNCHING SHEAR AND TORSION

8.1 Scope — Cl. 10.1

The scope is to cover the design provisions for shear in flexural members including arriving at the shear reinforcements.

The shear can arise out of (a) flexure, (b) Interface shear due to concrete cast at different times (c) shear between web and flange in flanged sections (d) due to punching (e) due to torsion. Different types of shear arising in flexural members can be represented in the following flow diagram:



The shear design has to be carried out under ultimate limit state only. When the member requires shear reinforcement, the reinforcement has to be calculated based on a truss model. For members without shear reinforcement the capacity of the section is estimated using empirical formula.

The adoption of higher grade of concrete with high permissible stress leads to slender web resulting in large height to thickness ratio giving rise to significant second order out-of-plane bending effect. This leads to failure of web at shear force less than sectional force predicted based on uniform crushing of concrete using the formula given in the code. Hence the code places restriction for shear strength of concrete grades higher than M60. The shear strength for the same shall be restricted to strength class M60 for design purposes.

8.2 Design of Flexural Members for Shear — Cl. 10.2

8.2.1 *Shear Design Model of Members without Shear Reinforcement* — Cl. 10.2.1

The sub clauses of the code are quite elaborate and the same can be followed by the designers without any difficulty.

Deck slab designed by using the effective width method as given in **Annexure B-3**, need not be checked for flexural shear.

8.2.2 *Zones of Shear Design* — Cl. 10.2.2.1

Members loaded beyond their shear capacity need to be provided with shear reinforcement to resist the full shear force. Members subjected to bending and shear has four distinct zones. The zone adjacent to the support does not develop any crack. Hence this zone is called no crack zone (Zone A). The next zone (Zone B) develops shear cracks but does not develop any flexural crack. In the next zone (Zone C) both flexural and shear cracks appear. This zone is further subdivided into two zones as zone C_1 and zone C_2. In the zone C_1 cracks, are parallel and in the zone C_2 the cracks converge. In zone D only flexural cracks appear. The appearance of cracks in different zones in shear is shown in **Fig. 10.1 (a)** of the Code.

The structure can be supported directly or indirectly as shown in **Figs. C10.1 (b) & (c)**. In zone A, the type of support affects the compression fields. In case of direct support, a fan like compression field develops. In the area confined by the beam end and the steepest inclination of compression field i.e. θ = 45º no shear reinforcement is required in this zone. However the shear reinforcement required at section ‘d’ away from support shall be extended in this region. As per this clause, loads located between ‘d’ and ‘2.5d’ from support, no shear reinforcement is required for the shear generated by these loads as the loads are carried to supports directly by the compression strut. This Clause is in contradiction to **Clause 10.3.3.3 (7)** and this is explained in detail in **Section 10.3.3.3 (7)**. The horizontal component of the compression strut will give raise to tension. To cater for this tension, additional tensile steel needs to be provided over and above the tensile steel provided for the bending effect. Reference shall be made to **Clause 10.3.3.3(6)** also for provision of additional longitudinal reinforcement.

In case of indirect support, a fan like compressive field does not exist. In the common intersection zone of supporting and supported beam, suspension reinforcements are required in addition to shear reinforcement. Preferably this reinforcement shall be provided in the supporting beam to resist the reaction transferred by the supported beam to the supporting beam.

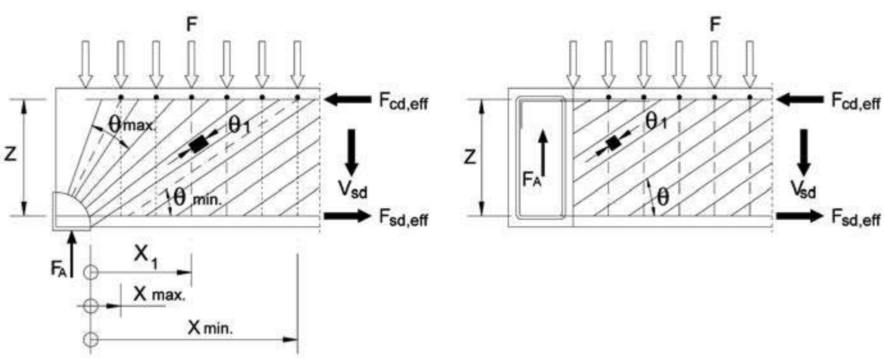


Fig. C10.1 Direct and Indirect Supports for Beams

Notations used in the above Fig. are as follows :

- X_i Distance corresponding to load position
- X_{max} Distance corresponding to max. strut angle (θ_{max} = 45º)
- X_{min} Distance corresponding to min. strut angle (θ_{min} = 21.8º)

8.2.3 *Shear Transfer Mechanism of Truss Model* Cl. 10.2.2.2

1) Beam with Constant depth

The design of shear resistance of members is based on truss model.

The Truss model can be described as shown in the sketch

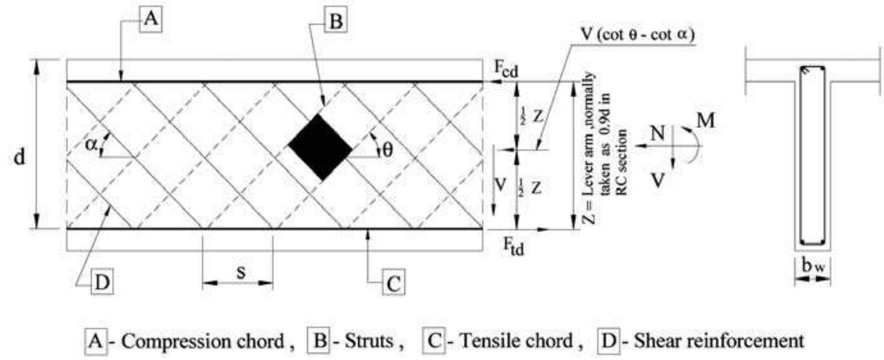


Fig. C10.2 Truss Model and Notations for Members with Shear Reinforcement

The truss model consists of compression chord, tension chord and diagonals. The web consists of compression strut diagonals of concrete and tension diagonals of steel, consisting of bent up bars, vertical stirrups. 'θ' is angle of compression strut and α is the inclination of steel stirrups. Angles are measured with respect to the axis of element.

The angle of compression strut θ shall be limited to minimum of 21.8º and maximum of 45º. Similarly the angle of shear stirrups shall be limited to minimum of 45º and maximum of 90º. As the angle of truss can be varied in the calculation, this method is called as "Variable Truss angle Method".

2) Beams of variable depth

The truss model shown in **Fig. 10.3** of the Code or any other truss model can be used for carrying out shear design of varying depth beams having both chords inclined. While designing local zone of beams of short length having inclined chords, the zone can be designed as a beam with parallel chords, provided the components of chord forces parallel to the shear force is taken into consideration as the components of inclined chord forces contribute towards shear.

8.2.4 *Design Shear Force*

As discussed earlier, the beams can be supported directly or indirectly. In case of direct support, the net shear force acting at a section, 'd' (effective depth away from the support), shall be used for design of shear reinforcement of the section and the same reinforcement shall be extended over the support. This will be applicable only if there are no concentrated loads located between the support and distance 'd' from support. However for checking the capacity of strut against crushing, the shear force acting at the face of support shall be considered.

In case of indirect supports the shear at the face of support shall be used for arriving at the reinforcement as well as for checking the capacity of strut.

In case of variable depth of beam, the compression chord or tension chord or both chords may be inclined. The inclinations of these chords generate chord forces. The components of these chord forces reduce the shear force acting on the section if they act favorably i.e. if the depth of section is increasing in the same direction as the bending moment increases. However in case where the depth is decreasing in the direction of increasing bending moment, the components of chord forces will add to the shear force. Depending upon the direction of the prestressing force it can offer relief or add to the shear force. In Fig 10.4 of the Code, the chord forces reduce the externally acting shear at the section.

In case of indeterminate structures, in addition to the above forces the shear due to hyper static effects shall also be considered.

The value of the prestressing force that is to be used in the calculation is as follows:

a) The Code allows the designer to use the full capacity of cable, if it can be mobilized.

b) If the prestressing force is already accounted for while arriving at the net shear force, duly accounting for all losses in the cable, then the increase in prestressing force due to cracking of concrete [Force at full capacity of cable if the corresponding strain could be achieved minus the force in the cable accounted for in the computation of shear force] only is to be taken as an additional force.

c) Component of this increase in force in the vertical direction can be taken as an additional resistance force as if contributed by the shear reinforcement, in case the prestressing force is 'favorable'. In case the prestressing force is 'unfavorable', the resistance shall be reduced.

If the strain in the cable is not attaining a value corresponding $\frac{f_{pd}}{E_p}$ then the force in cable shall be taken corresponding to the strain attained and the full capacity of the cable shall not be used.

Conservatively this increase in stress in the prestressing cable can be ignored and only the prestressing force available may be considered after accounting all losses. This method of design will lead to slight increase in shear steel.

For unbonded cable, at the ultimate limit state stage the force in the cable shall be assumed to be same as the initial prestressing force less the losses. Increase in cable force may not be considered. This will lead to slightly conservative design shear force.

In the development length portion, the prestressing force develops gradually in case of pre-tensioned girders. Hence the corresponding force shall be considered by taking a linear variation.

In case of beams of varying depth, the design shear force shall be calculated by ignoring the relief offered due to inclination of chords for justifying the no requirement of shear reinforcement. However, when the section needs to be provided with the designed shear reinforcement, the relief or additional force due to chord inclination shall be considered for arriving at the net shear force. The effect due to prestressing force shall always be considered.

V_{Ed} is defined as the shear force in the section resulting from external loading and prestressing. Chord forces also, if considered, then the net shear force is defined V_{NS} in the Code (i,e. $V_{NS} = V_{Ed} – V_{ccd} – V_{td}$).

8.3 Design Method **Cl. 10.3**

8.3.1 *Elements not Requiring Design Shear Reinforcement* Cl. 10.3

1) The shear capacity of concrete V_{Rdc} shall be greater than V_{Ed} which is the design shear force in the section considered resulting from external loading and prestressing. No relief due to inclined chord force shall be considered while arriving at V_{Ed}. For sections having shear capacity V_{Rdc} more than V_{Ed} no designed shear reinforcement needs to be provided. However minimum shear reinforcement needs to be provided as per **Clause 16.5.2**. This minimum reinforcement can be omitted in case of slab structures.

2) The formula given for calculating the shear capacity of section in the code is empirical only. The shear strength depends upon the tensile strength of concrete which in turn depends upon the compressive strength of concrete to the power 1/3, longitudinal reinforcement ratio and depth of the section. The longitudinal reinforcement contributes to the shear resistance in two ways viz. by dowel action and by controlling the crack width which will influence the amount of shear that can be transferred across the cracks by aggregate interlock. Shear strength increases with increase in reinforcement ratio but the rate of such increase reduces as the reinforcement ratio increases. Depth of section also plays significant role which is called as size effect on the shear strength, particularly for shallow depth members such as slabs.

3) The clause places a restriction that the formula given (Eq. 10.4) is applicable only to single span members. Reasons are not clear for this restriction. For other two types of construction viz, continuous and multiple span integral bridges this formula shall not be used and all sections shall be treated as cracked sections. Shear reinforcement shall be designed according to Eq. 10.7 after verifying the concrete capacity as per Eq. 10.8.

In any prestressed section there will not be appreciable longitudinal reinforcement. Hence the capacity of section to resist the shear without shear reinforcement as per Eq. 10.1 will be very negligible. Hence the designer can straightway use the Eq. 10.7 and 10.8 for design of prestressed concrete sections for shear, both in continuous and multiple span integral bridges.

In case of continuous bridges, for sections near the intermediate support, the bending moment will be very high and the section would automatically crack. Hence the Eq.10.4 cannot be used. The other affected sections in these structures are the contra flexure sections and sections near to contra flexure sections. The code has reservation with regard to applicability of this equation to these regions. Hence without giving reasons as to why this equation cannot be applied to the continuous structures, the restriction is placed. By the same argument for the single span integral bridge also this formula is not applicable as these sections (contra-flexure and adjacent sections) exist in said structures. However the code permits the use of this equation for single span integral bridges.

The other possibility is this formula is applicable only for one way spanning members because it does not take into account, the effect of two ways spanning of members.

In case of prestressed concrete members, at first the section needs to be checked whether cracked or uncracked. If the flexural tensile stress is less than $\frac{f_{ctk.05}}{\gamma_m}$ under maximum bending moment, then the section is deemed to be uncracked and the equation 10.4 of the Code shall be used to estimate the capacity of the section. This is generally applicable to zone B. In case the section turns out to be cracked, then equations 10.7 and 10.8 of code shall be used for calculating the capacity of section. In case of cracked sections and sections having inadequate capacity, the designed shear reinforcement needs to be provided. In prestressed beams the longitudinal tensile reinforcement may be absent or very minimal. Hence the capacity of section to carry the shear without shear reinforcement will be virtually absent. This is because V_{Rdc} in case of beam having no shear reinforcement directly depends upon A_{sl}, the longitudinal reinforcement which will be either absent or negligible.

Hence the designer, for most of the cases can straightway use Eq. 10.7 and 10.8, without checking the capacity of section to carry shear using Equation 10.1.

Derivation of Equation 10.4 for Un-cracked Prestressed Sections

For sections having no shear reinforcement, the shear failure is expected to occur when the principle tensile stress, anywhere in the section exceeds the tensile strength of concrete f_{ctd}, which shall be taken as $f_{ctk.05}/\gamma_m$. Taking tensile stress as negative, the minor principal stress.

$$-f_{ctd} = \frac{\sigma_{cp} + \sigma_{bending}}{2} - \sqrt{\left(\frac{\sigma_{cp} + \sigma_{bending}}{2}\right)^2 + \tau^2}$$

where,

σ_{cp} is the compressive stress at centroid due to axial loading or prestressing force (after all loss, including partial safety factor), taken as positive.

$\sigma_{bending}$ is the stress due to bending moment at the level considered in MPa, taking compressive stress as positive. This includes the bending stress due to prestress and all other design loads. The section is subjected to normal stress and shear stress.

τ is the applied shear stress = $\frac{V_{Rdc} A\bar{x}}{Ib}$

V_{Rdc} is the shear resistance of the concrete in the web from the shear force required to cause web cracking.

I is the second moment of area of section.

$A\bar{x}$ is the first moment of area of the concrete above/below the plane of consideration about the centroid of the section. This has been taken as S in the code.

Substitute $\left(\frac{\sigma_{cp}+\sigma_{bending}}{2}\right) = x$

$$-f_{ctd} = x - \sqrt{x^2+\tau^2}$$
$$-f_{ctd} - x = -\sqrt{x^2+\tau^2}$$
$$+f_{ctd} + x = \sqrt{x^2+\tau^2}$$

Squaring both sides:

$$(f_{ctd} + x)^2 = x^2 + \tau^2$$
$$+ (f_{ctd})^2 + x^2 + 2\, f_{ctd}x = x^2 + \tau^2$$
$$+ (f_{ctd})^2 + 2\, f_{ctd}x = \tau^2$$

$$\tau = \sqrt{(f_{ctd})^2 + 2\, f_{ctd}\frac{(\sigma_{cp}+\sigma_{bending})}{2}}$$
$$\tau = \sqrt{(f_{ctd})^2 + f_{ctd}\,(\sigma_{cp}+\sigma_{bending})}$$

Substituting for τ

$$\frac{V_{Rdc}A\bar{x}}{Ib} = \sqrt{(f_{ctd})^2 + f_{ctd}(\sigma_{cp}+\sigma_{bending})}$$
$$V_{Rdc} = \frac{Ib}{A\bar{x}}\sqrt{(f_{ctd})^2 + f_{ctd}(\sigma_{cp}+\sigma_{bending})}$$

In order to take care of the transmission length in pre-tensioned construction a constant k_1 is introduced.

$$V_{Rdc} = \frac{Ib}{A\bar{x}}\sqrt{(f_{ctd})^2 + k_1 f_{ctd}(\sigma_{cp}+\sigma_{bending})}$$

b is the web width at centroidal axis after allowing for deduction due to duct diameter as per **Clause 10.3.3.3 (5)** of the code.

At the centroidal axis σbending = 0

Hence the shear capacity at centroidal axis is

$$V_{Rdc} = \frac{Ib}{A\bar{x}}\sqrt{(f_{ctd})^2 + k_1 f_{ctd}\sigma_{cp}}$$

At times, the maximum principle tensile stress may occur at a section away from centroid and not at centroid. For any other section other than centroidal axis the shear capacity shall be calculated using the expression which includes the term $\sigma_{bending}$. The web width and the corresponding area shall be substituted accordingly as applicable for that section.

Additional equation for checking shear in section having precast beam supporting the deck slab

The uncracked shear resistance V_{Rdc} of such type of construction has to be worked out by limiting the principal tensile stress to f_{ctd}. If the loads Vc_1 acting on the precast beam alone produces the shear stress of τ_s, the additional shear force Vc_2 which can generate a shear stress of τ^1s,as the CG of the composite section. The principle tension under the combined shear and bending shall not exceed fctd. Hence the total shear capacity of section needs to be Vc_1 + Vc_2. Shear stress distribution will be as shown below.

The principal tensile stress can be checked at the composite centroid. In a given problem the other sections could be critical which should also be examined.

At the composite centroid the total longitudinal stress = σ (compression taken +ve)

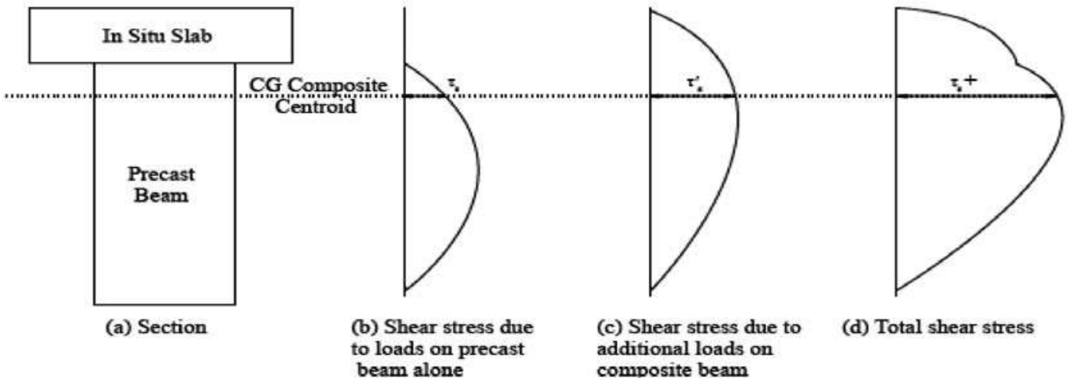


Fig. C10.3 Distribution of Shear Stress in Composite Beams

The shear stress due to loads applied on precast section = τ_s

The additional shear stress that can withstand after the section becomes composite = τ^1_s

The principal tensile stress at the composite centroid

$$=\frac{\sigma}{2} - \sqrt{\left(\frac{\sigma}{2}\right)^2 + (\tau_s + \tau_s^1)^2}$$

The principal tensile stress should not exceed = - f_{ctd}

$$-f_{ctd} = \frac{\sigma}{2} - \sqrt{(\frac{\sigma}{2})^2 + (\tau_s + \tau_s^1)^2}$$

$$-f_{ctd} - \frac{\sigma}{2} = -\sqrt{\left(\frac{\sigma}{2}\right)^2 + (\tau_s + \tau_s^1)^2}$$

$$(f_{ctd} + \frac{\sigma}{2})^2 = \left[\sqrt{(\frac{\sigma}{2})^2 + (\tau_s + \tau_s^1)^2}\right]^2$$

$$f_{ctd}^2 + \left(\frac{\sigma}{2}\right)^2 + 2f_{ctd}\frac{\sigma}{2} = \left(\frac{\sigma}{2}\right)^2 + (\tau_s + \tau_s^1)^2$$

$$\tau_s^1 = \sqrt{f_{ctd}^2 + f_{ctd}\sigma} - \tau_s$$

$$\tau_s^1 = \frac{Vc_2}{Ib} A_c\bar{x}$$

$$Vc_2 = \frac{\tau_s^1 Ib}{A_c\bar{x}}$$

Substituting for τ^1_s

$$Vc_2 = \frac{Ib}{A_c\bar{x}}\left(\sqrt{f_{ctd}^2 + f_{ctd}\sigma} - \tau_s\right)$$

$A_c\bar{x}$ is is the first moment of area of the concrete above/below the composite centroid about the CG of composite centroid.

'I' is the moment of Inertia of composite section b is the breadth of web

$$\tau_s = \frac{Vc_1}{b}\left(\frac{A_{pc}\bar{x}_{pc}}{I_{pc}}\right)$$

$A_{pc}\bar{x}_{pc}$ is the first moment area of the precast section above the composite centroid about the composite centroid.

4) The calculation of shear resistance as per above formulae for uncracked section is not required for cross sections between the support section and the section which contains the intersection point on the elastic centroidal axis and a line inclined from the inner edge of the support at an angle of 45° as shown below. However the reinforcement arrived at this section AA shall be continued in this region covering upto the end of the beam.

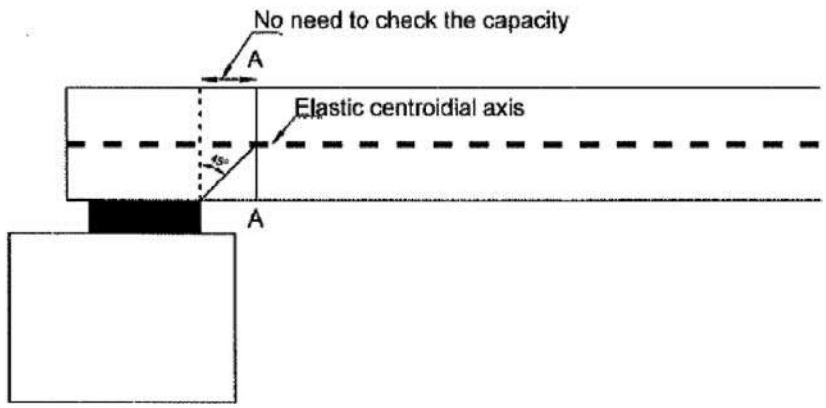


Fig. C10.4 Sections Requiring no Check for Shear Resistance

5) Members with loads applied on the upper surface (top) at a distance α_v where α_v is within 0.5d to 2d from the edge of support (or center of bearing when flexible bearings are used) the question arises, how much of this load will contribute towards shear, as part of the loads will be directly transmitted to the support by arch action without involving bending and shear actions and part of the load will be transmitted by truss action. Hence the code suggests reducing the contribution of this load towards truss shear by multiplying the shear due to this load by a factor $\alpha_v/2d$ (i.e. βV_{ED}). When loads are placed at less than 0.5d, a distance of 0.5d shall be assumed. This reduction in shear force is applicable for checking the capacity of section without shear reinforcement as per Equation 10.1 provided longitudinal reinforcement is anchored at the support. This reduction factor shall not be used while checking with equation 10.4. (While checking the uncracked section of prestressed concrete) For checking the adequacy of shear capacity of section as per Equation 10.5 this reduction factor shall not also be used and it shall be assumed that the entire load to be contributing towards the shear.

Designers can observe that the **Clause 10.2.2.1** stipulates that no shear reinforcement is required to be designed for a section within 'd' from support except the reinforcement designed at 'd' to be continued upto support and also for the shear generated due to loads placed in the zone covered by θ_{sup} and θ_{inf} (i,e. between 'd' and '2.5d' from support in case θ_{inf} is considered as 21.8°).

However if we consider the **Clauses 10.3.2(5) and 10.3.3.3(7)**, the requirement is some what different. Hence there is a contradiction among these clauses. The contradiction is, any load placed between ‘0.5d’ and ‘2d’, the reduced shear force due to these loads have to be worked out for arriving at the shear at any section between the support and the concentrated load and cannot be ignored as stated in **Clause 10.2.2.1**. Secondly, when the load is placed at 0.5d and if the shear force is to be calculated at ‘d’ distance away from the support, the effect of the load located at ‘0.5d’ will not appear in the shear force calculation.

The first para of the **Clause 10.2.2.1** is applicable only in case if there is no concentrated load occurring within a distance ‘d’ from support and the structure is subjected to uniformly distributed load. The second part of the clause covering the load placed between d and the distance corresponding to minimum strut angle 2.5 d shall not be operated. Instead **Clause 10.3.2 (5)** shall be followed.

Clause No.10.3.2 (5) is not suited for multiple concentrated loads or moving loads in case of bridges, indirect loads or subjected to uniformly distributed load. This clause is applicable only when the structure is subjected to single concentrated load. However if this clause is to the applied to the moving loads, then the following procedure shall be adopted:

a) Workout the support reaction and shear at various sections due to dead load.

b) It is assumed that there are three concentrated loads within a distance of 2d located at a distance of 0.5d, d and 1.5d. Any load occurring between the support and 0.5d the same may be considered to act at 0.5d.

c) Place the first concentrated load W_1 at 0.5d, W_2 at d and W_3 at 1.5d. The β_1, β_2 and β_3 factor will be 0.25, 0.5 and 0.75.

d) The support shear reaction

   $V_{support}$ = V_{DL} reaction + (reaction due to W_1) + (reaction due to (reaction due to W_2) + reaction due to loads beyond 2d.

e) Shear at section 0.5d (left of section load) V_{ED} = V shear due to dead load + β_1 (shear due to load W_1) + β_2 (shear due to load W_2 + β_3 (shear due to load W_3). + shear due to loads beyond 2d.

f) The shear worked out in the above step (e) should satisfy Equation 10.1.

g) Calculate the support reaction, as shown in (d) without introducing β factor and the support shear shall satisfy Equation 10.5 for crushing of strut.

h) If there is a reduction in cross-section within 2d or outside 2d this section shall also be checked against Equation 10.1 and 10.5.

i) Repeat the calculation by moving and placing the load W_1 and W_2 at d and 1.5d and at 2d for checking the section d.

j) When the loads reach section 2d, and beyond 2d, the reduction factor need not to be applied and the shear shall be checked against Equation 10.1.

k) If the sections are able to satisfy Equation 10.1, then no shear reinforcement is required to be provided. The support and the reduced sections are also able to satisfy equation 10.5 for crushing of strut, it can be concluded that the sections need no revision and the section is deemed to be safe in shear without shear reinforcement. However, the beams are to be provided with minimum shear reinforcement and the same will be dispensed with, in slabs.

6) For design of longitudinal reinforcement in the region cracked in flexure the M_{Ed} line shall be shifted over a distance of 'd' in the unfavorable direction or the tensile reinforcement can be increased due to additional chord force as explained later.

Worked Example 10.3-1: Elements not Requiring Shear Reinforcement

Estimate the shear capacity of RCC slab in which no shear reinforcement is provided. The particulars of slab are as follows:

Thickness of slab 750 mm, cover 50 mm concrete grade M30.

Reinforcement 25 dia 125 mm.

Area of reinforcement = 39.27 cm^2/m

Effective depth = 750 – 50 – $\frac{25}{2}$ = 687.5 mm.

Effective width = 1000 mm

As_l = 39.27 cm^2

$$\rho l = \frac{As_l}{b_w d} = \frac{39.27}{100 \, x \, 68.75} = .0057 \leq .02$$

$$k = 1 + \sqrt{\frac{200}{687.5}} = 1.53$$

Axial Load = 0 $\therefore$ σ_{cp} = 0.0

V_{Rdc} = [0.12 x 1.53 (80 x .0057 x 30)$^{0.33}$] x 1000 x 687.5 x 10^{-3}

= 302 kN/m

V_{Rdcmin} = .031 x $1.53^{3/2}$ x $30^{1/2}$ x 1000 x 687.5 x 10^{-3} = 221.0kN/m

The slab can with stand a shear of 302 kN/m without providing any shear reinforcement.

8.3.2 *Members with Vertical and Inclined Shear Reinforcements* Cl. 10.3.3.2 & 10.3.3.3

When the shear resistances of the members work out to be less than the shear to be resisted, then the members have to be provided with designed shear reinforcement. The required shear reinforcement shall be worked out using the truss model.

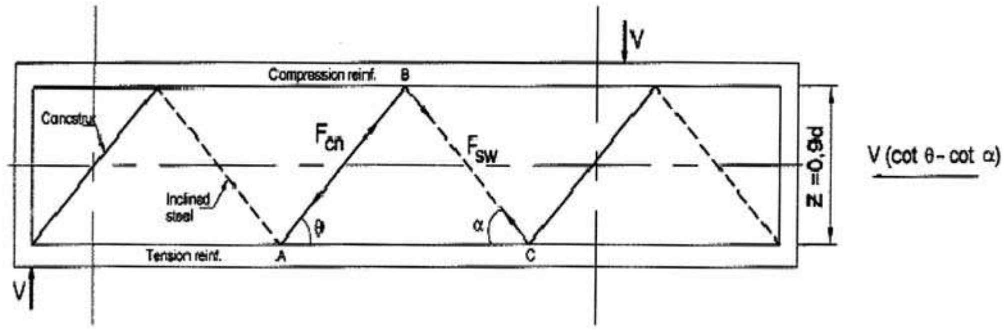


Fig. C10.5 Truss Model for Shear Resistance

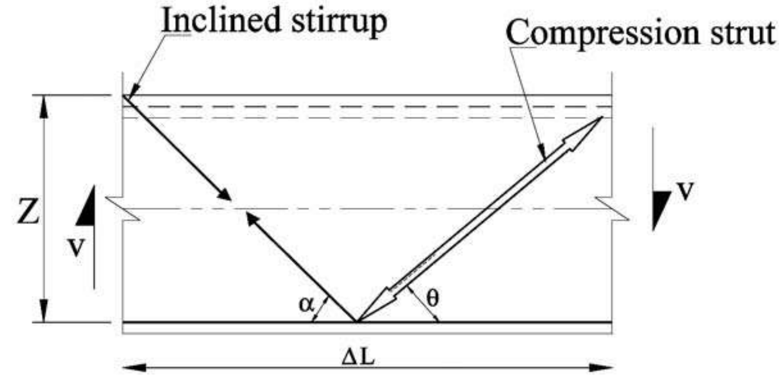


Fig. C10.6 Truss Model One Panel Length for Shear Resistance

Capacity of Shear Reinforcement and Concrete

One panel length of Truss $\Delta L = Z(\cot\alpha + \cot\theta)$

Spacing of inclined stirrup at an angle $\alpha = S$

Total number of stirrups $= \frac{Z(\cot\alpha + \cot\theta)}{S}$

Force in the vertical direction: (Codal Eq:10.11)

$$V_{Rds} = A_{sw} f_{yd} \frac{Z(\cot\alpha + \cot\theta)}{S} \times \sin\alpha = \frac{A_{sw} f_{yd}}{S} Z(\cot\alpha + \cot\theta)\sin\alpha$$

For vertical stirrups $\alpha = 90^\circ$, $\cot\alpha = 0$, $\sin\alpha = 1$ so $V_{RDS} = \frac{A_{sw} f_{yd}}{S} Z\cot\theta$ (Codal Eq:10.7)

Capacity of concrete strut

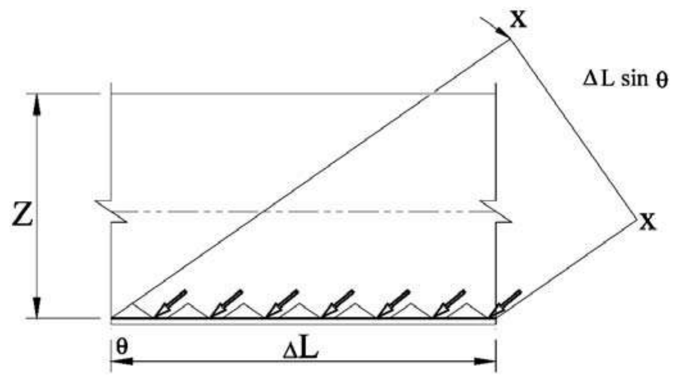


Fig. C10.7 Compression Strut Distributed over Panel Length ΔL

It σ_c is the allowable compressive stress

Total compressive force perpendicular to plane x-x = $\sigma_c b_w \Delta L\, Z \sin\theta$

= $\sigma_c b_w\, Z (\cot\theta + \cot\alpha) \sin\theta$

Total compressive force in vertical direction:

= $\sigma_c b_w\, Z (\cot\theta + \cot\alpha) \sin\theta \times \sin\theta$

= $\sigma_c b_w\, Z (\cot\theta + \cot\alpha) \sin^2\theta$

$$\frac{1}{1+\cot^2\theta} = \frac{1}{1+\frac{\cos^2\theta}{\sin^2\theta}} = \frac{1}{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}} = \sin^2\theta$$

$$\therefore \quad \sin^2\theta = \frac{1}{1+\cot^2\theta}$$

Total compressive force in vertical direction = $\frac{\sigma_c b_w Z(\cot\theta+\cot\alpha)}{(1+\cot^2\theta)}$

$$\sigma_c = \alpha_{cw} \nu_1 f_{cd}$$

$$V_{Rdmax} = \frac{\alpha_c v_1 f_{cd} b_w Z(\cot\theta+\cot\alpha)}{(1+\cot^2\theta)} \qquad \text{(Codal Eq:10.8)}$$

When stirrups are provided in vertical direction $\alpha = 90^\circ$ $\cot\alpha = 0$

$$V_{Rd\,max} = \frac{\alpha_{cw} b_w Z v_1 f_{cd}(\cot\theta)}{(1+\cot^2\theta)} = \frac{\alpha_{cw} b_w Z v_1 f_{cd}}{\frac{1}{\cot\theta}+\frac{\cot^2\theta}{\cot\theta}}$$

$$V_{Rdmax} = \frac{\alpha_{cw} b_w Z v_1 f_{cd}}{(\tan\theta+\cot\theta)} \qquad \text{(Codal Eq:10.8)}$$

For members with vertical shear reinforcements, the shear resistance is the smaller value of $V_{Rd,s}$ and $V_{Rd,max}$. The maximum effective cross sectional area of shear reinforcement for vertical stirrups can be found out by substituting $\theta = 45^\circ$ as the capacity due to reinforcement cannot exceed the capacity of concrete.

For Vertical Stirrups $\frac{A_{sw.max} f_{ywd} Z}{S} \le \frac{\alpha_{cw} b_w Z v_1 f_{cd}}{2}$

$$= \frac{A_{sw.max} f_{ywd}}{b_w S} \le \frac{1}{2} \alpha_c v_1 f_{cd} \qquad \text{(Codal Eq:10.10)}$$

As θ cannot be assumed more than 45º,the shear steel area cannot exceed the above shown value in a section. In case if it exceeds, it means, the section has failed in compression and hence need to redesigned

For $\theta = 45^\circ$ $\cot\theta = 1$

For Inclined stirrups

$$V_{Rds} = \frac{A_{sw.max} f_{yd} x Z}{S}(\cot\alpha + 1)\sin\alpha$$

$$V_{Rdmax} = \frac{\alpha_c b_w Z v_1 f_{cd}(1+\cot\alpha)}{(1+1)}$$

$$V_{Rds} \le \square V_{Rdmax}$$

$$\frac{A_{sw.max} f_{yd}}{s} \sin\alpha \le \frac{1}{2} \alpha_{cw} b_w v_1 f_{cd} \qquad \text{(Codal Eq:10.13)}$$

$$\frac{A_{sw.max} f_{yd}}{b_w s} \le \frac{\alpha_c v_1 f_{cd}}{2 \sin\alpha}$$

At any situation if the provided shear reinforcement $\frac{A_{sw} f_{yd}}{b_w s}$ works out to be greater than $\frac{\alpha_c v_1 f_{cd}}{2 \sin\alpha}$i it can be safely concluded the web has failed in shear and requires redesign.

It is necessary to restrict the shear resistance of concrete grades of higher than M60 to that of M60 grade for design purpose.

The adoption of higher grade of concrete with high permissible stress leads to slender web resulting in significant second of order out plane bending effect leading to failure at shear force less than the shear capacity calculated based on the formula given in the code applicable for crushing of concrete under axial compression. In order to avoid such failure the code has placed this restriction.

The reduction in the concrete strength due to cracking shall be calculated by using formula $v = 0.6\left(1 - \frac{f_{ck}}{310}\right)$, for calculating the reduction factor. The strength in steel reinforcement shall be taken as 0.87 f_y.

Evaluation of Inner Lever Arm

Regarding the use of lever arm 0.9 d, a word of caution is offered. The lever arm 0.9 d indicated in the code is only applicable for T-beams. In case of circular section it should be taken as $0.9d_e$ where $d_e = r + 2r_s/\pi$, where r = radius of circular section or r_s = the radius of the longitudinal reinforcement distributed over the circle. For the following cases, the actual lever arm should be derived.

1) Beams having varying width
2) Prestressed beams
3) Section having axial forces.
4) Cross section having no compression flange (Support section of continuous beam) i.e. rectangular beams.

In these cases after completing the flexural analysis the internal lever arm shall be worked out and the same may be taken as Z for arriving at the shear reinforcement.

In order to prove how erroneous will be if one assumes 0.9d for rectangular beam having no flange an example is shown below. Rectangular beam, Concrete grade M45, Breadth = 500 mm, Depth = 1000 mm, Effective depth = 950 mm, Area of steel = 10965 mm^2. The section is a balanced section.

From truss analysis depth of compression chord for yielding of steel = $\frac{10965\, x \frac{500}{1.15}}{0.45\, x\, 45\, x\, 500}$ = 471 mm
Lever arm = 950 – $\frac{471}{2}$ = 714.5 mm which is 0.75*d*.

Thus it can be seen that 0.9d always does not hold good. Hence the designer adopt the lever arm obtained in flexural analysis.

8.3.2.1 Additional tensile force Cl. 10.3.3.3 (6)

Additional force in tension and compression chord due to shear V_{Ed} may be worked out as shown below:

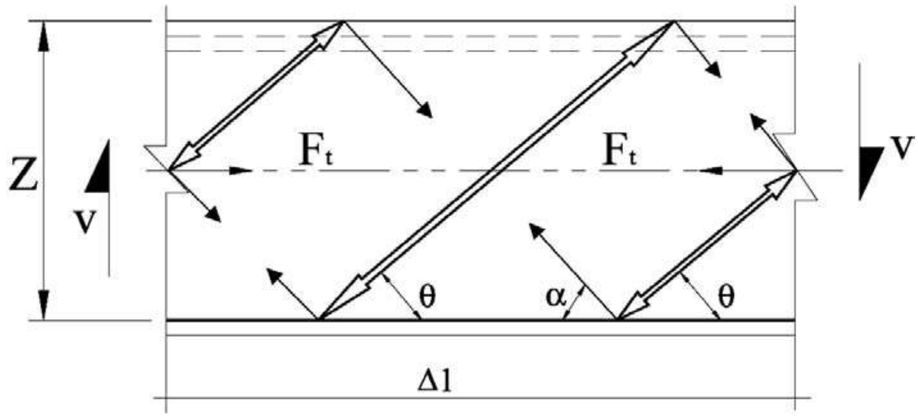


Fig. C10.8 Truss Model for Arriving at the Additional Tensile Force in Chords

Force in the strut = $\frac{V}{\sin\theta}$

Horizontal component of this force = $\frac{V}{\sin\theta}\cos\theta = V\cot\theta$

Similarly force in diagonal steel element = $\frac{V}{\sin\alpha}$

Horizontal component of this force = $\frac{V}{\sin\alpha}\cos\alpha = V\cot\alpha$

Unbalanced tensile force F_t = V (cot θ – cot α)

This force has been shown in **Fig. C10.8**. Distributing this force between tension and compression chords equally the additional force = $\Delta f_{td} = \frac{1}{2}V(\cot\theta - \cot\alpha)$. This tensile force shall be added to the tensile force generated due to flexure and the reinforcement to be provided accordingly. The total tensile force at a section $\frac{M_{Ed}}{z} + \Delta f_{td} \leq \frac{M_{Edmax}}{z}$ where $M_{Ed,max}$ is the maximum moment along the girder.

The angle of compression strut shall be limited to minimum of 21.8^0 and maximum of 45^0. Similarly, the angle of shear stirrups shall be limited to minimum of 45º and maximum of 90^0. Alternatively, shift rule can be applied as per **Clause 10.3.2 (6)**.

The spare capacity available in prestressing steel can be used to carry this additional tensile force arising out of shear. In case if the cable is draped the inclination has to be taken into account.

However, if it is decided to carry this additional longitudinal force both by un-tensioned steel and prestressing steel, the force shall be allocated in each of these reinforcement in the same ratio of their respective original forces arrived in the flexural analysis. This procedure will maintain the same lever arm arrived during the flexural analysis. However, additional longitudinal reinforcement can be provided to take care of this longitudinal force arising out of shear. As this reinforcement is located below the prestressing steel, the lever arm will be lowered. This increase in lever arm can be omitted in the analysis. Conservatively the prestressing steel only can be utilized for flexural and shear analysis.

Design Methodology of Arriving at the Shear Stirrups

In a design problem θ is to be assumed between 45º and 21.8º. The assumed strut angle will have to satisfy the shear capacity as per equation 10.8. i.e. the strut capacity should be equal to or more than the applied shear. If the concrete strut capacity is satisfied then, the assumed angle will be substituted in equation 10.7 and the stirrup requirement will be evaluated. If the assumption of maximum angle (θ = 45º) is not able to resist the applied shear, then the section needs to be revised. At lower θ value, the capacity of strut will be low and the stirrup requirement will also be low. For the higher side of θ, the strut capacity will be high and the shear stirrup requirement also will be high. For a given problem θ, the strut angle can be worked out as follows:

Take α_{cw} = 1.concrete less than 60 MPa.

$$f_{cd} = \frac{0.67\, f_{ck}}{\gamma_m} = \frac{0.67\, f_{ck}}{1.5} = 0.45\, f_{ck}$$

At θ = 45º Maximum permissible shear stress

$$= \frac{\alpha_{cw}.v_1.f_{cd}}{\cot\theta + \tan\theta} = \frac{1\,.v_1.0.45\, f_{ck}}{2} = 0.225\, v_1 f_{ck} \text{- (1)}$$

At θ = 21.8º Max. Permissible shear stress = $\frac{1\,.v_1.0.45\, f_{ck}}{(2.5+0.4)} = 0.155\, v_1 f_{ck}$- (2)

Shear stress due to applied shear force

$v_{Ed} = \frac{V}{b_w Z}$ should not be grater than $\frac{\alpha_{cw}.v_1.f_{cd}}{(\cot\theta + \tan\theta)}$- (3)

Applied shear stress should be less than maximum allowable shear stress

$$v_{Ed} = \frac{v_1\, 0.45\, f_{ck}}{(\cot\theta + \tan\theta)} = \frac{0.45\, v_1 f_{ck}}{(\cot\theta + \tan\theta)} \text{- (4)}$$

$$\cot\theta + \tan\theta = \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = \frac{\frac{1}{(\sin 2\theta)}}{2} = \frac{1}{0.5\sin 2\theta} \text{ - (5)}$$

$$(\cot\theta + \tan\theta) = \frac{1}{0.5\, sin2\,\theta}$$

$$\frac{1}{(\cot\theta + \tan\theta)} = 0.5\sin 2\,\theta \text{ - (6)}$$

Substituting Equation 6 in Equation – 4

$$v_{Ed} = 0.45\, \upsilon_1\, f_{ck}\, 0.5 \sin 2\,\theta = 0.225\, f_{ck}\, \upsilon_1 \sin 2\theta$$

$$\sin 2\theta = \left[\frac{V_{Ed}}{0.225\, v_1 f_{ck}}\right]$$

If $v_{Ed} = 0.225\, \upsilon_1\, f_{ck}$. then θ = 45º

$$\theta = 0.5 \sin^{-1}\left[\frac{V_{Ed}}{0.225\, v_1 f_{ck}}\right] = 0.5 \sin^{-1}\left[\frac{Appliedshearstress}{0.225\, v_1 f_{ck}}\right]$$

θ is the angle of strut for the given problem. The design procedure can be presented in the form of a flowchart for carrying out the shear design which is given below.

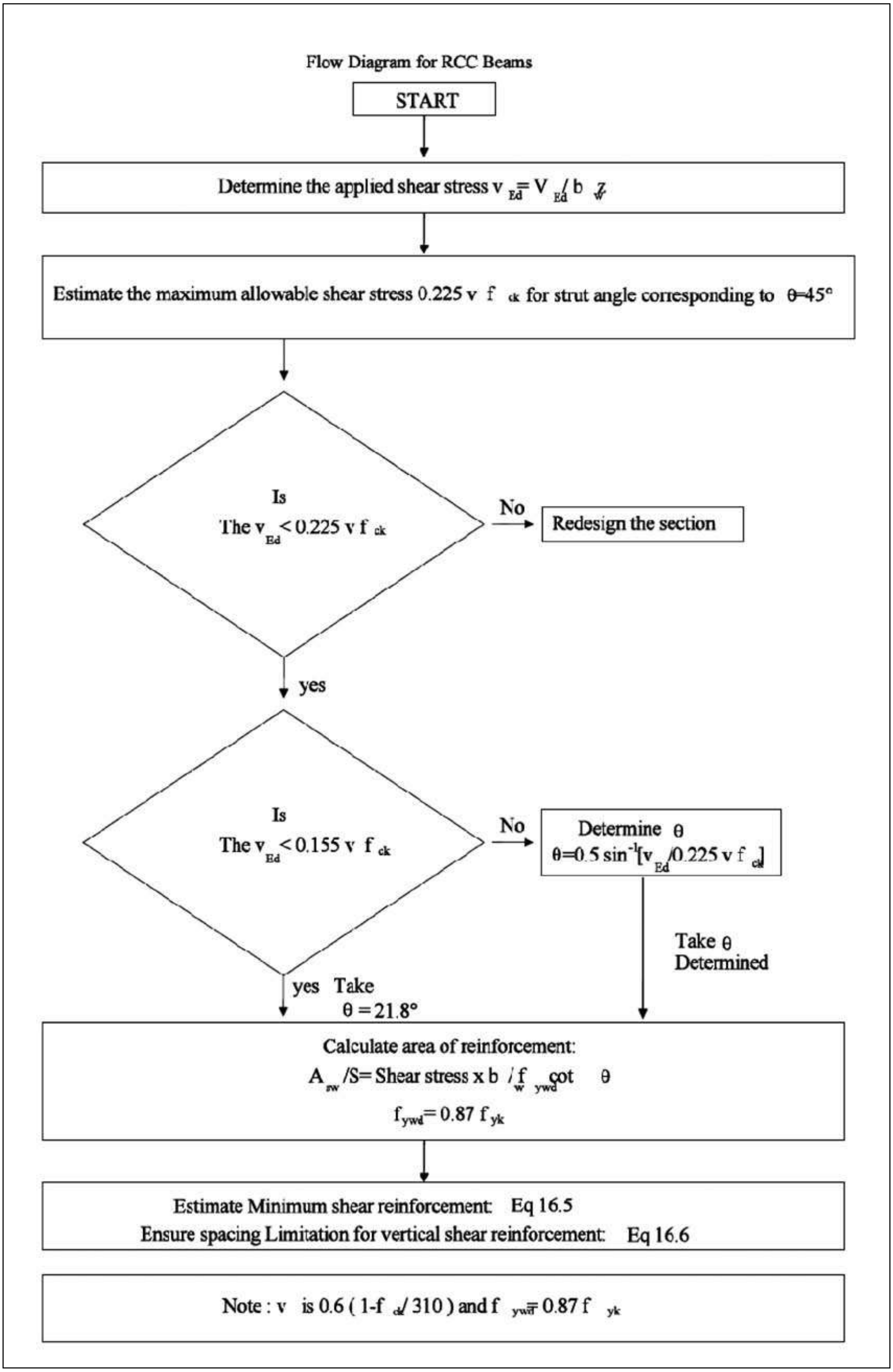


Fig. 10.9 Flow Diagram for Design Shear Reinforcement in RCC Beam

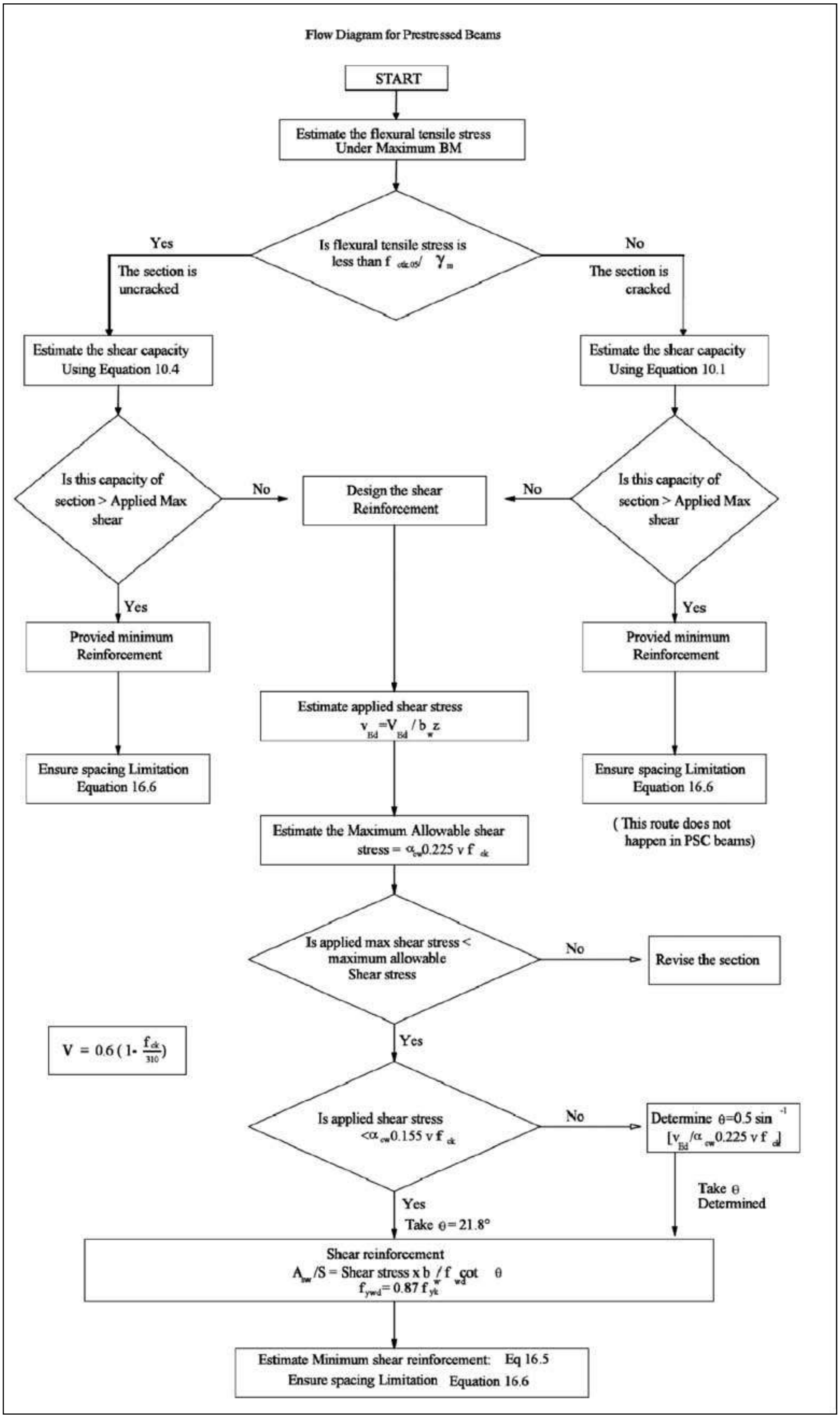


Fig. 10.10 Flow Diagram for Design shear Reinforcement Prestressed

8.3.2.2 Cl. 10.3.3.3(7) & (8)

When the loads are placed closer to the support, the part of the load will be carried to the support directly by the arch action and part by truss action, involving bending and shear. Closer the load to the support, greater is the portion of load which will be carried to the support directly. Hence a reduction factor is given in the Code to account for the reduction of this load for the direct transmission of shear while designing the shear reinforcement. In case of bridge superstructures, as the loads are moving loads, this provision may not be much useful. However this reduction is not applicable for verifying the concrete capacity as per Eq. 10.5. This clause is really applicable for single static load as explained before. While verifying the reinforcement requirement for resisting shear, only the shear reinforcement within central $0.75a_v$ shall be considered. This limitation is made because tests carried out indicated that the links adjacent to both load and support do not fully yield. The procedure of considering only the links between the load and the support works only for single loads. Where the structure is subjected to series of loads beyond '2d' and contribute to the shear, then the shear design for these loads should be carried out as outlined in earlier clauses for loads beyond '2d'. The link requirement from both the analysis should be added and provided. Similarly shear should be added from both the systems for checking the crushing. As pointed out earlier this reduction factor is not applicable while checking the capacity of strut (concrete crushing). For a_v less '0.5d', then a_v shall be taken as '0.5d' as minimum.

Following procedure may be adopted for sections within '2d':

a) Choose sections between '0.5d' and '2d' starting from 0.5d, 'd' and '1.5d'.

b) For explanation purpose it is assumed that there are three concentrated loads viz w1, w2 and w3 occurring between '0.5d' and '2d', at distances of av_1, av_2 and av_3 at '0.5d', 'd' and '1.7d', respectively from the support. Reduction factors to be used for these loads are β_1 = (0.5d/2d = 0.25); β_2 (d/2d = 0.5) and β_3 = (1.7d/2d = 0.85), respectively. The reduction factor is not applicable if loads are not acting from upper side of member, like suspended loads. The term "load acting from upper side" the intended load is one causing an additive support shear. A relieving load shall be considered as a suspended load.

c) Place w_1 at a_{v1} and arrive at the shear at support and the reinforcement between support and av_1 using Eq. 10.17. This reinforcement is to be provided within a distance of 0.75 a_{v1} between the load and the support. For convenience if the reinforcement is divided by 0.75, then the total reinforcement in the zone a_{v1} can be obtained.

d) Reduction factor shall be used to reduce the shear reinforcement. Let this be A_{sw1}.

e) Next place w_2 at a_{v2}. Repeat steps c and d and arrive at the reinforcement to be provided within a distance of av_2. Let this be A_{sw2}.

f) Similarly keep load w3 at a_{v3} and repeat the calculation. Reinforcement to be provided within a distance av3 can be worked out. Let it be A_{sw3}. Effects for each of the loads are to be evaluated separately.

g) Consider loads beyond '2d' and arrive at the reinforcement using support shear to be provided in zone av_1 assuming in equation 10.7. No reduction factors are to be used to arrive at the shear reinforcement due to these loads. Let this reinforcement be Asw4.

$$A_{sw4} = \frac{Shear\ force}{f_{ywd}}\left(\frac{av_1}{z}\right)$$

The dead load shear reinforcement can be added to these live load shears.

$$A_{sw5} = \frac{DL\ shear\ force}{f_{ywd}}\left(\frac{a_{v1}}{z}\right)$$

h) Add the shear reinforcements arrived in steps d, e, f and g as shown below. Arrive at the reinforcement to be provided in zone a_{v1}. In zone a_{v1}, the reinforcement will be

$$A_{sw1} + \frac{A_{sw2}}{av_2} x\ av_1 + \frac{A_{sw3}}{av_3} x\ av_1 + A_{sw4} + A_{sw5}.$$

i) Repeat the above calculations for other sections between '0.5d' and '2d'by placing the leading concentrated load at the chosen section viz'd', '1.5d' and '2d' (for reinforcements between '0.5d' and '2d') and arrive at the reinforcement. If '2d' distance is not significant, reinforcement provided from the support within'0.5d' can be continued upto '2d' to make it simple. When the process is repeated for section 'd' and reinforcement arrived, the reinforcement arrived from support to '0.5d' would be governed by earlier calculation as the shear at section would be severe by earlier placement of load. By placing the load at d the analysis would cover the region of between 0.5d and d. Further analysis can cover sections between d and 1.5d and 1.5d to 2d left.

j) The shear force also shall be arrived at all these sections without using reduction factor and the capacity of concrete shall be checked against Eq. 10.5 of the code.

However, the problem will arise as this shear has to be combined with the shear due to loads placed beyond 2d. For checking the crushing of strut due to loads between support and 2d and beyond 2d the following procedure may be adopted.

1) Estimate the shear force V_{ED1} due to loads placed within 2d without applying the reduction factor β and the shear stress $\frac{V_{ED1}}{b_w d}$.
2) Estimate the shear force V_{ED2} due to loads placed beyond 2d and the shear stress $\frac{V_{ED2}}{b_w z}$.
3) Estimate the shear force V_{EDL} due to dead load = $\frac{V_{EDL}}{b_w z}$.
4) Add the shear stress arrived in step 1, 2 and 3.

Combined shear stress = $\frac{V_{ED1}}{b_w d} + \frac{V_{ED2}}{b_w z} + \frac{V_{EDL}}{b_w z}$ which should be less than 0.5v f_{cd}.

In case if one wants to simplify the calculation for checking the crushing of concrete calculate the total shear force and calculate the allowable shear force by adopting 0.5 $b_w z$ v f_{cd} by substituting the d, by z in Eq. 10.5 and compare. This will simplify the elaborate calculation. It shall be kept in mind that for loads beyond 2d, the angle θ has been assumed as 45º. Under this condition only both struts effects can be superposed. Hence while arriving at shear reinforcement for loads beyond 2d in using equation 10.7, θ to be taken as 45º.

k) For sections beyond '2d', Eq. 10.7 and 10.8 of the Code, shall be used and the sections shall be designed.

l) If there is a reduction in cross section anywhere within the structure, the reduced section shall also be checked.

The method of transmission load is explained in the following diagram.

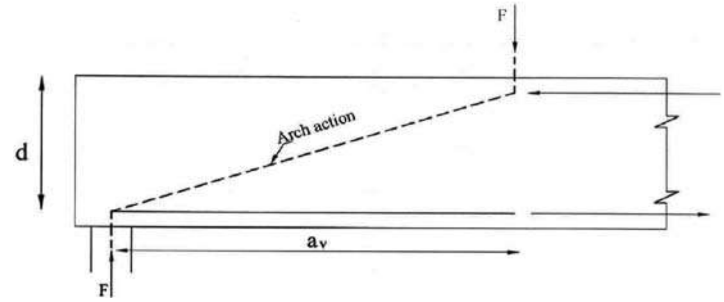


Fig. C10.11 (a) Force Transfer by "Direct Arch Action"

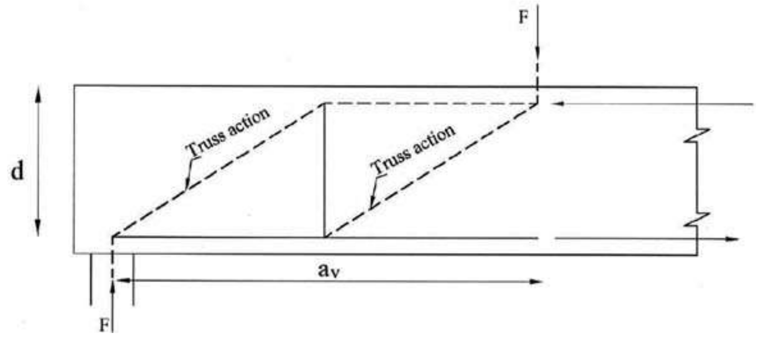


Fig. C10.11 (b) Force Transfer by "Truss Action"

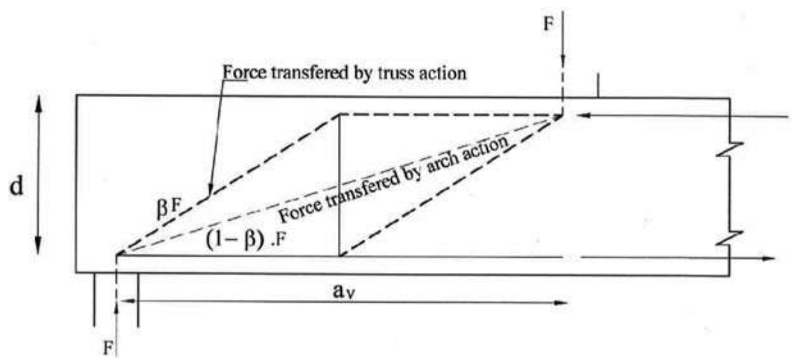


Fig. C10.11 (c) Transfer of Forces by Combined "Direct Arch Action" and "Truss Action"

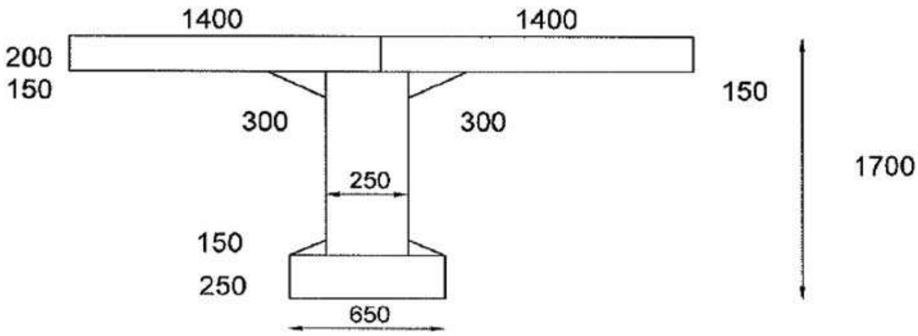
Worked Example 10.3-2

Elements Requiring shear reinforcement:

Determine the shear reinforcement in RCC Beam at Various sections. The particulars of the beam are as follows:

RCC girder M35 Grade concrete Span = 21.0M Girder cross section as shown.

Cross section of girder



Summary of Ultimate Shear Force (Value in kN)

Section Distance from Support	1M	2M	5M	6.71M	10.0M
Shear Force in kN	1234	1020	856	759	544
Width of Web in MM	650	250	250	250	250

Now Proceed to Flow Chart

Shear stress in N/mm^2 = $\frac{V_w}{0.9\,dxb}$ d = 1570 mm b = 250 mm	1.34	2.88	2.42	2.15	1.54

Max. Allowable shear stress 0.225 x 0.6 $\left[1-\frac{35}{310}\right]$ x 35 = 4.19 N/mm^2

As the shear stresses at the various sections are less than 4.19 N/mm^2 no redesign is required.

Allowable shear stress corresponding θ = 21.8º = 0.155 x 0.6 $\left[1-\frac{35}{310}\right]$ x 35 = 2.89 N/mm^2

As the shear stress at various sections are less than 2.89 N/mm^2

θ to be taken as 21.8º and Cot θ = 2.50

Working out of shear reinforcement:

$\frac{As_w}{s}=\frac{Shearstressxb_w}{f_{ywd}\cot\theta}$ $f_{ywd} = 0.87f_{yk}$	1M	2M	5M	6.71M	10M

Using f_{yk} 415 $\frac{As_w}{s} = \frac{Shearstressxb_w}{0.87\ x\ 415\ x\ 2.5}$	0.97 mm^2/ mm	0.8 mm^2/ mm	0.97mm^2/ mm	0.60 mm^3/ mm	0.43 mm^3/ mm
Providing reinforcement Reinforcement mm^2/m (2 legged)	12 @ 200 1.13 mm^2/ mm	12 @200 1.13 mm^2/ mm	12 @ 250 0.90 mm^2/ mm	10 @ 220 0.71 mm^2/ mm	10 @ 300 0.52 mm^2/ mm

The minimum reinforcement ratio as per equation 16.9

$$\frac{As_w}{b_w s} = \frac{.072\sqrt{f_{ck}}}{f_{yk}}$$

$$\frac{As_w}{b_w s} = \frac{.072\sqrt{f_{ck}}}{f_{yk}} \therefore \frac{As_w}{s} = 1.026\ x\ 10^{-3} x b_w$$

For b_w 650mm$\frac{As_w}{s} = 1.026\ x\ 10^{-3} x\ 650 = 0.667mm^2/mm$

For b_w 250mm$\frac{As_w}{s} = 1.026\ x\ 10^{-3} x\ 250 = 0.250mm^2/mm$

Design reinforcement is much higher.

Suppose at section 2M the shear force is made 1.40 times of the shear force worked out then the shear stress works out to 4.03 N/mm^2 > 2.8925 N/m^2 but less than 4.19 N/mm^2

Hence no redesign of section is required:

As the shear stress is more than 2.89 N/mm^2 the θ value has to be worked out.

$$.\theta = 0.5\ sin^{-1}\left[\frac{4.03}{4.19}\right] = 37.06^0$$

Cross checking from equation 10.8

$\therefore$ 4.03 = $\frac{1\ x\ 0.532\ x\ 0.45\ x\ 35}{\cot 37.06 + \tan 37.06} = \frac{1x0.532x0.45x35}{2.08} = 4.03$

LHS = RHS

Hence this proves that the formula shown for estimating the angle can be used.

$\therefore \frac{As_w}{s} = \frac{4.03\ x\ 250}{0.87\ x\ 415\ x \cot 37.06} = 2.107mm^2\ /mm$

The addition longitudinal tensile steel required at various sections for the original calculation.

Using the Eq. 10.16,

$\Delta F_{td} = 0.5\ V_{ED}\ (\cot\theta - \cot\alpha), \theta = 21.8°,\ \alpha = 90$

$$\Delta F_{td} = 0.5\ V_{ED}(2.5 - 0) = 1.25\ V_{ED} A_{st} required = \frac{1.25\ V_{ED}}{415/1.15} = 3.46\ x\ 10^{-3} V_{ED}$$

Section	1M	2M	5M	6.71M	10M
Addition steel mm^2	4.269	3.529	2.961	2.626	1.882

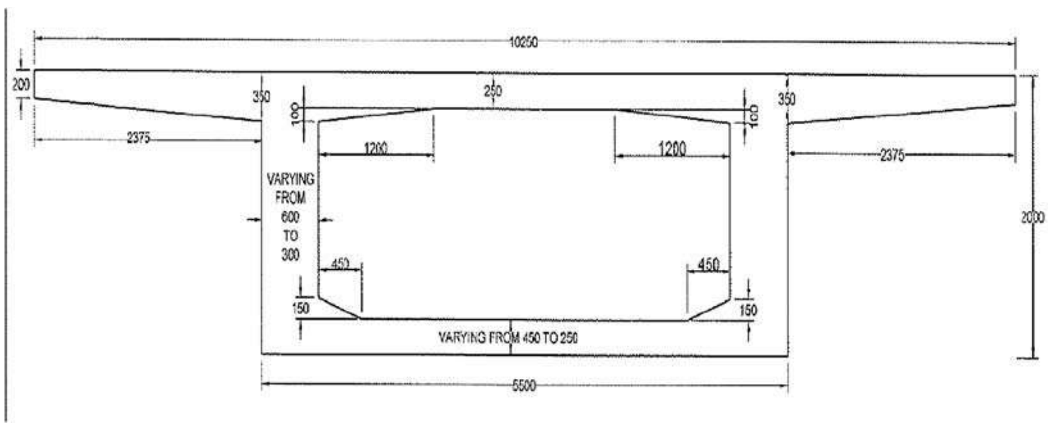
Additional longitudinal steel over and above that required to resist the moment has to be provided. It is to be noted that as θ becomes shallower, the longitudinal steel increases but the shear steel reduces. If θ increases, the additional longitudinal steel will reduce and shear steel will increase. Alternatively shift rule may be followed instead of providing additional longitudinal reinforcement.

Worked Example 10.3-3

Example of Prestressed Box Design for Shear:

Span of box 31.00 M, Grade of Concrete M40, Loading class 70R or Two lane of Class A. Prestressing, 10 Cables of 19 T 13 and 2 Cable, of 12 T 13. Cross section of box shown below.

Box Section



Cross Section of the Box Girder

The following are the Design Parameters: **Allowable tensile stress**

$$f_{ctd} = \frac{f_{ctd.05}}{1.5} = \frac{2.1}{1.5} = 1.4\text{MPa}$$

Section	1 – 1 Support	2.2 0.9 D	3.3 L/8	4.4 L/4	5.5 3L/8	6.6 L/2
Distance in mm from support	0	1.8	3.88	7.75	11.63	15.5
Cross sectional area of box m^2	6.90	6.90	6.90	5.38	5.14	5.14
CG of Section form bottom in m	1.09	1.09	1.09	1.2	1.22	1.22
Cable force after all losses in kN	20784	21040	21400	21838	22136	22192
Cable eccentricity from CG of section in m	0.32	0.5	0.62	0.83	0.9	0.91
Average compressive stress P/A in kN/m^2	3012	3049	3101	4059	4307	4318
Z_t m^3	4.16	4.16	4.16	3.99	3.96	3.96
Z_b m^3	3.5	3.5	3.5	2.68	2.52	2.52

$\frac{P}{A}+\frac{Pe}{Z_t}$ Top fiber stress due to prestress in MPa	1.41	0.52	-0.09	-0.48	-0.72	-0.78
$\frac{P}{A}+\frac{Pe}{Z_b}$ Bottom fiber stress due to prestress in MPa	4.91	6.05	6.89	10.82	12.21	12.33
I = Moment of Inertia in m^4	3.80	3.80	3.80	3.21	3.08	3.08
Ultimate shear force in kN	5640	5000	4100	3150	1750	620
vertical component of prestress force in kN	1825	1472	898	365	0	0
Net shear force in kN	3815	3528	3202	2785	1750	620
Ultimate moment in kNm	0	9400	18606	31720	38700	41147
Stress at bottom due to moment kN/m^2 In MPa	0	2685 2.69	5316 5.32	11836 11.84	15357 15.4	16328 16.32
Resultant Stress due to prestress effect and applied moment in MPa	4.91	3.36	1.57	-1.02	-3.19	-3.99
Comparing with allowable tensile stress of 1.4 MPa	Uncracked	Uncracked	Uncracked	Uncracked	Cracked	Cracked
S = $A\bar{x}$ = moment of the area above CG about CG in m^3	2.415	2.415	2.415	1.95	1.88	1.88
Breadth of web b_w in mm	2(600-0.5 x 90) = 1110	1110	1110	2(338-0.5 x 90) 586	2 x 300 = 600	600
$\frac{I\,b_w}{S} = m^2$	1.746	1.746	1.746	0.964	0.983	0.983
$\sqrt{f_{ctd}^2+\sigma_{cp}f_{ctd}}$ in kN/m^2. Note: σ_{cp} axial stress due to prestress at CG	2485	2495	2590	2764		
Shear capacity $\frac{Ib_w}{S}\sqrt{f_{ctd}^2+\sigma_{cp}f_{ctd}}$ in kN	4338	4356	4382	2664 Applied shear is greater. Hence section is cracked.	Formula not applicable section is cracked.	
Shear reinforcement	Not required Min to be Provided	Not required Min to be Provided	Not required Min to be Provided	Required		

In case of section not cracked but shear capacity is less than the applied shear adequate shear reinforcement needs to be provided which is to be based on Eq. 10.8.

If the section is cracked also shear reinforcement needs to be provided as the capacity of section to resist the shear without shear reinforcement will be virtually negligible due to absence of any appreciable amount of tensile reinforcement.

To calculate the shear reinforcement the most important parameter required is lever arm Z which is obtained from bending analysis.

Analyzing Section 4 – 4

Assuming the stress in the cable corresponding to the yield strain (Assumption steel yields)

Force in 10 cables of 19T13 = 0.87 x 3492 x 10 = 30380 kN

02 cables of 12T13 = 0.87 x 2205 x 2 = 3837 kN

Total Tensile force = 34217 kN

To balance this tensile force, the neutral axis will occur at 0.18 M for top

Lever arm = 0.80 + 0.83 - .09 = 1.54 M. Note: all cables are provided in one row.

Moment this force can resist = 34217 x 1.54 = 52694 kNm > 31720 kNm.

Z = 1.6 M for ease of calculations

Mean compressive stress at section 4 – 4 4059 kN/m^2.

Max. allowable stress

$$\frac{0.67\,x\,40000}{1.5} = 17866\,KN/m^2, \alpha_{cw} = \left(1+\frac{4059}{17866}\right) = 1.227\ \alpha_{cw} = 1.23\ \theta = 45^0$$

Capacity at θ = 45º = 1.23 x 0.118 x 40000 x 2 x 0.293 x 1.6 = 5443 kN and with θ = 21.8 capacity = 3736 kN. Actual shear force at section 4-4 is 2785 kN

Shear stress $= \frac{2785}{2\,x\,0.293\,x\,1.6} = 2970\,\frac{kN}{m^2}$: Taking cot θ = 2.5, min$\frac{As_w}{s} = \frac{2.97\,x\,293}{0.87\,x\,415\,x\,2.5} =$

0.964 mm^2/mm,10 @ 150 will give = 1.048 mm^2/mm

For section 5.5 mean compressive stress = 4307 kN/m^2

Allowable compressive stress = f_{cd} = 17866 kN/m^2 $(\frac{0.67}{1.5}\,x\,40000)$

$$\sigma_{cp} < 0.25\,f_{cd}, \alpha_{cm} = (1+0.24) \approx 1.25$$

Max. allowable shear force = 1.25 x 0.118 x 40000 x 2 x 0.30 x 1.6 = 5664 kN if θ = 45º

And with θ = 21.8 = 1.25 x 0.081 x 40000 x 2 x 0.3 x 1.6 = 3888 kN

Shear force at the section = 1750 kN

Shear stress $= \frac{1750}{2\,x\,0.3\,x\,1.6} = 1823\,kN/m^2$

Area of shear reinforcement $= \frac{As_w}{s} = \frac{1.823\,x\,300}{0.87\,x\,415\,x\,2.5} = .0.66\,mm^2/mm$

Adopting 2L10 @ 200 mm 2L reinforcement provided is =

$$\frac{0.7854\,x\,2\,x\,100}{200} = 0.785\text{mm}^2/mm$$

Hence adopt 10 @ 200: in each web and provide same reinforcement in the next section 6 – 6 also.

At other sections provide minimum reinforcement

Minimum reinforcement = $\frac{As_w}{s} b_w = \frac{.072\sqrt{f_{ck}}}{f_{yk}} b_w = \frac{300x\,.072\sqrt{40}}{415} = 0.329mm^2/mm$

By examining the capacity at a point 350 mm below deck slab. 350 form top (below cantilever)

Prestress effect and Moment effect are shown below.

Section	1 – 1	2.2	3.3	4.4	5.5	6.6
Effect of prestress in N/mm^2 at the above point	2.02	1.48	1.13	1.79	1.54	1.51
Moment effect at the above point	0	+1.38	+2.74	+4.44	5.40	5.75
Total effect at the above point	2.02	2.86	2.87	5.93	6.94	7.86
1st Moment of area of Deck and Haunch about CG	1.91	1.91	1.91	1.73	1.70	1.70
$\frac{I\,b_w}{s}$	2.20	2.20	2.20	1.086	1.087	1.087
$\sqrt{f_{ctd}^2 + \sigma_{cp} f_{ctd}}$	$\sqrt{1400^2 + 1400}$ = 2188	2442	2716	3203	3417	3274
$\frac{Ib_w}{s}\sqrt{f_{ctd}^2 + \sigma_p f_{ctd}}$	4813 kN	5372	5975	3478	Not applicable	

The section has more shear resisting capacity than at CG of section. Hence the designer has to check at other location on the cross section if required in case if there is a doubt that capacity may work out less than the capacity at the CG of section.

8.3.2.3 Diagonal stress fields for members having unbonded tendon Cl. 10.3.3.4

In case of pre-cast segmental construction where there is no bonded prestressing cable in the tension chord, the joints will open up after the decompression moment is reached at that section. The depth of opening will depend upon the depth of flexural compression block. The prestressing force should be assumed constant after the joint opening.

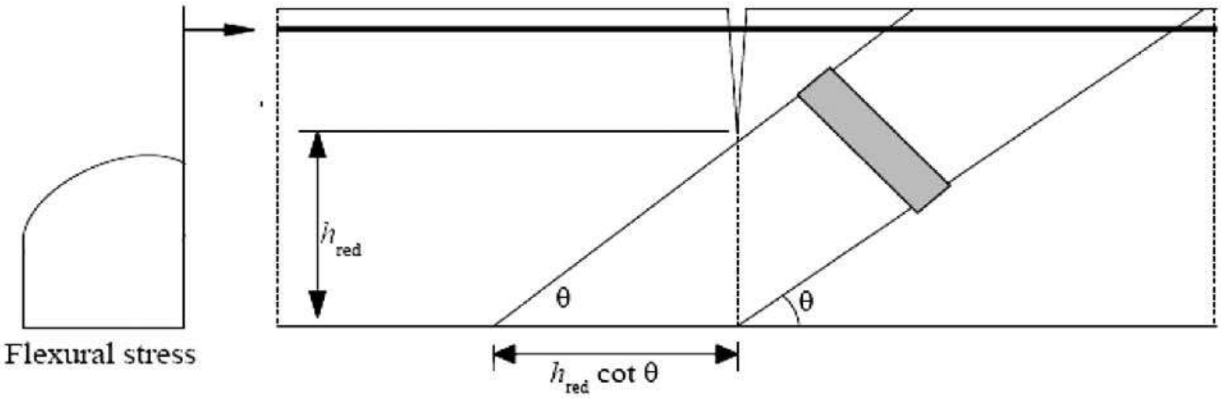


Fig. C10.12 Diagonal Stress Fields Across the Joint in the Web of Segmental Construction

Shear has to be balanced by the reduced depth. In order to avoid crushing of concrete it shall be ensured that, the compressive stress should be within allowable limit.

Taking equation 10.8 and sub-stituting z = h reduced and α_{cw} = 1.0

$$V_{ED} \leq V_{Rd,} V_{RD} = \frac{b_w . h_{reduced} . v . f_{cd}}{(\cot\theta + \tan\theta)} \;\; ; V_{Ed} = V_{NS}$$

$$h_{reduced} = \frac{V_{NS}}{b_w v f_{cd}} (\cot\theta + \tan\theta) \qquad \text{(Eq: 10.18)}$$

$h_{reduced}$ arrived form bending analysis shall be sub-stituted and θ shall be evaluated. If θ works out greater than 45º, then $h_{reduced}$ shall be increased by applying additional prestress. In case if 'θ' works out to be lesser than 45º, then the shear reinforcement can be worked out by substituting the 'θ' angle.

Taking equation 10.7

$$V_{Rds} = \frac{A_{sw}}{s} h_{reduced} f_{wd} \cot\theta$$

$$\frac{A_{sw}}{S} = \frac{V_{NS}}{h_{reduced}\, f_{wd} \cot\theta}$$

This reinforcement shall be provided within a distance h reduced cot θ adjacent to the joint but not greater than segment length. It shall be provided from both the edges of the joint. The opening up of joint shall be limited to 50 percent of the depth under the ultimate limit state check for flexure and shear. In case if the section opens up by more than 50 percent of the depth, the prestressing force shall be increased.

8.3.3 *Interface Shear* Cl. 10.3.4

When concrete elements are cast at different times, shear stress exists across the construction joint which is called as interface shear stress.

This shear stress has to be checked in order to ensure that elements act fully together.

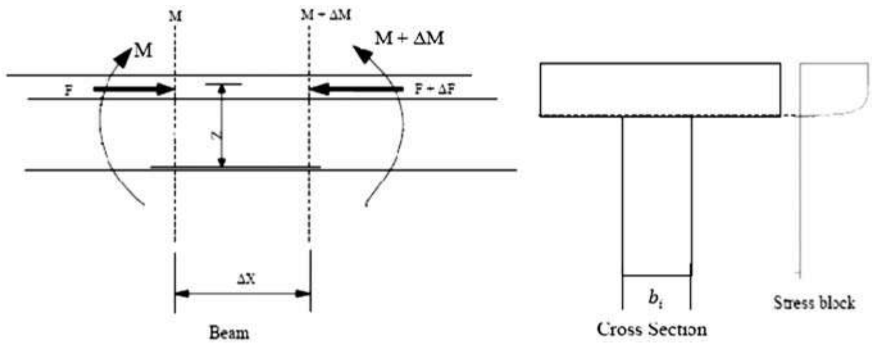


Fig. C10.13 Inter Face Shear between Web and Deck Slab Neutral Axis in Flange

The additional force ΔF has to be resisted by shear stress between the section across the construction joint:

$$v_{ED} = \frac{\Delta F}{b_i \Delta x}$$

$$\Delta F = \frac{\Delta M}{Z}$$

$$\therefore v_{ED} = \frac{\Delta M}{Z b_i \Delta x}$$

$\frac{\Delta M}{\Delta x}$ = Rate of Change of Bending Moment which is equal to shear force V_{ED}

Shear stress $v_{Ed} = \frac{V_{ED}}{Z b_i}$

Note: V_{Ed} is also V_{NS}, depending upon the case

This is true in case of neutral axis lies within the flange assuming the construction joint is at the top of web. In case if the neutral axis lies in the web the force diagram will be as follows:

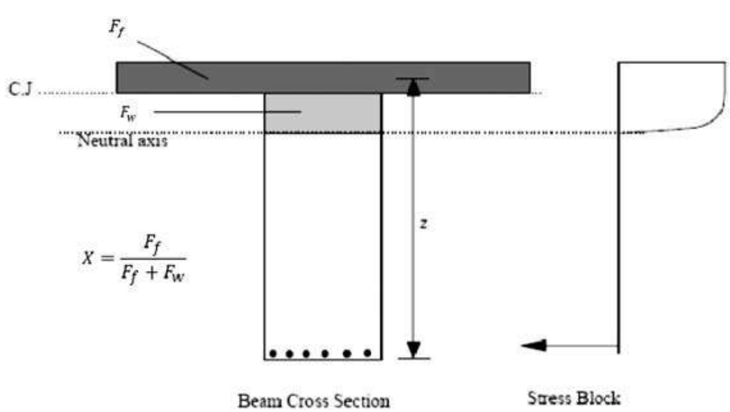


Fig. C10.14 Interface Shear: Neutral Axis in the Web

$$X = \frac{F_f}{F_f + F_w}$$

If the force in the flange is F_f and force in the web in F_w, the proportionate shear carried by flange is $X = \frac{F_f}{F_f + F_w}$ with respect to total shear. The corresponding shear stress = $XV_{Ed}/Z\ b_i$. If F_w = 0 then X = 1. As generally the force contribution by the web is small, it can be neglected. With the result X can be taken as 1.0. The shear force taken for checking the interface shear is the net shear at the section multiplied by ratio of longitudinal flexural force above construction joint to the total compressive or tensile longitudinal flexural force.

The lever arm used to compute the sectional resistance may be used to simplify the computation or it can be based on actual stress block for the particular loading case.

The first term of the equation 10.21 represents the frictional force across the inter face under the action of normal compressive force which is generally zero except in case of vertically prestressed sections and the second term corresponds to the mechanical resistance of reinforcement crossing the interface . The shear reinforcement provided in the section and continued across the interface having the adequate anchorage shall be considered for working out the reinforcement ratio ρ. In case if the additional reinforcement is required over and above the shear reinforcement, same shall be provided. The minimum reinforcement required to be provided across the horizontal interface shear will be 0.15 percent of interface area.

8.3.4 *shear in the flange portion of flanged beam and box section* Cl. 10.3.5

Reference shall be made to **Fig. 10.9** of the code. The longitudinal in plane force generated in the flange induces shear stress between at the flange and web junction which can be expressed as $\Delta F_d = v_{Ed} h_f \Delta x$

$$\therefore v_{Ed} = \frac{\Delta F_d}{h_f \Delta X}$$

ΔX is the length under consideration and ΔF_d unbalanced in-plane shear force in the flange at the interface section.

The maximum value of ΔX shall be assumed as the half the distance between the zero bending moment point and maximum bending moment section. When the structure is subjected to concentrated loads, the length shall not exceed the distance between concentrated loads. Alternatively, shear stress can be calculated using the formula from the previous section replacing b_i by h_f. The shear will be carried by web and also by flanges on either side of web. As we are interested in the shear transmitted by flange alone the following formula can be used for calculating the shear stress:

$$v_{Ed} = \frac{X.V_{ED}}{Zh_f} x \ \frac{b_{eff} \ of \ flange \ on \ one \ side}{b_{total \ effective \ width \ flange \ including \ web}}$$

where,

'X' is defined in **Clause 8.3.3** in case neutral axis is in web.

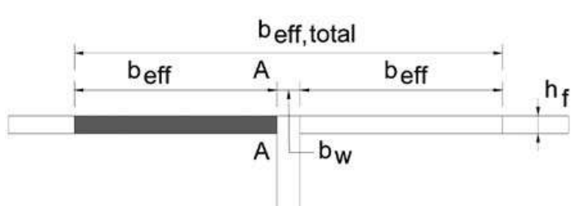


Fig. C10.15 Shear Check at Interface Showing the Section

h_f is the thickness of deck slab. In case if the deck slab is subjected to transverse bending as it happens in case of beam bridge and box girder bridge then the deck slab thickness shall be reduced by the depth of concrete in bending zone compression. Though the effects are occurring in two mutually perpendicular planes, in order to make the analysis simpler, the above simplification has been suggested.

The maximum compressive stress in the strut shall be checked in order to avoid the crushing of the compression struts.

$$v_{Ed} \leq vf_{cd} \sin \theta_f \cos \theta_f$$

for compression flanges $1.0 \leq \cot\theta_f \leq 2.0$ or $45° \geq \theta_f \geq 26.5°$

for tension flanges $1.0 \leq \cot\theta_f \leq 1.25$ or $45° \geq \theta_f \geq 38.60°$

The above formula can be derived as follows:

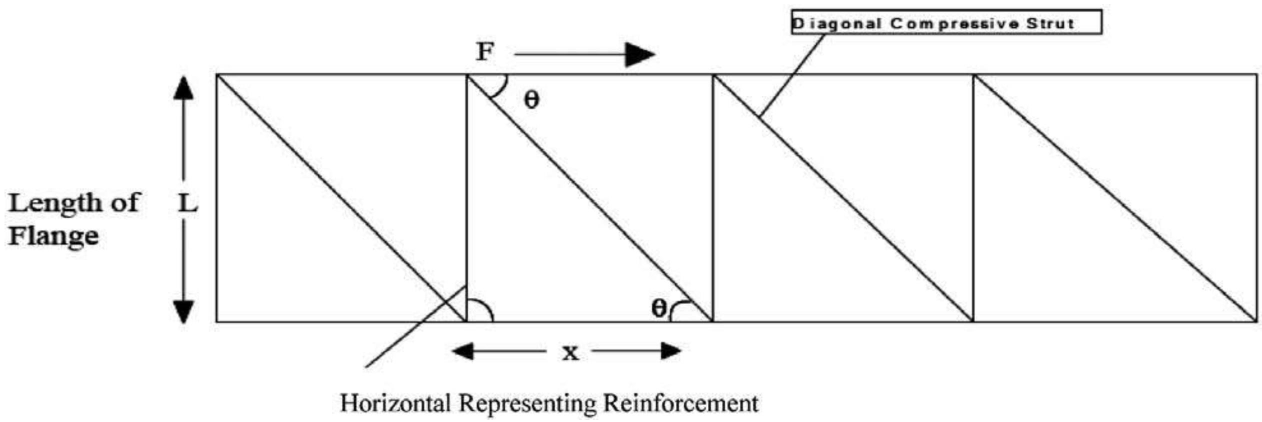


Fig. C10.16 Plan of Flange

Longitudinal shear force at the interface = F

$x = L \cot \theta$ $F = v_{Ed} \times h_f$

$F = v_{Ed} L \cot \theta h_f$ where v_{Ed} is the shear stress:

Force along the diagonal $\frac{F}{\cos\theta} = \frac{v_{Ed} L \cot\theta h_f}{\cos\theta} = \frac{v_{Ed} L h_f}{\sin\theta} - (1)$

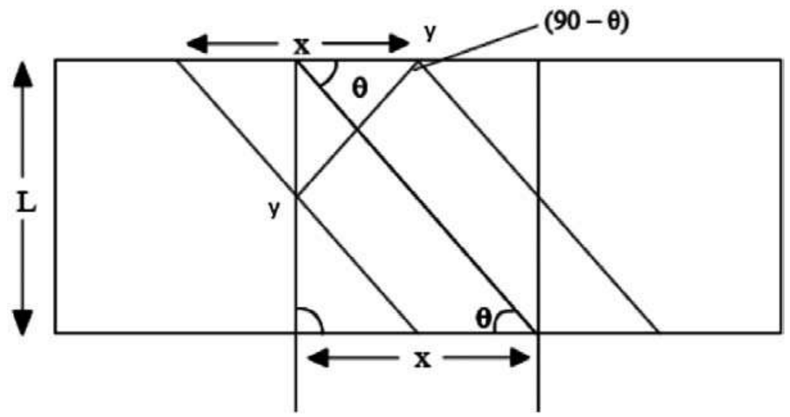


Fig. C10.17 Force in the Compression Strut of Flange

The diagonal compressive strut represents the compressive force in one panel as shown.

Length of the plane Y-Y over which the compressive force is acting = x cos (90 – θ)

Force this plane can support

$= vf_{cd}$ x cos (90 - θ) h_f

Substituting for x

$= vf_{cd}$ L cotθ cos(90-θ) $h_f = vf_{cd}$ L cosθ h_f - (2)

Equating (1) and (2)

$$\frac{v_{Ed} L h_f}{\sin\theta} = v f_{cd} L \cos\theta \, h_f$$

$v_{Ed} = vf_{cd}$ cosθ sinθ

Assuming the reinforcement is spaced at a spacing of S_f Resolving the longitudinal shear along the reinforcement. The force along the reinforcement is F tan θ

Force per unit length = v_{Ed} x h_f x 1 x tan θ = $\frac{v_{Ed} h_f}{\cot\theta}$

The force has to be resisted by reinforcement

$$\frac{As_f f_{yd}}{s_f} = \frac{v_{Ed} h_f}{\cot\theta}$$

$$\frac{As_f}{s_f} = \frac{v_{Ed} h_f}{f_{yd} \cot\theta}$$

This equation is given in the code as 10.23

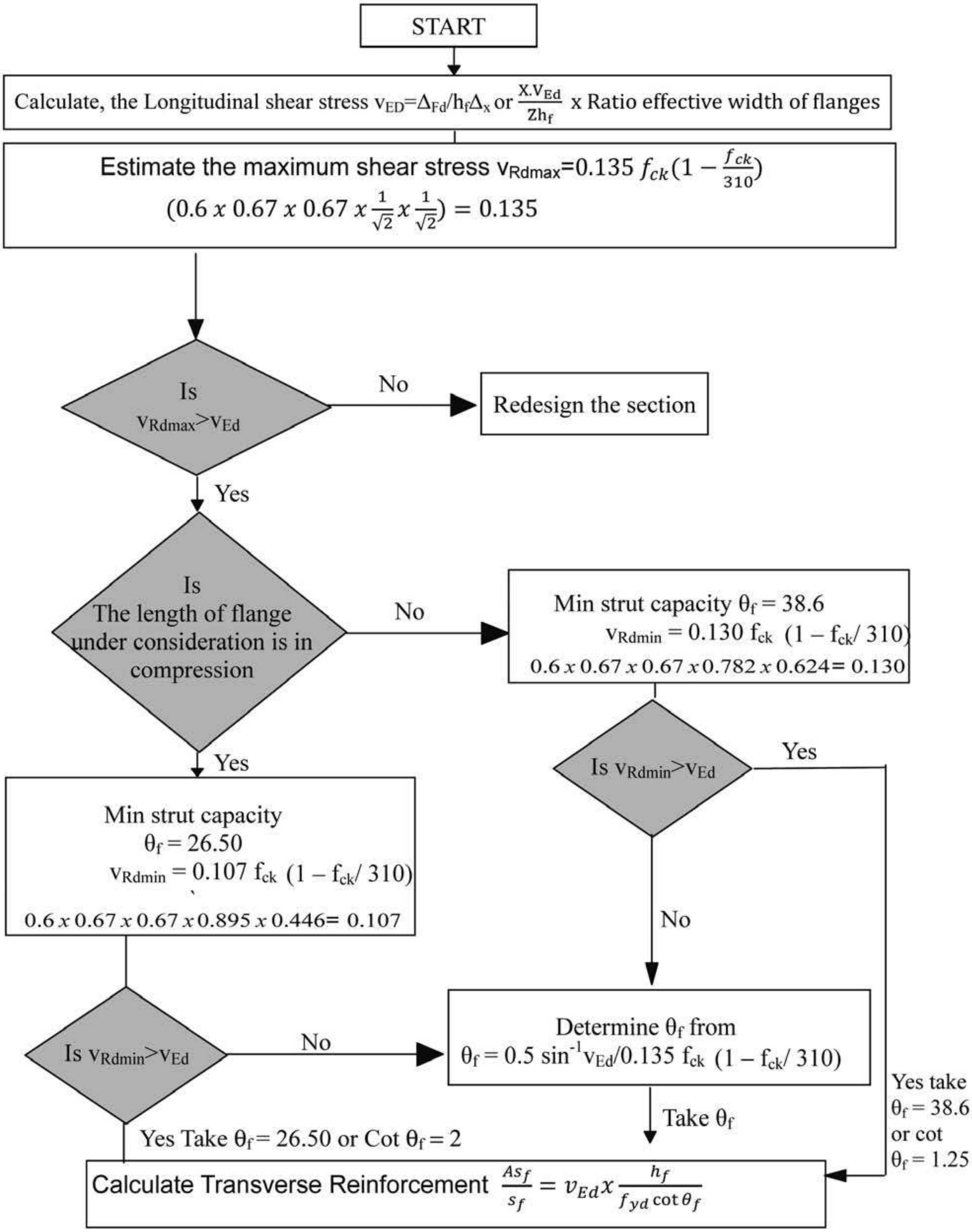
In case of combined shear between the flange and the web and transverse bending the area of steel should be greater of the following.

(1) $\frac{As_f}{s_f} = \frac{v_{Ed} h_f}{f_{yd} \cot\theta}$

(2) Half the above steel plus the steel required for transverse bending.

If v_{Ed} is less than 0.4 f_{cd} then (1) and (2) will not apply. Only flexural steel is sufficient.

The flow chart for the design check is presented in **Fig. C10.18** below.



Note: The term 'v_{Ed}' shall be taken as '$X.v_{Ed}$', in case the neutral axis lies in web.

Fig. C10.18 Flow Diagram for the Design of Flange of T Beam and Box Girders

8.4 Design for Punching Shear: Cl.10.4

8.4.1 *General* Cl.10.4

When a slab is subjected to localized concentrated force which acts over a small area the punching shear failure can occur. Common examples are, wheel load acting on deck slab, pile caps over piles open foundations supporting pier and well cap supporting the pier. Punching shear is resisted by the shear resistance of concrete through the depth of element over a perimeter. The section covers action of concentrated force over a small area on two dimensioned elements.

8.4.2 *Loaded Area and Basic Control Perimeter* Cl. 10.4.2

1) Punching shear stress should be evaluated at following perimeters:
   a) At the face of the loaded area
   b) At basic control perimeter
   c) At any other perimeter
2) The basic control perimeter U_1 to be chosen at a distance of '2d' form the face of the loaded area provided that the applied concentrated load is not opposed by any other upward pressure offering relief within this distance 'd' is the average of effective depth in both the direction i.e.

$$d = \left(\frac{d_x + d_y}{2}\right)$$

3) When the loaded area is situated near an edge or on the edge or at a Corner, the control perimeter shall be calculated based on the **Fig 10.11** of the Code.
4) The control perimeter should be chosen at a distance less than '2d' when the concentrated force on the loaded area is opposed by high pressure from soil (such as foundation) or by the effects of a load or reaction within distance of '2d'. The Code suggests that the relief offered by the opposing force should be minimized.

8.4.2.1 Punching shear design rules Cl. 10.4.2.1

Punching shear stress is to be calculated at two locations as stated above

a) At the face of the loaded area
b) Along the control perimeter

The reason for calculating at the face of the load area is to ensure that concrete strut does not get crushed. If the section is unable to salsify the shear capacity then one or more of the following actions are to be taken

i) Increase the depth of slab
ii) Increase the perimeter of the loaded area
iii) Increase the grade of concrete

The reason for checking at the control perimeter is to check whether the section can carry the load without of punching shear reinforcement.

8.4.3 *Checking of Punching Shear Stress at the Control Perimeter* Cl. 10.4.3.1

1) General

The punching shear stress generated on any control perimeter can be calculated using the following expression

$v_{Ed} = \frac{\beta V_{Ed}}{u_i d}$

V_{Ed} = Applied shear force

u_i = Control Perimeter

d = depth of element

β = Factor for Accounting Bending Moment

β = 1 for axial load with no bending moment

$\beta = 1 + k \left(\frac{M_{Ed}}{V_{Ed}}\right)\left(\frac{u_1}{W_1}\right)$ for axial load and bending moment

M_{Ed} is the ultimate moment and V_{Ed} is the ultimate shear force on the perimeter.

W_1 is the property which corresponds to a distribution of shear as shown in **Fig 10.12** of code and is a function of basic control perimeter u_1 and axis about which the moment acts. Theoretically, If the control perimeter changes or the axis about which the moment is acting changes, then the value of W_1 will have to be derived for the respective perimeter and direction. However, except for the foundation slabs, the value for W_1 and u_1, derived for the basic control perimeter can be used for any other perimeter. For foundation slabs, W_1 and u_1 shall be derived for each respective perimeters.

W_1 is defended as $W_1 = \int_0^{u_i} |e|\, dl$

dl is the length increment of the perimeter

"e" is the distance of *dl* from the axis about which the moment M_{Ed} acts. *k* is the constant depending the value of C_1 and C_2. C_1 is the dimension of the cross section perpendicular to the axis of bending and C_2 is the dimension parallel to the axis of bending.

β is a factor to take care of additional shear generated by moment. The magnitude of the additional shear force is dependent on the moment transferred and the distance of the perimeter from the loaded area and the shape of the loaded area. The enhancement of shear due to moment around the perimeter can be expressed as:

$$\beta \frac{V_{Ed}}{u_1} = \frac{V_{Ed}}{u_1} + \Delta V$$

ΔV is the additional shear due to moment distributed around the perimeter u_1.

$$\beta = 1 + \frac{\frac{\Delta V}{V_{Ed}}}{u_1} or\ 1 + \frac{\Delta V u_1}{V_{Ed}}$$

Referring to shear distribution around the perimeter **Fig. 10.2**

$$M_{Ed} = 4\,\Delta V\left[\frac{C_1}{2}x\frac{C_1}{4} + \frac{C_2}{2}\left(\frac{C_1}{2} + 2d\right) + \frac{2d\Pi}{2}\left(\frac{C_1}{2} + 2dx\frac{2}{\Pi}\right)\right]$$

$$M_{Ed} = \Delta V\left(\frac{C_1^2}{2} + C_1C_2 + 4C_2d + 16d^2 + 2\Pi dC_1\right) = W_1\Delta V$$

$$=\Delta V = \frac{M_{Ed}}{W_1}$$

Substituting in β $\beta = 1 + \frac{M_{Ed}u_1}{V_{Ed}W_1}$

Correcting the shear force aspect ratio of column

$$\beta = 1 + k\frac{M_{Ed}}{V_{Ed}}\frac{u_1}{W_1}$$

It is to be kept in mind that M_{Ed} is the unbalanced moment applied on column.

2) Derivation of for W_1 Internal Rectangular Column

The shear will be distributed along the periphery due to moment.

The control perimeter is ‘2d’ away from columns face.

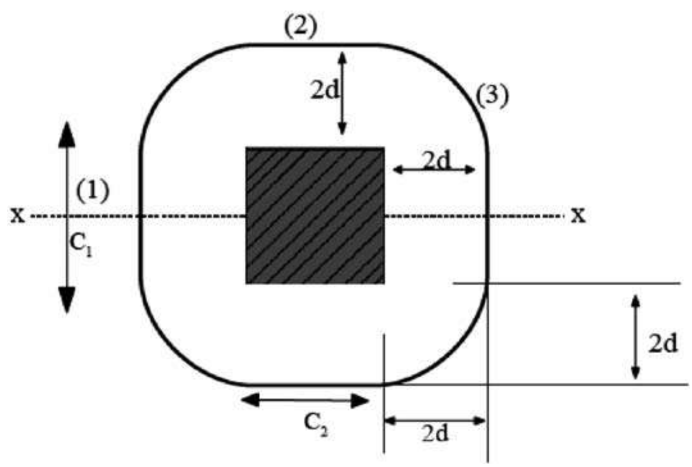


Fig. C10.19 Showing Control Perimeter

Taking moment about xx axis as the moment is acting about x – x axis.

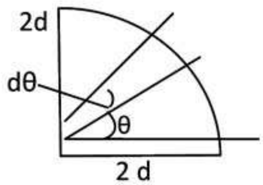
The moment contribution due to shear distributed on the length 1 for C_1

$= 4x\ \frac{C_1}{2}x\frac{C_1}{4} = \frac{C_1^{\ 2}}{2}$ for length C_1

The moment contribution due to shear distributed on the length 2 for C_2

$= 2\ x\ C_2\left(\frac{C_1}{2} + 2d\right) = \frac{2\,C_2C_1}{2} + 4C_2d = C_2C_1 + 4C_2d$

The moment contribution due to shear distributed on four quadrants on length 3



$$CG = \frac{\int 2dxd\theta d\ 2d\sin\theta}{\int 2dd\theta} = \frac{2d\int_o^{\Pi/2}\sin\theta\, d\theta}{\int_o^{\Pi/2} d\theta}$$

$$= \frac{2d\,[-\cos\theta]_o^{\Pi/2}}{[\theta]_o^{\Pi/2}} = \frac{2d}{\Pi/2} = \frac{2dx\ 1\ x\ 2}{\Pi} = \frac{4d}{\Pi}$$

= CG from axis = $\left(\frac{C_1}{2} + \frac{4d}{\Pi}\right)$

Length = $\frac{\Pi x\ 2d}{2} = \Pi d$

$= 4\left(\frac{C_1}{2} + \frac{4d}{\Pi}\right) x\Pi d = 4\Pi d\left[\frac{C_1}{2} + \frac{4d}{\Pi}\right]$

$= \frac{4\ \Pi dC_1}{2} + \frac{16\Pi d^2}{\Pi} = 2\Pi dC_1 + 16d^2$

$W_1 =$ (1) + (2) + (3) = $\frac{C_1^2}{2} + C_2C_1 + 4C_2d + 2\Pi dC_1 + 16d^2$

Incase if the shear is checked at the column face then $W_1 = \frac{C_1^2}{2} + C_2C_1$

For rectangular columns subjected to bi-axial moment

$$\beta = 1 + 1.8\sqrt{\left(\frac{e_y}{b_z}\right)^2 + \left(\frac{e_z}{b_y}\right)^2}$$

e_y and e_z are the eccentricities M_{Ed}/V_{Ed} along y and z axis respectively b_y and b_z are the dimensions of control parameter as in **Fig 10.10** of code.

It is to be understood that e_y results moment about Z axis and e_z moment about y axis

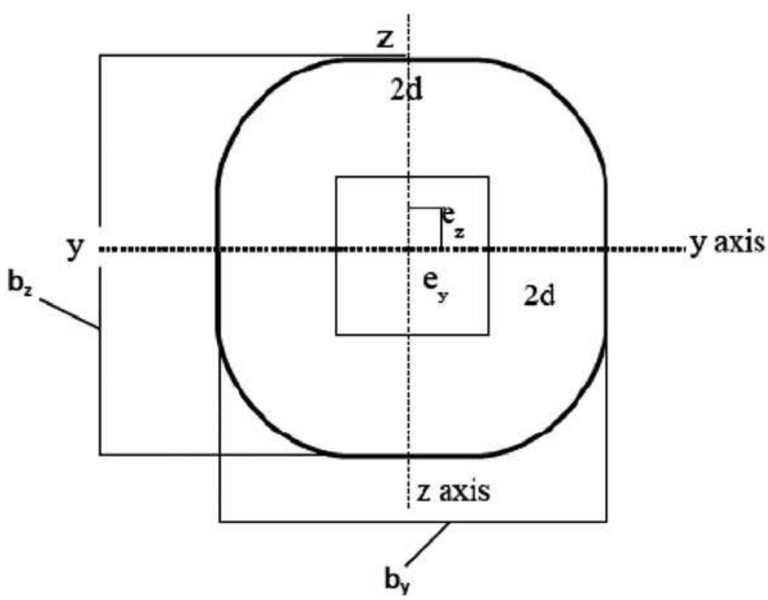


Fig. C10.20 Biaxial Eccentricity

3) For internal circular column: the value of β can be derived as follows

$$\beta = 1 + k\left(\frac{M_{Ed}}{V_{Ed}}\right)\left(\frac{u_1}{W_1}\right)$$

As $C_1 = C_2 \therefore C_1/C_2 = 1.0$ Hence $k = 0.6$, $\frac{M_{Ed}}{V_{Ed}} = e$

$$\beta = 1 + 0.6e\left(\frac{u_1}{W_1}\right)$$

Derivation W_1 for Circular column

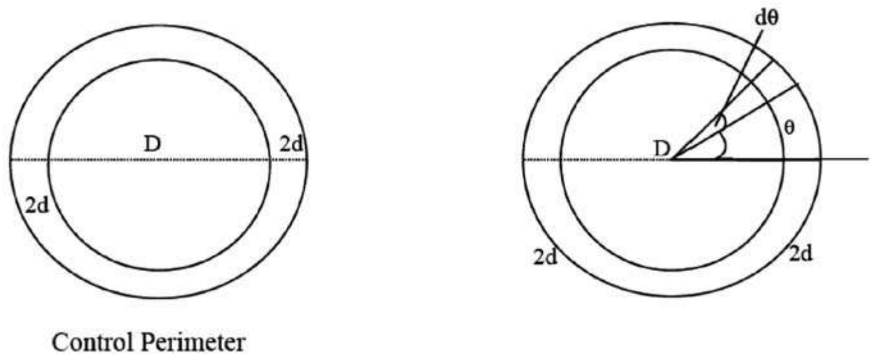


Fig. C10.21 Control Perimeter for Circular Column

$$u_1 = \Pi\,(D + 4d)$$

$$= W_1 = 2\int_0^{\pi} |e|dl$$

= Splitting into two halves.

$$CG = e = \frac{\int_0^{\pi}\left(\frac{D+4d}{2}\right)(2d + D/2)\sin\theta}{\int_0^{\pi}\left(\frac{D+4d}{2}\right)d\,\theta}$$

$$\frac{\left(\frac{D+4d}{2}\right)(2d + D/2)(-\cos\theta)_0^{\pi}}{\left(\frac{D+4d}{2}\right)\,[\theta]_0^{\pi}} = \frac{(2d + D/2)[+1+1]}{\pi}$$

$$CG = e = \frac{2(2d + D/2)}{\Pi}$$

$$W_1 = 2\left[\frac{\Pi(D+4d)}{2}\frac{2\left(2d + \frac{D}{2}\right)}{\Pi}\right] = 2\,(D+4d)(2d + \frac{D}{2})$$

$$\frac{u_1}{W_1} = \frac{\Pi(D+4d)}{2\,(D+4d)(2d + D/2)} = \frac{\Pi}{2(2d + D/2)} = \frac{\Pi}{(D+4d)}$$

$$\beta = 1 + 0.6e\frac{\Pi}{D+4d} = 1 + 0.6\Pi\left(\frac{e}{D+4d}\right)$$

4) For edge columns, when there is no eccentricity the basic control perimeter as shown shall be used.

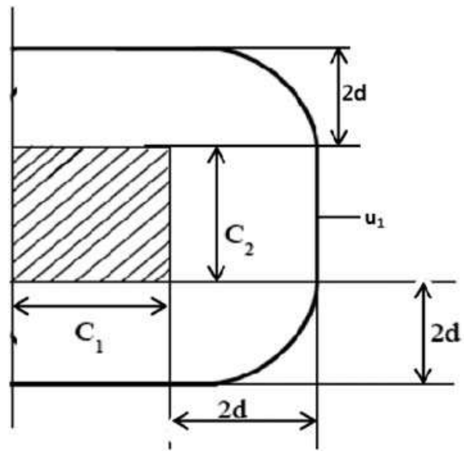


Fig. C10.22 Showing Basic Control Perimeter u_1

a) When there is no moment the punching shear stress $v_{Ed} = \frac{V_{Ed}}{u_1 d}$

where, $u_1 = 2C_1 + C_2 + 2\Pi d$.

b) When the edge column is subjected to a moment with respect to an axis parallel the slab edge (eccentricity perpendicular to slab edge) and is towards the interior and there is no eccentricity parallel to the edge. The punching shear stress can be estimate using $V_{Ed} = \frac{V_{Ed}}{u_2 d}$

where,

$$u_2 = Min of (C_2 + 3d + 2\Pi d) or \ (C_2 + C_1 + 2\Pi d)$$

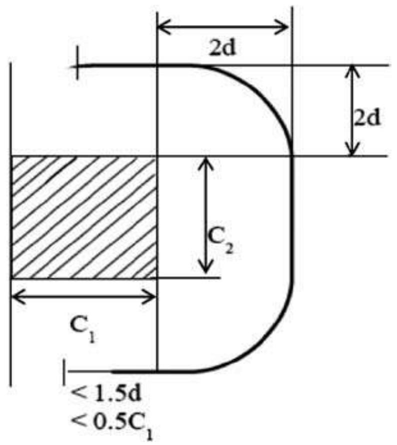


Fig. C10.23 Showing Reduced Control Perimeter u_2

c) When the edge column is subjected to moment about both axes and the eccentricity perpendicular to the slab edge towards interior, β shall be determined

$$\beta = \frac{u_1}{u_2} + k\frac{u_1}{W_1}e_{par} and V_{Ed} = \beta\frac{V_{Ed}}{u_1 d}$$

u_2 is the reduced basic control perimeter

k may be determined from table with C_1/C_2 replaced by $C_1/2C_2$

W_1 is the calculated for the basic control perimeter u_1

e_{par} = eccentricity parallel to slab edge causing moment about on axis per perpendicular to the slab edge. For rectangular columns

W_1 can be calculated as follows.

Derivation of W_1 for Edge Column

The shear will be distributed along the periphery due to moment and control perimeter is '2d' away from column face.

Taking moment about xx axis as the moment is acting about xx axis.

The moment contributed due to shear distributed on the length 1

$= 2\,x\,\frac{C_2}{2}\,x\,\frac{C_2}{4} = \frac{C_2^2}{4}$

The moment contributed due to shear distributed on the length 2

$= 2\,xC_1\left(\frac{C_2}{2} + 2d\right) = \; C_1C_2 + 4C_1d$

The moment concentrated due to shear distributed on the quadrants of length 3

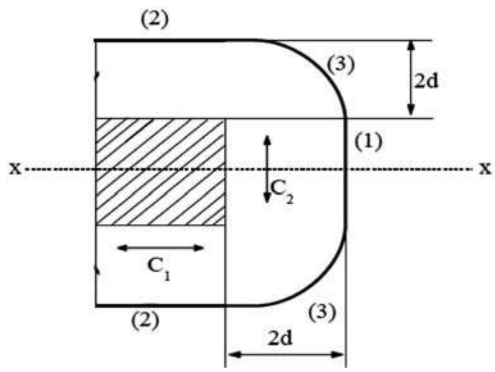


Fig. C10.24 Control Perimeter for Edge Column

50% of central column value $= \frac{16\,d^2}{2} + \frac{2\Pi dC_2}{2}$

$W_1 = (1) + (2) + (3) = \frac{C_2^2}{4} + C_1C_2 + 4C_1d + 8d^2 + \Pi dC_2$

d) If the eccentricity perpendicular to slab edge is towards exterior then the expression for β Eq. 10.25 of the Code is valid. However the value of W1 has to be worked out using the eccentricity emeasured from the centroid of the control perimeter.

5) For rectangular corner column, (**Fig. 10.11** of the Code) when there is no eccentricity

$$v_{Ed} = \frac{V_{Ed}}{u_1 d} wher\; eu_1 = (C_1 + C_2 + \Pi d)$$

For rectangular corner column, where the eccentricity is toward of the interior of the slab, it is assumed that the punching shear stress is uniformly distributed along the reduced control perimeter (**Fig. 10.13 (b)** of code).

$v_{Ed} = \frac{V_{Ed}}{u_2 d}$w Where u_2 = Min of (0.5 C_1 + 0.5 C_2 + ∏d) or (3d + ∏d)

In the above case if the eccentricity is towards exterior, then the expression for β Eq. 10.25 of the Code is valid

However the value of W_1 will have to be worked out. The eccentricity should be measured from the centroid of the control perimeter.

8.4.4 *Punching Shear Resistance of Slabs without Reinforcement* Cl. 10.4.4

Slabs without punching shear reinforcement, have to resist the punching shear stress purely by the tensile strength of concrete which is given by the following expression (Eq. 10.33 of the Code)

$$v_{Rdc} = (\frac{0.18}{\gamma_m} k\ (80\rho_1 f_{ck})^{\frac{1}{3}} + (0.1\sigma_{cp})) \geq (v_{min} + 0.1\sigma_{cp})$$

v_{Rdc} is in MPa., f_{ck} is in MPa.

$$k = 1\sqrt{\frac{200}{d}} \geq 2.0\ where\ d\ is\ the\ depth\ in\ millimeters$$

$$\rho_1 = \sqrt{\rho_{ly}\rho_{lz} \geq}\ .02\ v_{min} =\ .031\ k^{3/2} f_{ck}^{1/2}$$

ρ_{ly}, ρ_{lz} relate to the bonded tension steel in y and z directions. Taking a slab width of '3d' beyond the column face on each side the mean value of ρ_{ly} and ρ_{lz} shall be calculated.

$$\sigma_{cp} = \frac{(\sigma_{cy} + \ \sigma_{cz})}{2}$$

σ_{cy} and σ_{cz} are the axial concrete stress taking compression as positive.

$$\sigma_{cp} = \frac{N_{Ed.y}}{A_{cy}} and \sigma_{cz} = \frac{N_{Ed.z}}{A_{cz}}\ .$$

N_{Edy}, N_{Edy} are the longitudinal forces. The force may be either due to prestressing or axial force.

A_{cy}, A_{cz} are the corresponding cross sectional areas of concrete resisting the axial forces.

8.4.5 *Punching Shear for Foundation Slab and Pile Caps:* Cl. 10.4.5

The punching resistance of column bases for open foundations and pile caps shall be verified at control perimeters within 2d from the periphery of columns.

This is because the angle of punching cone will be steeper due to the favorable reaction from the soil. Checking the punching shear for open foundation with the basic control perimeter, ignoring the favorable reaction from the soil will lead to conservative assumption. When the foundation is subjected vertical loads only, the net shear force causing punching is calculated by subtracting the net relief offered by soil.

$$V_{Edred}. = V_{Ed} - \Delta V_{Ed}.$$

V_{Ed} is the applied shear force

ΔV_{Ed} is the net upward force within the control perimeter considered i.e. upward pressure from soil which shall be reduced by self-weight of the foundation.

The allowable shear stress in concrete v_{Rdc} without any punching shear reinforcement

$$v_{Rdc} = 0.12k\ (80\ \rho_1 f_{ck})^{1/3} x \frac{2d}{a} \geq v_{min} x \frac{2d}{a}$$

$$k = 1 + \sqrt{\frac{200}{d}} \geq 2.0\ and\ d\ in\ mm\ .$$

ρ_1 and v_{min} are as defined before.

a is the distance from the periphery of column to the control perimeter considered.

The shear stress at the control perimeter

$$v_{Ed} = \frac{V_{Edred}}{ud}$$

For eccentric loading or subjected to moment

$$v_{Ed} = \frac{V_{Ed.red}}{ud}\left[1 + k\frac{M_{Ed}u}{V_{Ed.red}W}\right]$$

Where k is defined in Eq. 10.25 or 10.30 of the Code as appropriate and W is similar to for the chosen control perimeter u.

For checking of punching shear in pile caps without shear reinforcement, no specific guidance is provided in the Code and the general rule as specified can be applied only in case edges of the pile cap are located further than '2d' from the face of pier, which is a rare situation in practice. In most of the situations, the pile edges are closer to the piers than '2d'. In such cases, part of the load will be transmitted directly into the support by way of strutting action. In such cases, it is suggested that punching be checked at a perimeter touching the pier/pile face (distance less than '2d') after allowing for enhancement in shear resistance.

8.4.6 *Design of Section for Punching Shear* Cl. 10.4.6

1) As it is difficult to provide punching shear reinforcement it is better to avoid this reinforcement. Hence the capacity of the slab in punching shear should be greater than the applied shear. This can be achieved by ensuring.

   $v_{Ed} \leq v_{Rdc}$

   Shear stress connected with punching shear is defined as below:

   v_{Ed}: Punching shear stress along the control perimeter.

   v_{Rdc}: Shear resistance of slab against punching without punching shear reinforcement.

   v_{Rdmax} : maximum punching shear resistance of slab.

2) Checking of punching shear stress around loaded area/column perimeters u_o.

Punching shear stress $v_{Ed} = \beta \frac{V_{Ed}}{u_o d}$

The perimeter u_o as follows

For central column $= 2 (C_1 + C_2)$

For edge column $= C_2 + 3d \leq C_2 + 2C_1$

For corner column $= 3d \leq C_1 + C_2$

d = depth of slab

C_1 and C_2 are dimensions of the loaded area as shown in **Fig. 10.12 and Fig. 10.13** of the Code.

β is the correction factor for application of moment.

β = 1 for purely axial load case without bending moment. For column with bending moment the value is to be calculated as given earlier.

Punching shear stress should be less than the allowable shear stress $v_{Rd,max}$.

The allowable shear stress = $v_{Rdmax} = 0.5\ v_{fcd}$.

$$v = 0.6\left(1 - \frac{f_{ck}}{310}\right)$$

$$v_{Rdmax} = 0.5 \times 0.6\left(1 - \frac{f_{ck}}{310}\right) \times \frac{0.67 f_{ck}}{1.5}$$

$$= 0.134\left(1 - \frac{f_{ck}}{310}\right) f_{ck}$$

It should be ensured that v_{Ed} is always less than v_{Rdmax}.

This provision makes sure that the concrete strut does not crush in punching.

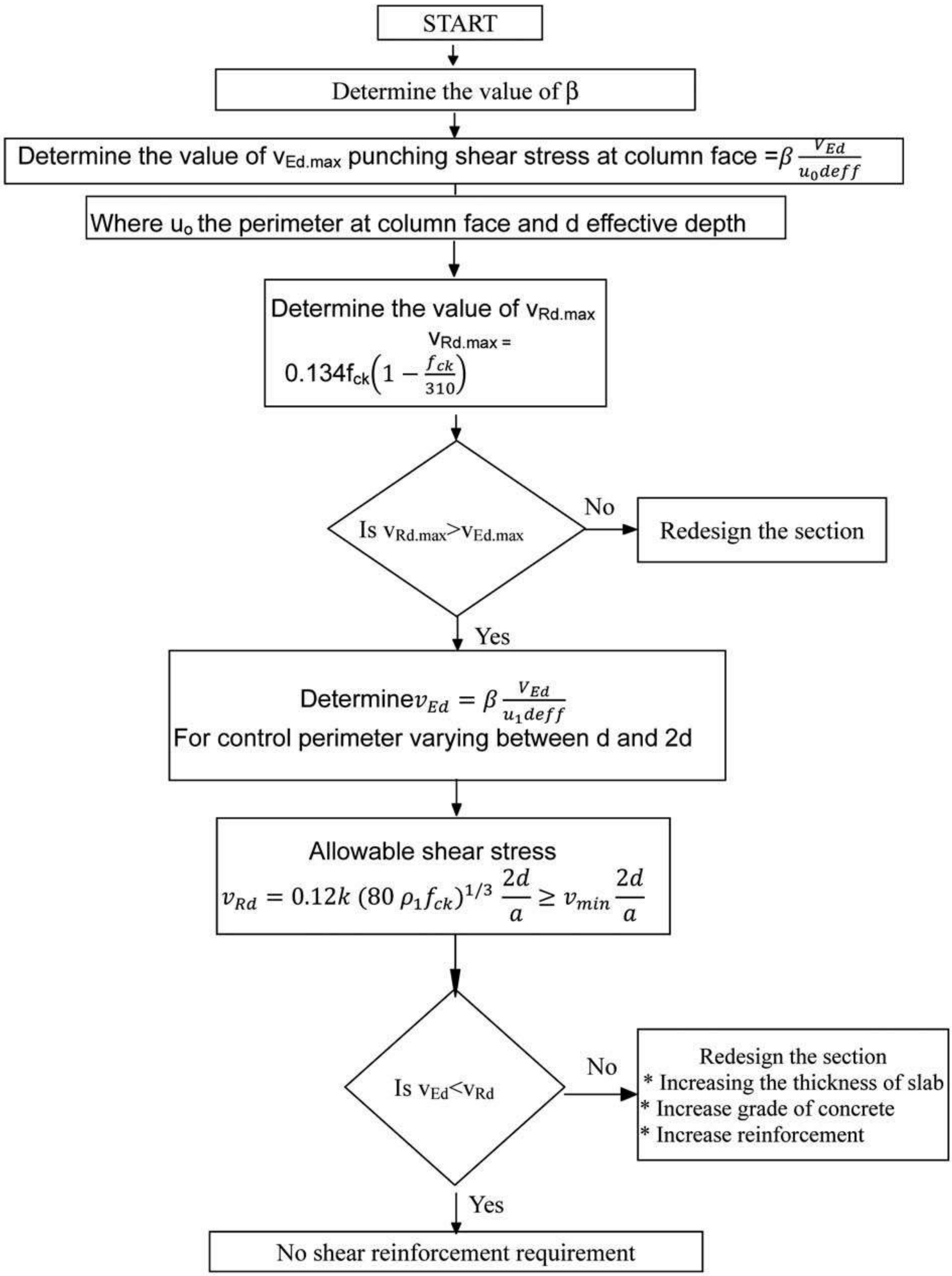


Fig. C10.25 Flowchart for checking Punching Shear

Worked example 10.4-1

Punching shear worked Examples:

Following are the details

1.	Open Foundation	:	Size	4 m x 4 m
2.	Column	:	Size	1 m x 1 m
3.	Load on Column	:		2000 kN
4.	Moment on Column	:		1000 kNm
5.	Material Properties	:		f_{ck} = 35 MPa f_{yk} = 500 MPa
6.	Footing thickness	:		0.7 M

Base pressure on foundation $\frac{2000}{4 \ x \ 4} \pm \frac{6 \ x \ 1000}{4^3}$

= 125 + 93.75 = 219 kN/m^2 31 kN/m

To resist the bending moment reinforcement provided is: 25 mm @ 200 c/c in both directions.

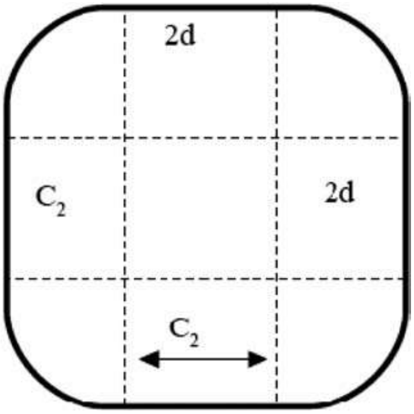
Reinforcement provided is 24.5 cm^2/m in each direction:

Effective depth d_y = 700 – 50 – 25/2 = 637.5 mm

Effective depth for other direction d_z = 637.5 – 25 = 612.5 mm

The effective depth for punching shear calculation

$$d = \frac{637.5+612.5}{2} = 625mm$$



Basic Control Perimeter u_i at 2d

$= 2\ (C_1 + C_2) + 4 \times \pi \times \frac{2d}{2}$

$= 2\ (C_1 + C_2) + 4\ \pi \times d$

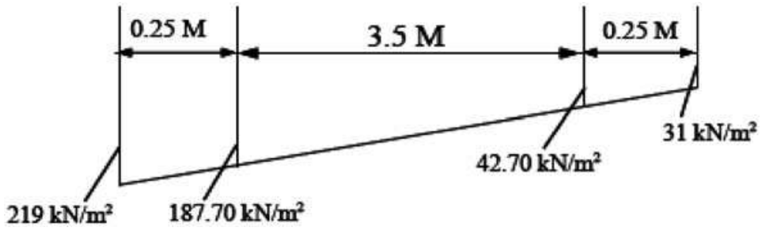
$= 2\ (1.0 + 1.0) + 4 \times \pi \times 0.625$

$= 11.85$ m

Area within perimeter = (4 x $\Pi \ x \ 0.625^2 + 4 \ x \ 0.625 \ x \ 2 \ x \ 1.00 \ x \ 1.00 \ x \ 1.0$ = 10.908 m^2

Relieving Pressure from this area:

on the conservative side



Base Pressure

Average pressure $\frac{187.70+42.7}{2} = 115.2\ kN/m^2$

Total upward force = 10.90 x 115.2 = 1255 kN

Net shear force = $V_{Ed.red} = V_{Ed} - \Delta V_{Ed}$

$V_{Ed.red}$ = 2000 – 1255 = 745 kN

% Steel in longitudinal direction = $\frac{2450}{637.5\ x\ 1000} = 0.00384$

% Steel in Transverse direction = $\frac{2450}{612.5\ x\ 1000} = 0.004$

$$\rho_L = \sqrt{0.00384\ x\ 0.004} = 0.00392$$

$$k = 1 + \sqrt{\frac{200}{625}} = 1.562$$

$$V_{Rdc} \frac{0.18}{0.5} x 1.565\ (80x0.00392x35)^{1/3} = 0.417 MPa$$

$$V_{min} = 0.031x1.565^{3/2}x35^{1/2} = 0.359\ MPa$$

Governing V_{Rdc} = 0.417 MPa

For an Internal columns subject to moment

$$W_1 = \frac{(C_1)^2}{2} + C_1C_2 + 4\ C_2d + 16d^2 + 2\Pi dC_1$$

$= \frac{1}{2} + 1 + 4\ x\ 1\ x\ 0.625 + 16\ x\ 0.625^2 + 2\Pi x\ 0.625\ x\ 1.0$

$= 0.5 + 1 + 2.5 + 6.25 + 3.92 = 13.17\ m^2$

$V_{Ed} = \frac{V_{Ed.red}}{u_1 d}\left[1 + k\frac{M_{Ed}}{V_{Ed.red}}\frac{u}{W}\right] = \frac{745\ x\ 10^3}{11.85\ x\ 1000\ x\ 625}\left[1 + \frac{0.6\ x\ 1000\ x\ 10^6 x\ 11.85\ x\ 1000}{745\ x\ 10^3 x\ 13.17}\right]$

$= 0.100\ [1 + 0.724] = 0.172\ MPa < V_{Rdc}$

As per clause 10.4.2 (4) and 10.4.5 (1) the punching shear should also be verified at a distance less than ‘2d’. Checking at a section at ‘d’ distance away from the column face,

Perimeter length u = 2 Π x 0.625 + 2 (1.0 +1.0) = 7.925 m

Area within this perimeter (Π x 0.6252 + 4 x 0.625 x 1.0 + 1.02) = 4.7 m^2

Relieving Average Pressure: 125 kN/m^2

Total upward face = 125 x 4.7 = 587 kN

Net shear force= 2000 – 587 = 1413 kN

Shear resistance of concrete = 0.417 x$\frac{2d}{a} = 0.417x2 = 0.834\ MPa$

The value of W what was estimated earlier cannot hold good as the plane has come closer; the W applicable for this plane is

$$W = \frac{1.0^2}{2} + 1\,x\,1 + 4\,x\,1.0\,x\frac{0.625}{2} + 16\,(\frac{0.625}{2})^2 + \ 2\Pi x\,1.0\,x\frac{0.625}{2}$$

$= 0.5 + 1 + 1.25 + 3.125 + 1.963 = 6.838m^2$

$= V_{Ed} = \ \frac{V_{Ed.red}}{ud}\left[1 + k\frac{M_{Ed}}{V_{ED.red}}\frac{u}{W}\right]$

$=\frac{1413\,x\,10^3}{7.925\,x\,10^3 x\,625}\left[1 + \frac{0.6\,x\,1000\,x\,10^6 x\,7.925\,x\,10^3}{1413\,x10^3 x\,6.838\,x\,10^6}\right]$

0.285 [1+0.492] = 0.425 MPa< 0.834 MPa

Checking the punching shear stress at face of column as per clause 10.4.6.(2)

u_o = 4 x 1.0 = 4.0m

W$_o$ $=\frac{1.0^2}{2} + 1.0\,x\,1.0 = 1.5m^2$

$$v_{Ed} = \frac{V_{Ed}}{u_o d}\left[1 + k\frac{M_{Ed}}{V_{Ed.red}}\frac{u_o}{W_o}\right] = \frac{2000\,x\,10^3}{4000\,x\,625}\left[1 + \frac{0.6\,x\,1000\,x10^6 x\,4000}{2000\,x10^3 x\,1.5\,x10^6}\right]$$

= 0.8[1+0.8] = 1.44 MPa

Allowable shear stress $0.3\,x\left[1 - \frac{35}{310}\right]x\frac{0.67\,x\,35}{1.5} = 4.16\ MPa > V_{ED}$

Hence the section is safe.

8.5 Torsion **Cl. 10.5**

8.5.1 *General* Cl. 10.5.1

When a concrete element is subjected to Torsion the longitudinal fibers are free to undergo deformation. Torsion can be of classified into Equilibrium Torsion and Compatibility Torsion. Equilibrium Torsion is that torsional resistance which is required to keep the structure in equilibrium and is essential for the basic stability of the element or structure. A few examples are canopy cantilevering off an edge beam, Beams/Box girders curved in plan. Element subjected to Equilibrium Torsion has to be designed for full torsional resistance in the ultimate limit state.

Compatibility torsion arises out of compatibility of displacement/ rotations to be maintained in the connected elements. Generally it occurs in monolithic construction. Compatibility torsion can be released without causing collapse. It is not necessary to consider this torsion at ultimate limit state. At serviceability limit state cracks may occur in the absence of sufficient reinforcement. The cracked torsional stiffness of elements subjected to torsion is only about 25% of the uncracked value. Low torsional stiffness significantly reduces torque resisting ability of the beams. Hence if the torsional rigidity is ignored in the analysis it would not make much difference. To limit the crack width under limit state of serviceability, check as per clause 12.3.5 of the Code shall be carried out and suitable reinforcement as per clause 16.5.3 shall be provided.

When the longitudinal fibers are restrained against deformation by an external element warping torsion arises.

The torsional resistance of a closed section may be calculated on the basis of a thin walled closed section. The equilibrium is satisfied by closed shear flow. Solid sections can be modeled by equivalent thin walled sections.

In case of complex shapes such as T section, the section shall be divided into series of subsections. The acting torsional moments on subsections can be distributed in proportion to the uncracked torsional stiffness. Each of the subsections can be modeled as thin walled section and the torsional resistance can be computed.

When hollow sections are modeled as thin walled section, the thickness of section shall be taken as A/u which will neither be less than twice the axis distance of longitudinal bars from the outer surface nor greater than the actual thickness. For conversion of solid section to equivalent hollow section **Fig. C10. 27** shall be referred to.

In the analysis the torsional stiffness may be based on uncracked sectional stiffness for equilibrium torsion and 25% of the uncracked sectional stiffness for compatibility torsion to allow for torsional cracking.

8.5.2 *Design Procedures:* Cl. 10.5.2

The shear stress in a wall of section subjected to pure torsional moment can be derived as follows:

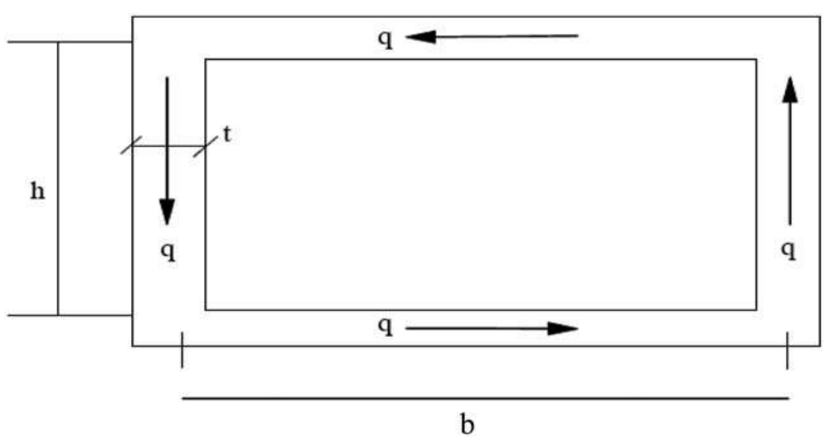


Fig. C10.26 Torsional Shear Flow

Shear Flow = q

Total Torsion Moment that can be resisted is q x b x h + q x h x b = 2 qhb

If applied torsion at moment is T_{ED}

Then $T_{ED} = 2\ qhb$

Shear flow $q = \frac{T_{Ed}}{2hb}$

A_k= hb ∴ $q = \frac{T_{Ed}}{2 \text{ Area of core}}$

Shear stress = $\frac{q}{t_{efi}} = \frac{T_{Ed}}{2\ A_k t_{efi}}$

Shear force in a wall = $\frac{T_{Ed}}{2 \text{ Area of core}}$ x length of wall

Shear force in a wall = $\frac{T_{Ed} x\ (h or b)}{2 \text{ Area of core}}$

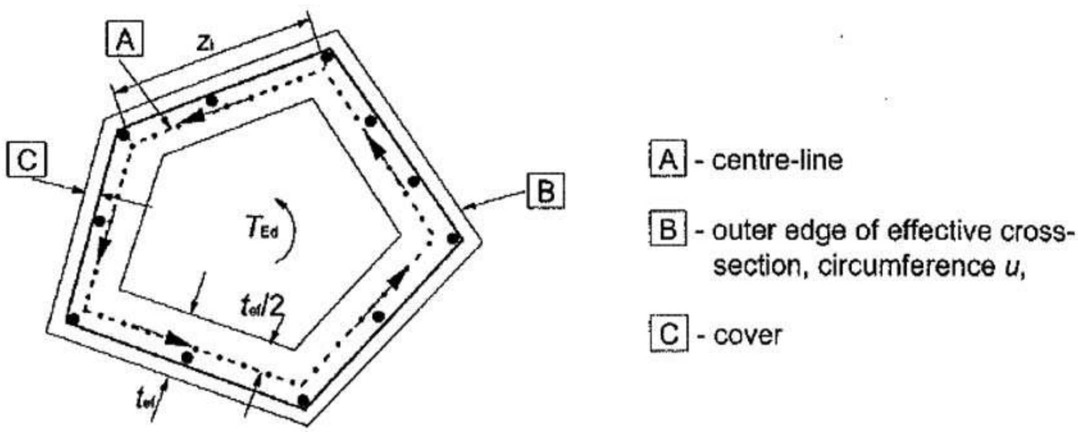


Fig. C10.27 Notations and Definitions Used

A_k	is the area enclosed by the centerlines of the connection walls, including inner hollow areas.
$\tau_{t,i}$	is the torsional shear stress in wall i
$t_{ef,i}$	is the effective wall thickness. It may be taken as A/u, but should not be taken as less than twice the distance between edge and centre of the longitudinal reinforcement. For hollow sections the real thickness is an upper limit.
A	is the total area of the cross-section within the outer circumference, including inner hollow areas
u	is then outer circumference of the cross-section
z_i	is the side length of wall I defined by the distance between the intersection points with the adjacent walls

The shear force generated due to torsion shall be calculated using the following formula θ angle shall be same as what has been assumed in shear analysis.

$V_{Torsion} = \frac{T_{Ed}}{2\ A_k} z_i$ Which can be equated to $\frac{A_{st}}{S_t} z_i f_{yd} \cot\theta$ (Eq.10.7 of the Code to get the transverse reinforcement)

$\therefore \frac{A_{st}}{s_t} = \frac{T_{ED}}{2\, A_k f_{yd} \cot\theta}$ where Ast is area of transverse reinforcement in thickness t_{efi} with a spacing of s_t.

The transverse reinforcement required shall be arrived based on **Clause 10.3.3.2** when vertical strips are provided. It shall be kept in mind that each wall has to be designed separately. The concrete capacity shall also be checked using the following equation.

$$\frac{T_{Ed}}{T_{Rdmax\,(c)}} + \frac{V_{NS}}{V_{Rdmax\,(c)}} \leq 1.0$$

T_{Ed} and V_{NS} Design Torsion and Shear forces

$V_{Rdmax\ (c)}$ is the maximum concrete capacity of member in shear as given in equation 10.8.

$T_{Rdmax\ (c)} = 2\ v.\alpha_{cw}.f_{cd}.A_k.t_{efi}.\sin\theta.\cos\theta.$

T_{Rdmax} is the torsional capacity of concrete

v is the strength reduction factor α_{cw} is the coefficient. Both the factors are as defined in earlier sections.

"θ" shall be taken the same value as assumed in shear design.

An additional longitudinal steel also needs to be provided for resisting torsion. The reinforcement can be calculated using the expression.

$$\frac{\Sigma A_{sl} f_{yd}}{u_k} = \frac{T_{Ed} \cot\theta}{2\, A_k}$$

u_k is the perimeter of area A_k.

A_{sl} is the area of Longitudinal reinforcement

f_{yd} is the yield strength of longitudinal reinforcement

T_{Ed} = Torsion applied on the section

θ = is the angle of compression strut

The longitudinal reinforcement generally has to be distributed uniformly along each wall. However in case of small sections the reinforcement can be concentrated at corners. Designers can calculate the requirement of this longitudinal reinforcement in a wall and the above equation can be modified as follows.

$$\frac{A_{sl}}{s} = \frac{T_{Ed} \cot\theta}{2\, A_k f_{yd}}$$

Where A_{sl} is the area of longitudinal reinforcement requirement in a wall with the spacing s of the reinforcement.

The above reinforcement shall be in addition to flexural and shear reinforcement in tensile zones. In the case of compression zone this longitudinal reinforcement can be reduced in proportion to the compressive force available in the compression zone. The compression zone shall be taken as twice the cover to the torsional links.

In case of precast segmental construction where there is no continuity of longitudinal reinforcement and tension due torsion and bending exceeds the compression due to prestress and bending, additional tendons to counter the tension need to be provided. The additional tendons shall be distributed around the precompressed tensile zone inside the closed stirrups. At least one tendon shall be placed at each corner.

Warping Torsion

For closed thin walled sections and solid sections warping torsion may be ignored since warping torsion is not necessary for equilibrium. Hence in these sections the torsion is equated to St. Venant torsion or circularity torsion. For open sections having very slender cross section, the warping torsional effects can be evaluated using the reference "Roarks Formula for stresses and strain" by W.C Young.

Worked Example 10.5-1

The box girder shown in example 10.3.3 is subjected to torsion of 5000 kNm at support. Design the shear and longitudinal reinforcement.

$$A_k = (2 – 0.25) \times (5.5 – 0.6) = 8.57 \text{ m}^2$$

Referring Equation 10.46 of the Code

$$\tau_{ti} = \frac{T_{Ed}}{2\,A_k t_{efi}}$$

$$V_{Edi} = \tau_{ti} \times t_{efi} \times Z_i$$

Substituting for τ_{ti} in the above equation

$$V_{Edi} = \frac{T_{Ed}}{2A_k t_{efi}} x t_{efi} x Z_i = \frac{T_{Ed}}{2\,A_k} Z_i$$

V_{Edi} is the design shear force due to torsion

But $\frac{T_{Ed} Z_i}{2\,A_k} = \frac{A_{st}}{s_t} Z_i f_{yd} \cot\theta$

$$\therefore \frac{A_{st}}{s_t} = \frac{T_{Ed}}{2\,A_k f_{yd} \cot\theta}$$

θ should be same as what has been assumed in shear analysis.

$\theta = 21.8^0$ $\quad f_{yk} = 0.87 \times 415 = 361 \text{ N/mm}^2$

$$\frac{A_{st}}{s_t} = \frac{5000\,x\,10^6}{2\,x\,8.57\,x 10^6 x\,361\,x\,2.5} = 0.32\,mm^2/mm$$

Adopting 2L – 10 MM @400 c/c will give a reinforcement of $\frac{78.5\,x\,2}{400} = 0.39 mm^2/mm.$

Longitudinal Reinforcement

$$\therefore \frac{A_{sl}}{s_l} = \frac{T_{Ed} \cot\theta}{2\,A_k f_{yd}}$$

$$\frac{A_{sl}}{s_l} = \frac{5000\ x\ 10^6}{2\ x\ 8.57\ x 10^6 x\ 361} x\ 2.5 = 2.02\ mm^2/mm$$

on each face $\frac{2.02}{2}$ = 1.1 mm^2/mm 12 @ 100 mm c/c will give 1.13 mm^2 /mm

Checking for T_{Rdmax} for web capacity

$$2\ xvx\alpha_{cw}xf_{cd}xA_kxt_{efi}\sin\theta\cos\theta$$

$$2(0.6)\left(1-\frac{40}{310}\right)x\ 1.0\ x\ 0.67\ x\frac{40}{1.5}x\frac{8.575\ x 10^6 x\ 600\ x\ 0.371\ x\ 0.928}{10^6} = 33077\ kNm$$

$$> 5000\ kNm.$$

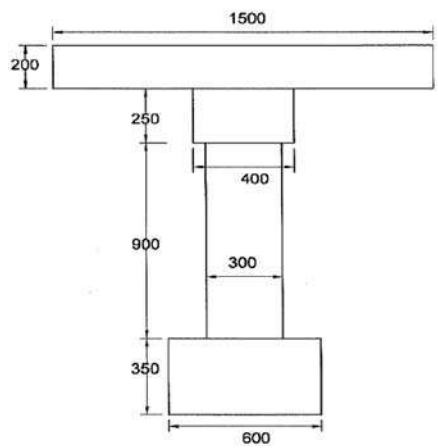
$$\frac{T_{Ed}}{T_{Rdc}} = \frac{5000}{33077} = 0.150$$

15% of allowable torsion the section is carrying. Hence $\frac{V_{Ed}}{V_{Rdc}}$ will workout to 0.85. Hence, the section can be subjected 85% allowable shear.
Similar exercise has to be done for other sections.

Worked Example 10.5-2

I beam is subjected To Torsion of 50 kNm. Design the section for torsion. Concrete is M40 grade. The Reinforcement f_{yk} is 415 N/mm^2

I idealized section is show below.



Cross Section

As a first step torsional inertia of each rectangular is to be evaluated. The torsional constant can be obtained from any standard reference book.

Torsional Inertia of Deck Slab:

b = 1500 mm t = 200 mm $\frac{b}{t}$ = 7.5 k (torsional constant) = 0.305

$I_{xx} = \frac{1}{2}$ x 0.305 x 1500 x 200^3 = 18.3 x 10^8 mm^4

As the deck slab is acting in two directions the torsional inertia has been halved.

Torsional Inertia of Top flange:

b = 400 mm t = 250 mm $\frac{b}{t}$ = 1.6, k = 0.203

I_{xx} = 0.203 x 400 x 250^3 = 12.68 x 10^8 mm^4

Torsional Inertia of web

b = 900 mm t = 300 mm $\frac{b}{t}$ = 3.0 k = 0.263

I_{xx} = 0.263 x 900 x 300^3 = 63.9 x 10^8 mm^4

Total Torsional Inertia of Section = 18.3 x 10^8 + 12.68 x 10^8 + 63.9 x 10^8 +53.25 x 10^8
= 148.13 x 10^8 mm^8

The torsion will be shared in proportion to their torsional stiffness.

Deck slab T_{Ed} $\frac{18.3}{148.13}$x 50 = 6.2 kNm

Top flange T_{Ed} $\frac{12.68}{148.13}$ x 50 = 4.28 kNm

Web T_{Ed} $\frac{63.9}{148.13}$ x 50 = 21.57 kNm

Bottom Flange T_{Ed} $\frac{53.25}{148.13}$x 50 = 17.97 kNm

Design of Reinforcements for Torsion:

Deck slab torsion should be combined with Deck slab design.

Effective thickness of the members can be found as follows.

For top flange

$$t_{efi} = \frac{A}{W} = \frac{400\ x\ 250}{2(400+250)} = 76.9mm.$$

But this thickness should not be taken less than 2 twice the distance of longitudinal bar from the surface [effective cover].

Taking cover as 40mm and dia for longitudinal and transverse bars of 10 mm,

Effective cover 40 + 10 + 5 = 55 m. Twice the 110 mm.

As 110 > 76.9 mm consider 110 mm.

Ak = (400 – 110) (250 – 110) = 40600 mm^2

The value of θ will be same as what has been worked out in shear analysis. However for simplicity assume θ = 45º

$$\frac{A_{st}}{s_t} = \frac{T_{Ed}}{2\ A_k f_{yd} \cot\theta} = \frac{4.28\ x 10^6}{2\ x\ 40600\ x\ 361\ x\ 1} = 0.146\ mm^2/mm$$

Adopt 8 M @ 300 two legged stirrup = Reinforcement provided

= 50/300 = 0.166 mm^2/mm

Longitudinal reinforcement = $\frac{As_L}{s_L} = \frac{T_{Ed}}{A_k f_{yd}} \cot\theta$. Since θ = 450 reinforcement will be same. Hence provide same reinforcement.

The can be provided at corner of links instead of distributing it throughout the section.

Checking the crushing resistance = $T_{Rdmax} = 2\nu\alpha_{cw} f_{cd} A_k t_{efi} \sin\theta \cos\theta$.

$$T_{Rdmax} = 2\,x\,0.6\left(1-\frac{40}{310}\right)1.0\,x\,0.67\,x\frac{40}{1.5}x\,40600\,x\,110x\frac{1}{\sqrt{2}}x\frac{1}{\sqrt{2}}x\frac{1}{10^6} = 41.67\,kNm$$
$$> 4.28\,kNm$$

$\frac{T_{Ed}}{T_{Rd}}\frac{4.28}{41.67} = 0.102$ (ie) 10% has been stressed against crushing strength against torsion.

Design of Web:

Thickness $t_{efi} = \frac{A}{w} - \frac{900\,x\,300}{2(900+300)} = 112.5\,mm$

Twice the distance of longitudinal bar = 110 mm

$$A_k = (300 - 112.5)(900 - 112.5) = 147656\,mm^2$$

$$\frac{A_{st}}{s_t} = \frac{T_{Ed}}{2A_k f_{yd}\cot\theta} = \frac{21.57\,x10^6}{2\,x\,147656\,x\,352\,x\,1} = 0.22mm^2/mm[\theta\ is\ taken\ as\ 45^0]$$

The reinforcement in each vertical leg is to be combined with shear reinforcement and provided. The same amount of longitudinal steel to be provided as θ has been taken as 45º.

Torsion Resisting Capacity

$$T_{Rdmax} = 2v\sigma_c f_{cd} A_k t_{efi} \sin\theta\cos\theta$$

$$= 2x0.6\left(1-\frac{40}{310}\right)x\frac{0.67\,x\,40}{1.5}x\frac{147656\,x\,112.5}{10^6}x\frac{1}{\sqrt{2}}x\frac{1}{\sqrt{2}} = 155\,kNm$$

$$=155\,kNm > 21.57\,kNm$$

Similarly bottom flange can be analyzed.

References

1. Christian Menn, Prestressed Concrete Bridges, published in 1986 by Springer - Verlag.

2. C.R. Hendy and D.A. Smith, Designer's Guide to EN 1992-2, Thomas Telford, 2006.

3. R.S. Narayanan & A.B. Beeby, Designers Guide to EN 1992-1-1 and EN1992-1-2, Thomas Telford, 2005.

4. PD-6687-2 : 2008, Recommendations for the design of structures to BS EN 1992-2:2005, published by BSi – British Standards.

CHAPTER 9

SECTION 11 : ULTIMATE LIMIT STATE OF INDUCED DEFORMATION

9.1 General Cl. 11.1

(1) This Section deals with structural members and structures whose load deformation behaviour and ultimate capacity is significantly affected by second order effects. Second order effects are additional action effects caused by the interaction of axial forces and deformations under load (Refer **Fig. C11.1**). First order deformations induce internal forces which in turns lead to further deformations. Sometimes these effects are also called P-Δ effects as they are the products of axial forces and deformations of the elements or system. Normally second order effects are calculated by second order analysis.

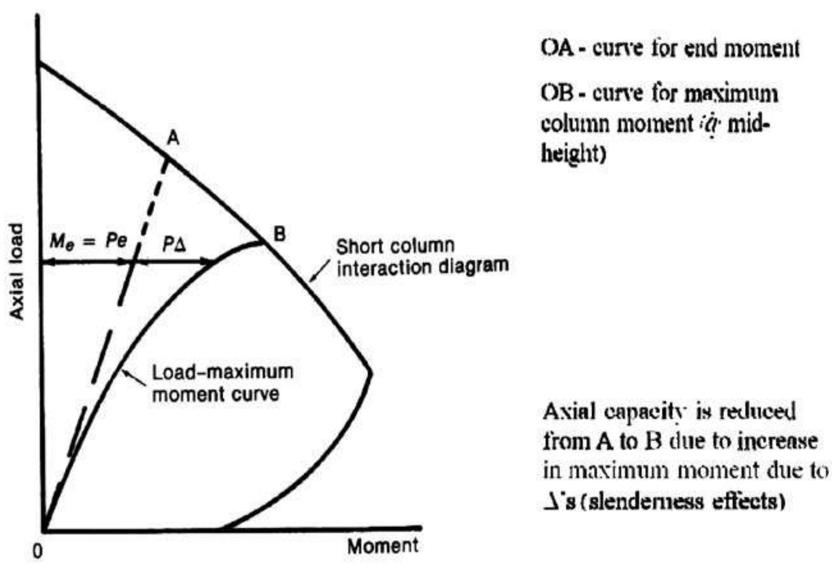


Fig. C11.1 Interaction Chart showing Second Order Effect[1]

Design with second order analysis is not commonly warranted. The Code provides guidelines to consider these effects in design, either by empirical approach or by accurate analysis. This shall be decided by the designer. Generally complexities are involved in this analysis. Significant aspects of second order analysis are:

a) The principle of superposition is not valid in second order analysis and all actions must be applied to the structure or element together with all respective loads and combination factors.

b) The flexural rigidity (EI) of the reinforced concrete structure is not constant. EI reduces with increasing moment due to cracking.

The code provides relaxation in cases where second order effects are less than 10 percent of the first order effects. In such cases, second order analysis can be done away with or by following the alternative methods and provisions as in sub **Clauses (4) & (5) and Clause 11.2.1** (in case of isolated members of uniform cross section).

(2) Classical buckling, defined as sudden failure due to instability of perfectly axially loaded members without horizontal load does not usually occur in practical reinforced/prestressed concrete members. However, long slender members after a particular load exhibit large and dis-proportionate increase of deformations due to combined effects of geometric non-linearity (P-Δ effect), non-linear structural response due to material non-linearity, progressive cracking and local plasticity. This reduces the ultimate load carrying capacity as compared to the short members of identical cross-section and steel ratio.

Therefore, long members have lower capacity as compared to that of short members of identical sectional details.

Second order effects are more pronounced in slender compressive elements. The well-known elastic buckling theory by Euler determines the extent of slenderness and quantum of second order effects. The elastic buckling itself has little relevance in the design, however the same gives good indication about the susceptibility to second order effects and can be used as a parameter in determining second order effects from the results of first order analysis as explained in **Section 11.3.2**.

Majority of commercially available structural soft ware have the capability to carry out second order analysis. In the second order analysis in addition to the invalidity of the principle of superposition, the flexural rigidity of reinforced concrete structures EI is not constant. As the moment increases for the same load, EI reduces due to cracking of concrete and inherent non-linearity in the concrete stress-strain response also increases. Thus it involves both geometrical and material non-linearity for RC elements and has to be taken in to account while choosing the method for 2^{nd} order analysis.

The slender piers of the bridges are commonly affected by 2^{nd} order analysis while the provisions related to same are also applicable to other slender members with significant axial loads like pylons and decks of cable supported bridges.

(3) Where second order effects are significant, these must be taken in to account in the analysis by linear elastic method in conjunction with further magnification of moments and reduced stiffness properties, accounting for cracking and creep.

However, for the analysis at the ultimate limit states, section properties used shall be similar to that at serviceability limit states as per the **Clauses 7.2 and 7.3** of the Code.

(4) This clause clearly defines conditions under which the second order effects are to be considered. The clause requires that the structural behaviour to

be considered in all directions in which significant second order effects can occur and bi-axial bending to be taken into account when necessary. Quite often the deformations in orthogonal directions need to be considered in bridge design though some times, one direction moment is negligible.

The distinction between long and short columns is made on the basis of 10% difference criteria as given in the **Clause (5)** below.

(5) This clause permits second order effects to be ignored if the same are less than 10 percent of the corresponding first order effects. When column is within range of short column parameters the effects are deemed to be within 10%. For all other cases the effects will need to be analysed to determine their magnitude.

(6) For the analysis, if the column length is modified to effective length as per the criteria for stiffness at column ends, column is also deemed as short column. Second order effects can be ignored in this case. Specific second order analysis is not necessary as well. The **Clause 11.2.1** provides a simplified criterion for isolated members with uniform cross sections based on limiting slenderness for the same.

(7) Second order effects apply to both 'isolated' members as well as 'group of components', which can sway involving several members. Piers with variable cross-section and piers located in river and creeks where substructure is supported by pile foundations with free length due to the scouring action will be an example of the latter. Normally, the pier/piers is/are connected to piles by a rigid pile cap. In some cases pier/piers are having variable sections. This clause emphasizes that the idealisation of the composite structure shall consist of varying sections for pier, pile cap and piles with appropriate un-supported lengths below pile cap, as the secondary effects may be quite significant in such structures. Where the piles are embedded fully in to soil, the clause permits the independent slenderness analysis for the 'isolated' pier.

9.2 Simplified Slenderness Criteria — Cl. 11.2

9.2.1 *Slenderness Criteria for Isolated Members (Columns) of Uniform Cross-Section* Cl. 11.2.1

While determining the second order effects by simplified methods instead of non-linear second order analysis, the effective length concept can be used to determine slenderness. On determination of slenderness, the requirement of second order analysis itself may be deduced. According to **Clause 11.2.1 (1)**, the slenderness ratio is defined as $\lambda = l_e/i$ where 'l_e' is effective length and 'i' is the radius of gyration of the uncracked concrete section.

The **Clause 11.2.1 (2)** provides simplified criteria when second order analysis is not required by limitation of slenderness value λ as follows:

$$\lambda_{\lim} = 20.A.B.C/\sqrt{n}$$

n = $N_{ED}/(A_c\ f_{cd})$ is the relative normal force. As the axial force 'n' becomes greater, the section becomes more susceptible to development of second order effects and consequently limiting slenderness value becomes lower. Higher limiting slenderness can be achieved where:

- there is low creep (because the stiffness of the concrete part of the member in compression is then higher)
- there is a high percentage of reinforcement (because total member stiffness is then less affected by the cracking of the concrete)
- The location of the peak first order effect is not the same as the location of peak second order moment. These effects are accounted for by the terms A, B and C respectively where:

  A = $1/(1 + 0.2\phi_{ef})$ φ_{ef} is effective creep ratio

  $$\varphi_{ef} = \phi(\infty, t_o).\frac{M_{oEqp}}{M_{oEd}}$$

  M_{oEqp} = First order B.M. in quasi-permanent load combination in S.L.S.

  M_{oEd} = First order B.M. in design load combination in U.L.S.

Value of φ_{ef} is not explicitly defined in the Code. Simplified value of 'A'0.7, which corresponds to φ_{ef} = 2.0 would be typical of concrete loaded at relatively young age with a loading being entirely quasi permanent. Using the default value of A = 0.7 is reasonably conservative as the same in any case is not sensitive to realistic variation of φ_{ef}.

$$B = \sqrt{1+2\omega}$$

Where $\omega = A_s\ f_{yd}/(A_c\ f_{cd})$ is the mechanical reinforcement ratio. If the same is not known, 'B' is recommended as 1.1 for first iteration, that is equivalent to ω = 0.1. This value would usually be achieved in a slender column; however this is generous in comparison to minimum reinforcement **Clause 16.2.2**. As such refining the design with actual value is imperative.

$$C = 1.7 - r_m$$

Where $r_m = M_{01}/M_{02}$; moment ratio. M_{01},M_{02} are the first order end moments at two ends of member as calculated from the analysis of structure, where $|M_{02}| \geq |M_{01}|$. If the end moments M_{01} & M_{02}; give tension on the same side, r_m should be taken as positive (i.e. C $\leq$ 1.7), otherwise negative (i.e. C > 1.7).

If 'r_m' is not known, C may be taken as 0.7 which corresponds to uniform moment throughout the member. C also should be taken as 0.7 where there is transverse loading, where first order moments are predominantly due to imperfections and where the members are not braced. As such refining the design with actual value is imperative.

Before carrying out the nonlinear analysis as per **Clause 11.3** and simplified method as per **Clause 11.3.2**, it is usual to check whether such effects can be ignored by achieving λ below the limiting value as $\lambda_{\lim}$.

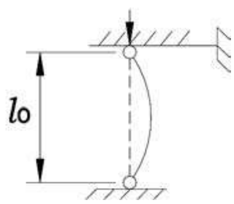
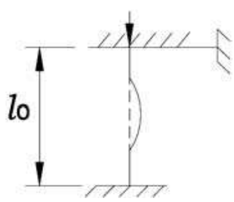
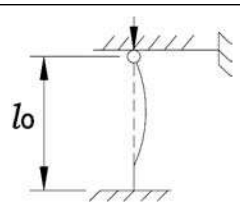
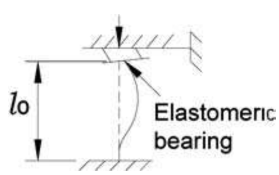
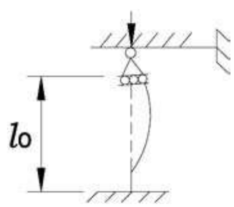
The **Clause 11.2.1 (3)** permits the independent verification in each direction for slenderness criteria. On the basis of limiting λ value, further exploration of requirement of second order effects can be decided. Depending upon the value of λ for each of the directions the slenderness effects can be neglected respectively i.e. if within individual values of λ_{lim}.

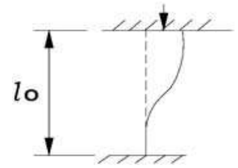
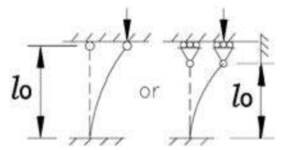
For bending in two orthogonal directions, the use of simplified interaction diagram may be adopted as shown below:

$$\left(\frac{M_{sd,x}}{M_{Rd,x}}\right)+\left(\frac{M_{sd,y}}{M_{Rd,y}}\right)\leq 1$$

9.2.2 *Effective Length (height) and Slenderness Ratio of Columns and Piers with Bearings* Cl. 11.2.2

Table 11.1 (Reproduced from the Code), Effective Length, l_e for Columns/Piers

Case	Idealised Column and Buckling Mode	Restraints in Plane of Buckling			Effective Length, l_e
		Location	Position	Rotation	
1.		Top	Full	None	$1.0l_o$
		Bottom	Full	None	
2.		Top	Full	Full *	$0.70\ l_o$
		Bottom	Full	Full	
3.		Top	Full	None	$0.85l_o$
		Bottom	Full	Full *	
4.		Top	None*	None*	$1.3l_o$
		Bottom	Full	Full*	
5.		Top	None	None	$1.4l_o$
		Bottom	Full	Full*	

Case	Idealised Column and Buckling Mode	Restraints in Plane of Buckling			Effective Length, l_e
		Location	Position	Rotation	
6.		Top	None	Full*	$1.5l_o$
		Bottom	Full	Full*	
7.		Top	None	None	$2.3l_o$
		Bottom	Full	Full*	

* Positional restraints are given for directions at right angles to the member

Parameters for calculating effective lengths for isolated members are available in this clause. Multiplication factors for calculating effective lengths are given on the basis of end conditions in **Table 11.1** of the Code which has been reproduced here above for the purpose of illustration.

For the cases from (2) to (6) assumption is that the restraint at one or both ends are infinitely stiff for determining rotational stiffness. It is difficult to achieve this boundary condition in reality. As such the effective lengths for the rigid restraints will always be somewhat greater.

Mathematical approach for determination of the rotational flexibility by equations 11.2 and 11.3 are available in **Clause 11.2.2 (1)** of the Code, which are reproduced here below:

For compression members in regular frames, the effective length l_e is determined in the following way:

Braced Members:

$$l_e = 0.5 l_o \sqrt{\left(1+\frac{k_1}{0.45+k_1}\right)*\left(1+\frac{k_2}{0.45+k_2}\right)}$$

Unbraced members:

$$l_e = l_o * \max of \left\{ \sqrt{\left(1+10.\frac{k_1.k_2}{k_1+k_2}\right)}\ ;\ \left(1+\frac{k_1}{1+k_1}\right)*\left(1+\frac{k_2}{1+k_2}\right) \right\}$$

where,

k_1 and k_2 are the relative flexibilities of rotational restraints at ends 1 and 2 respectively.

$k = \frac{(\theta)}{M}.\frac{EI}{l_o}$

θ/M = is the rotation of restraining members at a joint for unit bending moment M

EI = is the bending stiffness of compression member

l_o = is the clear height of compression member between end restraints.

For the unbraced members with rotational restraint at both ends, the second equation above can be used. For the theoretical case of a member with ends built in rigidly for positional restraint without rotational restraint i.e.($k_1 = k_2 = 0$), the effective length l_0 = l.

Effective length is determined in relation of flexural stiffness of compression member with that of rigidity of restraint. This means using un-cracked stiffness value of the member will not be on conservative side for reducing the buckling length to suggested value for which the restraint should be relatively stiffer. It is relevant to note here that this is in line with the definition of radius of gyration, 'i', given in the **Clause 11.2.1 (1)** based on the un-cracked section. Further the note 1 under the **Clause 11.2.1 (1)** implies that for determination of the stiffness of restraint like pier base cracked properties of the compression member or pier should be considered as it affects overall stiffness of restraint to a considerable extent. Alternatively the section shall be ensured to be un-cracked in ULS.

The 2nd note recommends that minimum value of the 'k' as 0.1, as the condition of fully restrained joint is hypothetical. In case of integral bridges where deck is connected to piers rigidly, the end stiffnesses to be used for piers can be determined by applying a corresponding deflection to pier to match the relevant mode of buckling and finding out moment and rotation at the connection of deck and piers in a plane frame model. Alternatively the effective lengths can be directly deduced by elastic buckling method provided under **Clause 11.2.2 (2)**.

9.3 Non-Linear Analysis of Structure and Elements **Cl. 11.3**

9.3.1 *General* Cl. 11.3.1

1) In principle on the basis of the **Clause 11.1 (1)**, RCC sections with axial loads need to cater for material as well as geometric non-linearity. This clause recommends non-linear analysis considering different types of loadings such as permanent and quasi-permanent loads leading to creep effects and short term loads such as live load. Use of generalized analysis methods is allowed.

   The **Fig. C11.2** illustrate the effects of non-linearity of a cantilever pier. When there is an axial load P, there is displacement resulting in a moment deflection curve, from which the curvature can be arrived at. In a cantilever pier, the maximum moment occurs at the base of the pier varying along its height from which an approximate relationship can be deduced. So the deflection $\Delta = kL^2/2$ where k is the curvature at the bottom of the pier. Therefore the total moment including second order moment = $M_0 + P\Delta$ where M_0 is the first order moment including moment from imperfections. When this is plotted as shown in **Fig. C11.3** on pier moment deflection curve, the same can be used for ascertaining compatibility and equilibrium.

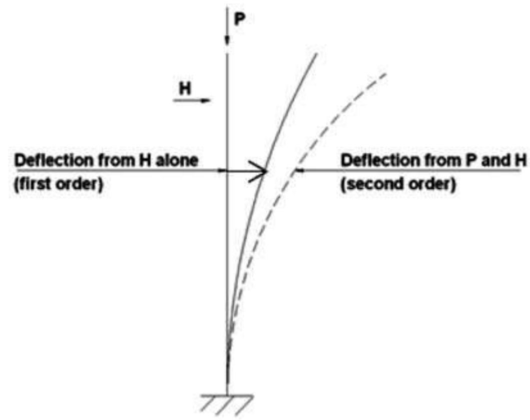


Fig. C11-2 Deflections for an Initially Straight Pier

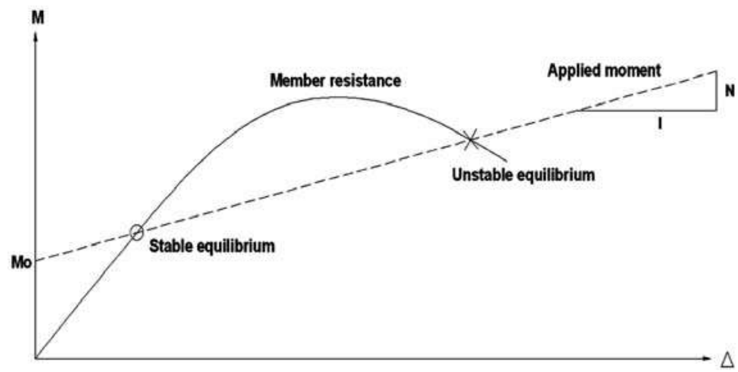


Fig. C11-3 Moment Deflection Relationship for Cantilever Pier

2) For the isolated members, the Code recognises the method based on nominal curvature which is explained in detail in the **Clause 11.3.2**. However, the elastic theory based analysis i.e. nominal stiffness method which has been elaborated in other international standards such as EN-1992 is done away within this Code. Generally, isolated member may be defined as a "member within nodes of the frame or pier with bearings at one end and without any bracings till its base and alike".

3) Though the **Clauses (1) and (2)** account for structural behaviour in general, these do not provide guidance regarding the material properties to be used at different stages. As per this clause, stress strain curves for concrete and steel given in **Annexure-A2.7 and Section 6 (Figs. 6.2 and 6.4)** respectively for concrete and steel are to be used as long as creep effect is considered in overall analysis. The resistance of local sections are governed by design values of the material strengths. Design values of the ultimate loads are obtained directly from the analysis. In equation Eq. A2-26 and in the calculation of k-value, f_{cm} is then substituted by the design compressive strength f_{cd} and E_{cm} is substituted by:

$E_{cd} = \dfrac{E_{cm}}{\gamma_{cE}}$, where γ_{cE} is taken as 1.2

As per the clause, the realistic stiffness has to be accounted for in analysis which warrants elaborate verification format. This clause simplifies the requirement by providing alternative that as long as material properties given in **Annexure-A 2.7** are used in the analysis for equilibrium and compatibility, no further local checks are required. The Code provides this alternative as the same is on conservative side. This is conservative because of uniformly reduced stiffness is implicit in the method leading to higher P-Δ effects and also method neglects the effect of tension stiffening.

4) If design properties use dare from concrete stress-strain relationships given in 6.4.2.5.4(iii) and creep is accounted for by multiplying all strain values in the above concrete stress-strain diagram by a factor ($1 + \text{Ø}_{ef}$), where Ø_{ef} is the effective creep ratio, then further checks of local sections may not be required, as strength and stability are verified directly by the analysis. However, additional attention is called for in cases where imposed displacements may make the overall stiffer system to attract more reaction components at the design section despite the reduction in P-Δ effects.

9.3.2 *Method Based on Nominal Curvature* Cl. 11.3.2

9.3.2.1 General Cl. 11.3.2.1

In this method an estimate of the maximum possible curvature is used to calculate the second-order moment. The clause specifies the limitation that the method is applicable for isolated members of the bridge with constant normal force and the defined effective lengths of the members that will depend upon the boundary conditions. The first-order moment, including that from initial imperfections, is added to the moment from the additional maximum deflection according to the expression in 11.3.2.2 (1) for design moment value.

9.3.2.2 Design Bending Moments Cl. 11.3.2.2

The Design moment $M_{Ed} = M_{0Ed} + M_2$

where,

M_{0Ed} is the first-order moment, including the effect of imperfections

M_2 is the second-order moment with actual boundary condition duly accounting for effective length, especially for indeterminate structures.

The Note (2) of this clause gives method to evaluate equivalent 1st order end moment M_{0e} for differing end moments. M_{0e} for differing end moments can be

$$M_{0e} = 0.6\ M_{02} + 0.4\ M_{01} \geq 0.4\ M_{02} \qquad \text{where} \qquad |M_{02}| \geq |M_{01}|$$

Sign convention is important here for the moments. If the moments are giving tensions on the same side they have to be given the same sign, otherwise opposite signs shall be assigned. It is to be noted, however that this note does not apply to bridge piers having bearings on top of the piers as the system is not braced system.

The nominal second-order moment is given under the note (3) of the clause which is as follows:

$$M_2 = N_{Ed}e^2$$

M_2 is determined by calculating e_2 from the estimated curvature at failure, 1/r, according to the formula, $e_2 = (1/r)l^2{}_0/c$. c depends on the distribution of curvature in the column. The definition of c depends on the shape of the total curvature, not just the curvature from first-order moment. For sinusoidal curvature, $c = \pi^2$ and for constant curvature, c = 8.

The latter value of c is best illustrated by considering a free-standing pier of length L with rigid foundations and hence l_0 = 2L. For constant curvature, 1/r, the deflection is obtained by integration of the curvature as follows:

$$\Delta = \int^L_0 \int^x_0 (1/r)\ dx;\ dx = (1/r)\ L^2/2$$

From the above formula for e_2, with c = 8 and l_o = 2L, the deflection is:

$$e_2 = (1/r)\ lo^2/c = (1/r)\ 4L^2/8 = (1/r)\ L^2/2$$

which is the same result as that in the earlier equation,

$$\Delta = \int^L_0 \int^x_0 (1/r)\ dx\ dx = (1/r)\ L^2/2$$

The value of $c = \pi^2$ is recommended in 11.3.2.2 (4) but care should again be taken when reinforcement is curtailed continuously to match the moment capacity envelope. In that situation, it will be more appropriate to use c = 8.

9.3.2.3 Curvature Cl. 11.3.2.3

The value of the curvature '1/r' depends upon the extent of axial load and the creep. The method is applicable for members with constant symmetrical cross-section (including reinforcement) only. The curvature can be determined according to 11.3.2.3.

$1/r = K_r K_{ø}\ 1/r_0$ where

$1/r_0$ is the basic value of curvature

K_r is a correction factor depending on axial load

$K_{ø}$ is a factor for taking account of creep

The above equation is applicable to a uniform symmetric section having symmetric reinforcements throughout.

As explained under **Clause 11.3.1 (2)**, there are "nominal stiffness method" and "nominal curvature method" under simplified methods. In the nominal stiffness method, the curvature is expressed in terms of estimated nominal stiffness EI, as $^1/_r = {}^M/_{EI}$

However, in nominal curvature method as explained below, the curvature $1/r$ is estimated on the basis of yield strain in tensile and compressive reinforcement.

The curvature $1/r_0$ is based on a rectangular beam with symmetrical reinforcement and strains of yield in reinforcement at each fibre separated by a lever arm z = 0.9d, where d is the effective depth (the compression and tension reinforcement thus being considered to reach yield). Hence the curvature is given by:

$$1/r_0 = \varepsilon_{yd}/0.45d$$

For situations where the reinforcement is not just in opposite faces of the section, d is taken as h/2 + i_s in accordance with 11.3.2.3 (2) where is is the radius of gyration of the total reinforcement area. This expression is again only applicable to uniform symmetric sections with symmetric reinforcement.

In the absence of symmetrical reinforcement distribution, normally other codes follow the method as explained below. The tension steel yields at ε_{yd} and the extreme fiber in compression reaches its failure strain ε_c, so the curvature $1/r_0$ would be given approximately by:

$$1/r_0 = (\varepsilon_{yd} + \varepsilon_c)/h$$

Where h is the depth of the section in the direction of bending used as an approximation to the depth to the outer reinforcement layer. The concrete strain can conservatively be taken as $\varepsilon_c = \varepsilon_{cu2}$. If the above expression is used, the factor K_r below should be taken as 1.0.

K_r is a factor which accounts for the reduction in curvature with increasing axial load and is given as $(n_u - n)/(n_u - n_{bal})$ <1.0, n_u is the ultimate capacity of the section under axial load only, N_u, divided by $A_c f_{cd}$. N_u implicitly includes all the reinforcement area, A_s, in calculating the compression resistance such that $N_u = A_c f_{cd} + A_s f_{yd}$ so that

$$N_u = \frac{(A_c f_{cd} + A_s f_{yd})}{A_c f_{cd}} = 1 + \underline{A_s f_{yd} / } A_c f_{cd}$$

As given in **Clause 11.3.2.3 (3)** n_{bal} is the value of design axial load, divided by $A_c f_{cd}$, which would maximize the moment resistance of the section as can be seen in **Fig. C11-4**.

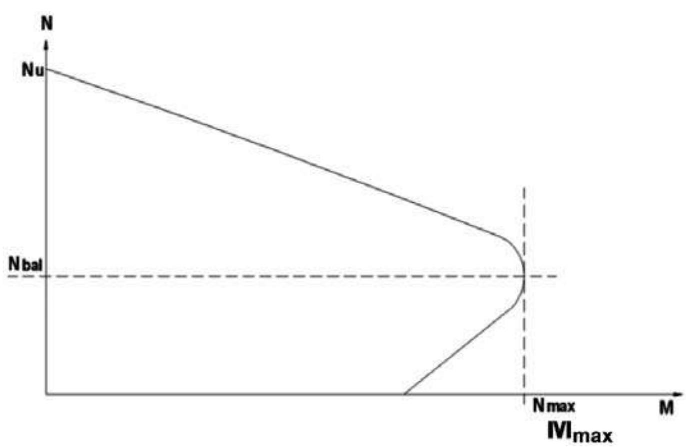


Fig. C11-4 Axial Force for a Balanced Section

The clause allows a value of 0.4 to be used for n_{bal} for all symmetric sections. In other cases, the value can be obtained from a section analysis. K_r may always be conservatively taken as 1.0 (even though for $n < n_{bal}$ it is calculated to be greater than 1.0), and this approximation will usually not result in any great loss of economy for bridge piers unless the compressive load is unusually high.

$K_\varnothing$ is a factor which allows for creep and is given by 11.3.2.3 (4) as follows:

$$K_\varnothing = 1 + \beta\varnothing_{ef} > 1.0$$

where,

$Ø_{ef}$ is the effective creep ratio

$\beta = 0.35 + f_{ck}/200 - \lambda/150$ and λ is the slenderness ratio

For braced members (held in position at both ends) which do not have transverse loading, an equivalent first-order moment for the linearly varying part of the moment may be used according to 11.3.3.2 (2). The final first-order moment M_{0Ed} should comprise the reduced equivalent moment from $M_{0e} = 0.6\ M_{02} + 0.4\ M_{01} \geq 0.4\ M_{02}$ (Eq 11.15 in the code) added to the full first-order moment from imperfections.

9.3.3 *Biaxial Bending* Cl. 11.3.3

When the columns are bent bi-axially slenderness for the same can be ascertained as described in the **Clause 11.3.1**. This clause is applicable when simplified methods are used.

Also the approximate method described in the **Clause 11.3.2** can also be used. In both of these methods 2nd order moments are determined in independently for both directions including imperfections. It is to be noted however, imperfections should be considered in only one direction which is most unfavourable. The clause cautions to explore the critical combination as it is function of the moments as well as dimensions in each of the directions and reinforcement. The critical section may not be identical for both directions. Critical section for each of the direction needs to be analysed for forces at that section in other direction and adequacy shall be established each section.

Sub-Clause (2) specifies to refer to the **Clause 8.3.2** for simplified methods where bi-axial moments take second order deformation into account. Here the interaction between moments in both direction are not required to be considered if the slenderness ratios in the two principle directions are less than or equal to 2 and the 'relative eccentricities' satisfy one of the criteria in 8.3.2 (3). In case the slenderness ratio in any principle direction exceeds 2.0, section design under the bi-axial moments and axial force shall be done by a rigorous cross-section analysis using the strain compatibility method.

9.4 Lateral Instability of Slender Beam **Cl. 11.4**

9.4.1 *General* Cl. 11.4.1

This clause discusses the lateral stability of concrete beams. The concrete girders are vulnerable to instability due to out of plane bending resulting in lateral and torsional displacements. The lateral and torsional instabilities are essentially to be checked during prestressing, side shifting, transportation and erection or launching as the beams are normally safe once deck slab is in place.

If for any reasons the requirements as specified in 11.4.1 (3) are not met then second-order analysis needs to be carried out to determine the additional transverse bending and torsional moments developed. Geometric imperfections must be taken into account and the clause requires a lateral minimum deflection of *l*/300 to be accounted as a geometric imperfection, where l is the total length of the beam. It is not necessary to include an additional torsional imperfection as well. Irrespective of the method used, the supporting structures and restraints must be designed for the resulting torsion as per **Clause 11.4.1 (4)**.

9.4.2 *Slenderness Limits for Beams* Cl. 11.4.2

This clause gives criteria for ignoring 2nd order effects in beams.

The above lateral stability formulae based on numerical study (3) don't cover for lateral stability of prestressed beams during transportation, handling and even during prestressing, arising out of casting imperfections, inclinations etc.

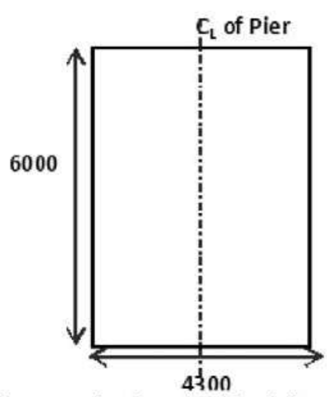
For the factor of safeties against lateral buckling of beams during prestressing, transportation and handling etc., special literature may be referred to and one of the papers could be (4) under references.

REFERENCES

1. C.R. Hendy and D.A. Smith, Designer's Guide to EN 1992-2, Thomas Telford, 2006.
2. CEB-FIP Model code 1990 : Design Code (For Concrete Structures).
3. Eurocode 2 : Design of Concrete Structures : 1992-1-1:2004.
4. V.N. Heggade, R.K. Mehta & R. Prakash, 'Design & Construction of Pre tensioned Sutlej Bridge Punjab', Paper No. 524, volume 67-2, July-September 2006, Journal of the Indian Roads Congress.
5. British Standards Institution (1990), Steel, Concrete and Composite Bridges Part 4 : Code of Practice for the Design of Concrete Bridges, London, BSI 5400.

Worked out Examples:

1. Effective Length of Cantilever Pier:



A bridge pier with monolithic connection with superstructure is 46m tall and has cross-section dimensions, as shown in Fig.11.5. The pier base has a rigid connection at the pile cap location because of rocky foundation material

Elastic Modulus of Concrete : $E_{cm} := 38.729 \cdot 10^3 \frac{N}{mm^2}$

Width in one direction: $b_1 := 6.0m$

Width in Other direction: $b_2 := 4.3m$

Clear Height between end restraints : $l_0 := 46 \cdot m$

Moment of Inertia about the transverse Axis : $I := \frac{\left[b_1 \cdot \left(b_2{}^3\right)\right]}{12} = 3.975 \times 10^{13} \cdot mm^4$

$$\frac{\left(E_{cm} \cdot I\right)}{l_0} = 3.347 \times 10^7 \cdot kN \cdot m$$

At the top of Pier, $k = \left(\frac{\theta}{M}\right) * \frac{\left(E_{cm} \cdot I\right)}{l_0}$ $\qquad k_2 := \frac{\left(4.579 \cdot 10^{-9}\right)}{kN \cdot m} \cdot \frac{\left(E_{cm} \cdot I\right)}{l_0} = 0.153$

where, $\left(\frac{\theta}{M}\right)$ = is the rotation of restraining members for unit bending moment (M= 1unit)

$E_{cm} \cdot I$ = is the bending stiffness of compression member

l_0 = is the clear height of compression member between end restraints.

Here k denotes the relative flexibility of rotational restraints.

This is greater than lowest recommended value of 0.1 given in (Clause 11.2.2).
Thus the value of k_1 taken as 0.153

At the base of pier $k_1 := 0.1$ as the connection is fixed, but the code recommends a minimum value of 0.1

From Clause 11.2.2 Equation 11.3, for unbraced members, the effective length is then:

$$\text{Hence } l_c = l_o * \text{Maximum}\left[\sqrt{\left[1 + 10 \cdot \frac{\left(k_1 \cdot k_2\right)}{\left(k_1 + k_2\right)}\right]}, \left(1 + \frac{k_1}{1 + k_1}\right) \cdot \left(1 + \frac{k_2}{1 + k_2}\right)\right]$$

$$\sqrt{\left[1 + 10 \cdot \frac{\left(k_1 \cdot k_2\right)}{\left(k_1 + k_2\right)}\right]} = 1.267 \qquad \left(1 + \frac{k_1}{1 + k_1}\right) \cdot \left(1 + \frac{k_2}{1 + k_2}\right) = 1.236$$

Maximum of (1.267,1.236)

$l_c := 1.267 l_0$

Alternatively, In the example above, if the deck were to be supported on a sliding bearing, the effective length calculation will be as below,

At the base of the pier, $k_1 = 0.1$

At the top of the pier, $K_2 := \infty$

$$\text{Hence } l_c = l_o * \text{Maximum}\left[\sqrt{\left[1 + 10 \cdot \frac{\left(k_1 \cdot k_2\right)}{\left(k_1 + k_2\right)}\right]}, \left(1 + \frac{k_1}{1 + k_1}\right) \cdot \left(1 + \frac{k_2}{1 + k_2}\right)\right]$$

The effective length is therefore close to $2l_o$ for a completely rigid support.

2. Check for the limiting slenderness:

The bridge pier in Worked example 1 has concrete with characteristic strength 60 MPa and carries an axial load of 81268 kN (74966 kN Long term and 6302 kN short term). Calculate the slenderness about the longitudinal axis and determine whether second order effects may be ignored. Take the effective length as 1.267 times the height. The moment of inertia of the inertia of the cross-section about the minor axis = 39.7535 m*4.

The area of the cross-section = 25.8 m^2.

The limiting slenderness is determined from Clause 11.2.1 (2) Eq. 11.1 as follows:

$$\lambda_{lim} := \frac{(20 \cdot A \cdot B \cdot C)}{\sqrt{n}}$$

The reinforcement ratio is known as 0.835 %. This leads to a value of

$$\alpha := 0.67$$

$$\omega = \frac{(A_s \cdot f_{yd})}{(A_c \cdot f_{cd})} \qquad f_{ck} := 60 \frac{N}{mm^2} \quad \frac{A_s}{A_c} = \blacksquare \cdot \frac{0.835}{100} \quad f_{yd} := 434.78 \frac{N}{mm^2}$$

$$\gamma_m := 1.5$$

where $f_{cd} := \frac{(\alpha \cdot f_{ck})}{\gamma_m} = 26.8 \cdot \frac{N}{mm^2} \qquad \omega := \left(\frac{0.835}{100}\right) \cdot \left(\frac{434.78}{34}\right) = 0.107$

$$B := \sqrt{(1 + 2 \cdot \omega)} = 1.102$$

Since the pier is free to sway, this is an un-braced. Hence, C = 0.7 (which also corresponds to equal moments at each end of a pier that is held in position at both ends). As per Clause 11.2.1

$$A = \frac{1}{(1 + 0.2 \cdot \Phi_{ef})} \qquad C := 0.7$$

Where, Effective Creep Ratio, $\Phi_{ef}\ \phi(\propto, t_0) \cdot \frac{M_{oEqp}}{M_{oEd}}$

M_{oEqp} is the Quasi permanent load combination in SLS = DL + SIDL + PS (in this case)

$$M_{oEqp} := 46475 kN \cdot m$$

M_{oEd} is the Design load combination in ULS = 1.35DL + 1.75SIDL + 1.5PS +1.5 LL (in this case)

$$M_{oEd} := 39363 kN \cdot m$$

$$\Phi_{ef} := 2.597 \quad \text{Thus} \quad A := \frac{1}{(1 + 0.2 \cdot \Phi_{ef})} = 0.658$$

hence

$$N_{ed} := 81268 kN \qquad A_c := 25.8 m^2 \qquad f_{cd} = 26.8 \cdot \frac{N}{mm^2}$$

The relative normal force is given by: $n := \frac{N_{ed}}{(A_c \cdot f_{cd})} = 0.118$

Hence $\lambda_{lim} := \frac{(20 \cdot A \cdot B \cdot C)}{\sqrt{n}} = 29.608$

The radius of gyration of the pier cross-section $i := 1.241m$

and the effective length $l_c := 1.267 \cdot 46m = 58.282 \cdot m$

so the slenderness $\lambda := \frac{l_c}{i} = 46.964$

Hence, $(\lambda > \lambda_{lim})$ Second order Effects cannot therefore be ignored for this pier.

3. Check for the limiting slenderness:

The pier of Worked example 1 & 2 is subjected to a 75521 kN axial load and a 2461 kN lateral load about the minor axis at the ultimate limit state. The main vertical reinforcement is 240 No. 32 mm diameter bars with yield strength 500 MPa. The effective creeo ratio Φ_{ef} is 1.6 and E_{cm} = 38.729* 10*3 MPa. Calculate the final moment at the base of the pier.

Since the section is symmetrical (with respect to cross-section and reinforcement), the method of 11.3.2 can be used without modification

The radius of gyration, is of the reinforcement was calculated to be 1789 mm so the effective depthe, d is found from 11.3.2.3 (2)/Eq. 11.8:

$h := 4300mm \quad i_s := 1789mm$

$d := \left(\frac{h}{2}\right) + i_s = 3939 \cdot mm$

Compression reinforcement is included in the above - see the discussion on compression reinforcement in section 11.3.2 above.)

The curvature (1/r) is then calculated from 11.3.2.3(1) / Eq 11.7 :

$\alpha := 0.85 \qquad E_s := 200GPa \qquad l_0 := 46.000m$

$f_{ck} := 60 \frac{N}{mm^2}$ where $A_c := 25.8m^2 \qquad \Phi_{ef} := 1.6$

$N_{Ed} := 75521kN \qquad A_s := 240 \cdot \frac{3.1414 \cdot 32mm \cdot 32mm}{4} = 0.193 \cdot m^2$

$\gamma_m := 1.5$

$f_{cd} := \frac{|\alpha \cdot f_{ck}|}{\gamma_m} = 34 \cdot \frac{N}{mm^2} \qquad f_{yd} := 434.78 \frac{N}{mm^2} \qquad \varepsilon_{yd} := \frac{f_{yd}}{E_s} = 2.174 \times 10^{-3}$

$\frac{1}{r_0} := \frac{\varepsilon_{yd}}{0.45 \cdot d} \qquad r_0 := \frac{1}{\left(\frac{\varepsilon_{yd}}{0.45 \cdot d}\right)} = 8.154 \times 10^5 \cdot mm \qquad \frac{1}{r_0} = 1.226 \times 10^{-6} \cdot \frac{1}{mm}$

The relative normal force is given by: $n := \frac{N_{Ed}}{|A_c \cdot f_{cd}|} = 0.086$ which is less than n_{bal} which may be taken as 0.4.

$$\omega := \left(\frac{A_s}{A_c}\right)\cdot\left(\frac{434.78}{34}\right) = 0.096 \quad n_u := 1 + \omega = 1.096 \quad n_{bal} := 0.4$$

According to Clause11.3.2.3 (3) / Eq 11.9, $k_r := \min\left[\frac{|n_u - n|}{|n_u - n_{bal}|}, 1\right] = 1$

In order to calculate k_Φ, the parameter, β must first be calculated taking the slenderness .

$\lambda := 46.964$ from Worked example 2

As per Equation 11.11, $\beta := 0.35 + \left[\frac{\left[\frac{f_{ck}}{\left(\frac{N}{mm^2}\right)}\right]}{200}\right] - \left(\frac{\lambda}{150}\right) = 0.337$

$k_\Phi := \max(1 + \beta\cdot\Phi_{ef}, 1) = 1.539$ From Clause 11.3.2.3 (4) / Eq 11.10

The nominal curvature according to Clause 11.3.2.3 (1) / Eq 11.7 is then:

$$\frac{1}{r} := k_r \cdot k_\Phi \cdot \left(\frac{1}{r_0}\right) \qquad r := \frac{1}{\left[k_r \cdot k_\Phi \cdot \left(\frac{1}{r_0}\right)\right]} = 5.298 \times 10^5 \cdot mm \qquad \frac{1}{r} = 1.888 \times 10^{-6} \cdot \frac{1}{mm}$$

The effective length of buckling $l_e := 1.267 \cdot l_0 = 58.282\,m$

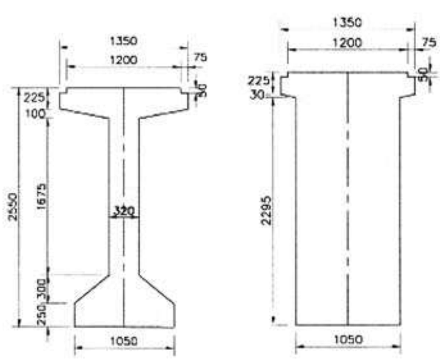
$c := 10 \cdot \pi^2$ for a sinusoidal distribution of curvature From Clause 11.3.2.2 (3) / Eq 11.6

$$e_2 := \left(\frac{1}{r}\right)\cdot\frac{l_e^{\,2}}{c} = 0.065\,m \qquad M_2 := N_{Ed}\cdot e_2 = 4906 \cdot kN \cdot m$$

The initial imperfection displacement at the pier top is obtained from as

4. Lateral instability of Slender beam:

The I Girder shown in the sketch below has a total span of 40 m. It is a precast PSC I Girder, which shall be cast, launched and erected without any persistent lateral support. After casting of deck and diapharagms. it will attain intermediate supports in lateral direction. Thus, for the transient condition, this girder is laterally unsuporrted. Check if this beam is slender and if second order effects are to be considered.



In the transient condition, the unsupported span is 40m.

As per the clause 11.4.1 (3) , Eq 11.13, the requirement for ignoring the second order effrects is

n transient situations: $\left(\frac{l_{ot}}{b}\right) \le \frac{70}{\left(\frac{h}{b}\right)^{\frac{1}{3}}}$ $\left(\frac{h}{b}\right) \le 3.5$

Where

Total depth of the beam at mid span $h := 2.55m$

Effective width of Compression flange $b := 1.35m$

Distance between torsional restraints $l_{ot} := 40m$

$\frac{l_{ot}}{b} = 29.63$ $\frac{70}{\left(\frac{h}{b}\right)^{\frac{1}{3}}} = 56.628$ Thus $\frac{l_{ot}}{b} < \frac{70}{\left(\frac{h}{b}\right)^{\frac{1}{3}}}$

$\frac{h}{b} = 1.889$ Thus $\frac{h}{b} < 3.5$

Even though both the conditions are satisfied in this case, the girder is a pre- stressed girder and thus has an axial force during this transient condition, i.e Compression due to Pre-stressing cables. This in conjuction with minimum geometric imprefections i.e lateral deflection due to geometric imprefection is also to be considered.

Geometric imperfection is also to be considered in this case as per Clause 11.4.1 (2)

$\frac{l_{ot}}{300} = 0.133\,m$ of lateral deflection due to geometric imperfection is also to be considered.

Since the section area and Second Moment of Area are changing along the span, the method based on nominal curvature will not be accurate in this case. A commercially available software package should be used to do a non-linear analysis of this problem.

As per the clause 11.4.2, for the slenderness limits of the beams, the clear distance between lateral restraints should not exceed 60b or 250b2/h whichever is lesser.

$60 \cdot b = 81\,m$

$250 \cdot \frac{(b \cdot b)}{h} = 178.676\,m$

Whereas the clear distance between the lateral restraints 9span) is 40m. Thus OK.

CHAPTER 10

SECTION 12 : SERVICEABILITY LIMIT STATE

10.1 General — Cl. 12.1

(1) The verification of SLS is performed under service load conditions.

The excessive cracking or stresses under SLS may affect the durability of structure, its appearance, its air/water tightness etc. Excessive deflections may cause ugly appearance and inefficient use.

(2) This clause permits an un-cracked concrete cross-section to be assumed for stress and deflection calculation provided that the flexural tensile stress under the relevant combination of actions considered does not exceed f_{ctm} or $f_{ctm,fl}$ but should be consistent with the value used in the calculation of minimum tension reinforcement.

For the purpose of calculating crack widths and tension stiffening effects, f_{ctm} should be used. Where the maximum tensile stress in the concrete calculated on the basis of uncracked section exceeds f_{ctm} or f_{ctm}. fl, the crack state should be assumed. Where the section is assumed to be uncracked, whole concrete section is considered to be active and both concrete and steel are assumed to be elastic, both in tension and compression.

10.2 Stress limitation — Cl. 12.2

10.2.1 *Allowable Compressive Stress in Concrete* — Cl. 12.2.1

(1) The limitation of compressive stress in concrete aims at controlling excessive compression, producing irreversible strains and longitudinal cracks (parallel to the compressive strains) within limits. The clause addresses this my limiting the stress level under the rare combination of actions to value of 0.48 f_{ck}.

(2) This clause specifies distinction between linear and non¬linear creep behaviour of concrete. Under the quasi–permanent combination, if the compressive stress in concrete exceeds 0.36 f_{ck} non-linear model should be used for the assessment of creep effects.

10.2.2 *Allowable Tensile Stress in Steel* — Cl. 12.2.2

This clause is provided to ensure stress in reinforcement is in elastic range by limiting the stress to 0.8 f_{yk} under the rare combination of loads. It is not necessary to carry out the check for fatigue, if the tensile stress in the reinforcement does not exceed 300 MPa under Rare Combination of Serviceability Limit State.

The stress in prestressing steel shall not exceed the limits prescribed in **Clause 7.9.2** after allowance for losses. If it is ensured that for prestressed concrete structures, under the frequent combination of action and prestressing force, only compressive stresses occur at the extreme concrete fibres, under Serviceability Limit State fatigue check is not required.

10.3 Limit State of Cracking **Cl. 12.3**

10.3.1 *General* Cl. 12.3.1

By limiting crack width, the functioning and durability of the structure is ensured. In the reinforced concrete structures cracks are inevitable due to tension, bending, shear and torsion which is a result of either direct loading or restraint of imposed deformations. This may not necessarily impair serviceability or durability.

Cracks may also occur due to other causes like plastic shrinkage or chemical reactions accompanied by expansion of hardened concrete. The measures to control width of such cracks are covered in **Section 14**. Design crack widths are specified to satisfy requirements of functionality, durability and appearance.

10.3.2 *Limiting Crack Width* Cl. 12.3.2

(1) This clause requires that the design crack width to be chosen such that cracking does not impair the functioning of the structure. Cracking normally 'impairs' the function of the structure by either helping to initiate reinforcement corrosion or by spoiling its appearance. The size of the cracks also has an influence on the time to initiation of reinforcement corrosion. Noticeable cracking in structures causes concern to the public and it is therefore prudent to limit crack widths to a size that is not readily noticeable. The above considerations have led to the crack width limitations specified in **Table 12.1** (Reproduced for ready reference.). Crack width is to be generally checked for quasi-permanent load combinations for reinforced concrete members and prestressed members with unbounded tendons. For prestressed members with bonded tendons check shall be for frequent load combinations, unless a different load combination is specified.

Table 12.1 Recommended Values of w_{max}

Condition of Exposure[1]) As per Clause 14.3.1	Reinforced members and prestressed members with un-bonded tendons	Prestressed members with bonded tendons
	Quasi-permanent load combination[2]) mm	Frequent load combination mm
Moderate	0.3 [2)]	0.2
Severe	0.3	0.2[3)]
Very Severe and Extreme	0.2	0.2[4)] and decompression

(1) The condition of exposure considered applies to the most severe exposure the surface will be subjected to in service.

(2) For moderate exposure class, crack width has no influence on durability and this limit is set to guarantee acceptable appearance.

(3) For these conditions of exposure, in addition, decompression should be checked under the quasi-permanent combination of loads that include DL+ SIDL + Prestress including secondary effect + settlement + temperature effects (0.5).

(4) 0.2 applies to the parts of the member that do not have to be checked for decompression.

(Not a part of (1) and hence is deleted.)

As a rule the limit state of decompression is required, if cracking or re opening of cracks have to be avoided for a given combination. The margin between zero stress and tensile strength may be reserved for self equilibrating stresses not quantified in the analysis. In flexure of a prestressed beam the state of decompression is reached when the section under consideration is fully in compression and extreme fibre concrete will normally have compressive stress in service.

The decompression limit deck requires that no tensile stresses occur in any concrete within a distance of 100 mm, of the tendon duct. This ensures that there is no direct crack path to the tendon for contaminants. The 100 mm requirement is not a cover requirement. It simply means that if the cover is less than 100 mm. If cover is less than 100 mm it must be fully in compression. Tensile stresses are permitted in larger cover as long as the concrete within 100 mm of the tendons or ducts is in compression. If, while checking decompression, the extreme fibre is found to be cracked, the check of decompression at the specified distance from the tendons, 100 mm becomes imperative.

(2) In the combinations where temperature distributions (temperature gradient along the member depth) are involved, gross properties should be taken in to consideration for calculating crack width and self-equilibrating stresses shall be ignored. For cracked sections, the analysis to determine self-equilibriating stresses is complicated and highly iterative. However, since cracking results in a reduction in stiffness of section, cracking of a section will lead to a substantial relaxation of the stresses induced by temperature. It is therefore generally satisfactory to ignore temperature-induced self-equilibriating stresses in cracked sections as well and to consider only the secondary effects.

(3) The PSC members with unbounded tendon alone are treated like RCC member as specified in the clause. However, for prestressed members having combination of bonded and unbounded tendons, limits of bonded presstressing apply.

(4) An alternative for crack width calculations out as specified in the **Clause 12.3.4** or by providing bar spacing and sizes in accordance with the **Clause 12.3.5** is available. For latter it is not necessary to calculate the crack widths as crack control criteria is deemed to be satisfied by adopting this clause.

10.3.3 *Minimum Reinforcement for Crack Control* Cl. 12.3.3

(1) A minimum amount of untensioned reinforcement is required to control cracking in areas where tension due to external loadings or external restraints is expected. The amount of such reinforcement may be estimated from equilibrium between the tensile force in concrete just before cracking and tensile force in steel at yielding.

Fundamental principle of crack width calculations and spacing is that the reinforcement remains elastic. If the steel yields, excessive deformation will occur at the location of the crack and the formulae used in the calculation would get rendered invalid.

For a section subjected to uniform tension, the force necessary for the member to crack is $N_{cr} = A_c f_{ctm}$, where N_{cr} is the cracking load, A_c is the area of concrete in tension and f_{ctm} is the mean tensile strength of the concrete. The strength of the reinforcement is $A_s f_{yk}$. To ensure that distributed cracking develops, the steel must not yield when the first crack forms hence:

$$A_s f_{yk} > A_c f_{ctm}$$

(2) The above equation needs to be modified for the stresses other than uniform tension. When the section is subjected flexure for example, the stresses vary across the depth reducing tension consequently the requirement of reinforcement. Thus a factor k_c is introduced in the above equation to account for the same. Another factor k is introduced to account for self equilibrating stresses arising out of variation in strain along the depth of the member. The shrinkage and temperature differences are the common cause of non-linear strain variation. For instance surface shrinkage occurs faster than interior and similarly the surface which gets heated up faster also cools faster than interiors. The factor k therefore reduces the reinforcement necessary where self-equilibriating stresses can occur. These stresses are more pronounced for deeper members and thus k is smaller for deeper members.

σ_s is the absolute value of the maximum stress permitted in the reinforcement immediately after formation of crack.

$f_{ct,eff}$ is the mean value of tensile strength of concrete effective when the cracks are first expected to occur. In calculating minimum reinforcement to cater for shrinkage $f_{ct,eff}$, should be taken as greater of 2.9 MPa and $f_{ctm(t)}$.

(3) The bonded tendons normally contribute towards the minimum reinforcement requirement. However, the clause recommends neglecting the same in crack control as a conservative assumption. The other international codes particularly Euro code allows any bonded tendons located within the effective tension area to contribute to the area of minimum reinforcement required to control cracking provided they are within 150 mm of the surface to be checked.

(4) This clause allows minimum reinforcement to be omitted for prestressed concrete members, wherein the stress at the extreme tensile fibre is compressive, under the rare combination of actions and the characteristic value of prestress. This does not however do away with the need to consider the provision of reinforcement to control early thermal and shrinkage cracking prior to application of the prestressing.

10.3.4 *Calculation of Crack Width* Cl. 12.3.4

(1) The crack width calculations are based on the basic case of a prismatic reinforced concrete bar, subjected to axial tension. With regard to the behaviour under increasing tensile strain four stages are distinguished (as shown in the **Fig. C12-1)**:

- the uncracked stage,
- the crack formation stage,
- the stabilized cracking stage,
- the steel yielding stage.

Fig. C12-2 shows a simplified representation of the load-deformation behaviour of a centrally reinforced member subjected to tension or imposed deformation. According to the simplification, in the crack formation stage (2) the axial tensile force does not increase. When enough cracks have been formed to ensure that no undisturbed areas are left, the tensile strength of the concrete cannot be reached any more between the cracks, so that no new cracks can appear. This is the start of the stabilized cracking stage (3). In this stage no new cracks are formed but existing cracks widen. Finally the steel will start yielding at stage (5)

For all stages of cracking, the design crack width W_k may be calculated by:

$W_k = S_{r,\ max}\ (\varepsilon_{sm}\text{-}\varepsilon_{cm})$ Where:

$S_{r,\ max}$ is the maximum crack spacing calculated depending upon the conditions in Equations 12.8, 12.11 or 12.12 of the code.

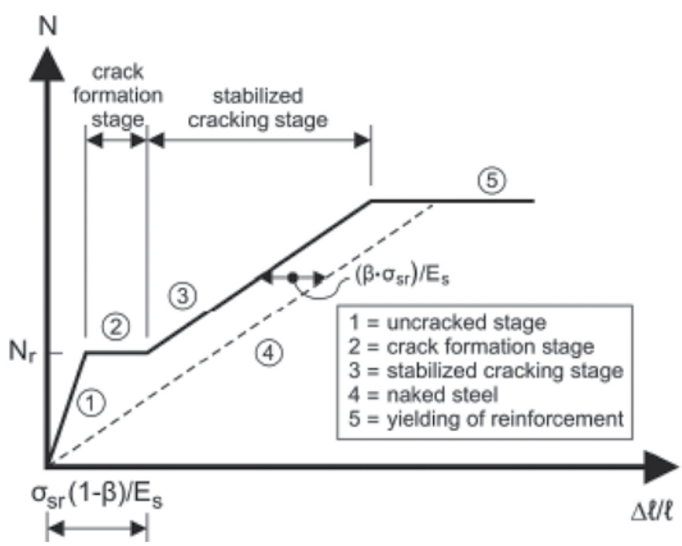


Fig. C12-1 Crack Formation Stages

ε_{sm} is the mean strain in the reinforcement under the relevant combination of loads, including the effect of imposed deformations, restrained thermal and shrinkage effects and taking into account the effects of tension stiffening. For prestressed members only the additional tensile strain beyond the state of zero strain of the concrete at the same level is considered.

ε_{cm} is mean strain in the concrete between cracks.

(2) Relative mean strain in the above clause is given by :

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_{sc} - k_t \dfrac{f_{ct,eff}}{\rho_{p.eff}}\left(1 + \alpha_e \rho_{p,eff}\right)}{E_s} \geq 0.6 \frac{\sigma_{sc}}{E_s}$$

where,

σ_{sc} is the steel stress in the crack

α_e is the modular ratio E_s/E_{cm}

$\rho_{p,eff} = A_s / A_{c,eff}$; $A_{c,eff}$ is the effective area of concrete in tension surrounding the reinforcement, of depth $h_{c,eff}$, where $h_{c,eff}$ is the lesser of 2.5 (h-d), (h-x)/3 or h/2 (refer **Fig. C12.2**).

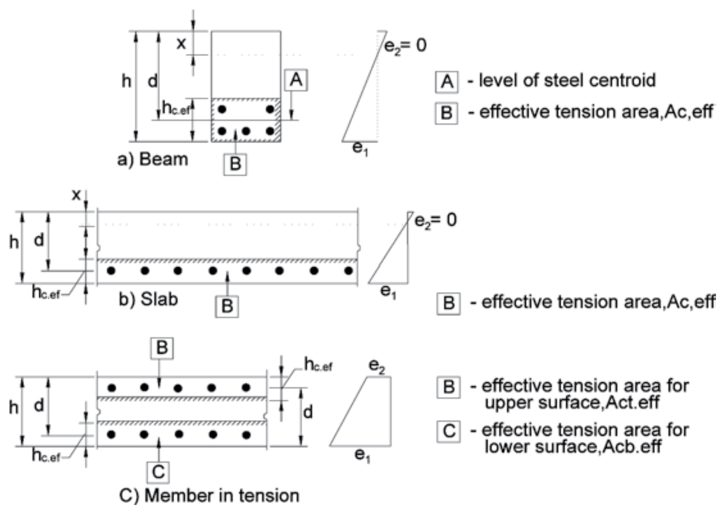


Fig. C12.2 Effective Tension Area (Typical Cases)

k_t is an empirical coefficient to assess the mean strain depending on the type of loading taken as 0.5.

(3) This clause elaborates the crack spacing $S_{r.max}$ for various situations as enumerated below are self explanatory in the code as such is not reproduced here :

- Spacing of bonded reinforcement within the tension zone is reasonably close.
- Case of deformed bars associated with pure bending.
- Spacing of the bonded reinforcement is more or where there is no bonded reinforcement within the tension zone.

(4) If the cracks in a member reinforced in two orthogonal directions are expected to form at an angle which differs substantially (> 15°) from the direction of

the reinforcement, the approximation by following equation may be used to calculate $S_{r,max}$ and W_k.

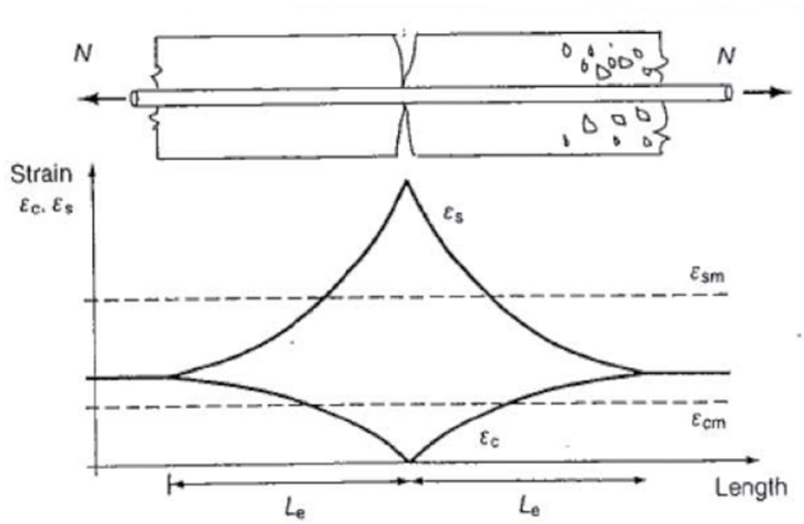
$$S_{r,max} = \frac{1}{\dfrac{\cos\theta}{S_{r,\max.y}} + \dfrac{\sin\theta}{S_{r,\max.z}}}$$

where,

θ is the angle between the reinforcement in the y direction and the direction of the principal tensile stress.

$S_{r,max,y}$ $S_{r,max,z}$ are the crack spacings calculated in the y and z directions respectively, according to **Clause 12.3.4 (3)**.

Derivation for crack width and crack spacings



Strain Adjacent to Crack

- The member will first crack when the tensile strength of weakest section is reached.
- Cracking leads to local redistribution of stresses adjacent to the crack by strain distribution.
- At crack the entire tensile force is carried by reinforcement.
- Moving away from the crack tensile stress is transferred from the reinforcement by bond to the surrounding concrete.
- At some distance L_e from crack the distribution of stress is unaltered from that before the crack formed.
- At this location the strain in concrete and reinforcement is equal and the stress in the concrete is just below its tensile strength.

- The redistribution of stress, local to the crack results in an extension of member which is taken up in the crack causing it to open.
- With increase in tension the crack will form in the next weakest section.
- This will not be within distance of L_e of the first crack due to reduction in concrete stress in that region associated with first crack.
- With further increase in tension more cracks will develop until the maximum crack spacing is 2 L_e
- No further cracks then form but further loading will cause existing cracks to widen.
- This is called stabilized cracking.
- The member stiffness will continue to reduce tending towards fully cracked section
- The crack spacing will lie between Le and 2L_e

Assuming Constant bond strength T along the length Le

$$T\Pi\phi L_e = f_{ct} A_c$$

$$L = \frac{f_{ct} A_c}{T\Pi\phi}$$

$$\rho = \frac{\Pi\phi^2}{4A_c}$$

$$\frac{A_c}{\Pi\phi} = \frac{\phi}{4\rho}$$

$$L_e = \frac{f_{ct}}{T} x \frac{\phi}{4\rho} = \frac{1}{4}\frac{f_{ct}}{T}\frac{\phi}{\rho}$$

L_e – 0.25 K_1 Ø/ρ K_1 bond properties of reinforcement

This does not fit well with the experimental data.

So a term Kc is added to relate it to cover.

K shall be taken as 2

Le = 2c + 0.25 K_1 Ø/ρ

This is valid for pure tension. For bending case another constant K_2, is included to take into account the variation in stress distribution

L_e = 2c + 0.25 K_1K_2 Ø/ρ

S_{rm} = 2c + 0.25 K_1K_2 Ø/ρ

K_1 = is 0.8 for HYSD bars; K_2 = 1.0 pure tension and 0.5 for bending cases.

L_e – mean crack spacing

The crack width will give only the mean crack width if this expression used.

As we are interested characteristics crack width. Experimentally it has been found; 1.7 times this distance gives the maximum crack spacing for characteristics crack width.

Hence maximum crack spacing =

S_{rmax} = 1.7 (2c + 0.25 K_1 K_2 Ø/ρ)

= 3.4c + 0.425 K_1 K_2

K_1 for HYSD Bars is 0.82; K_2 for bending is = 0.5

= 3.4c + 0.425 x 0.8 x 0.5 Ø/ρ

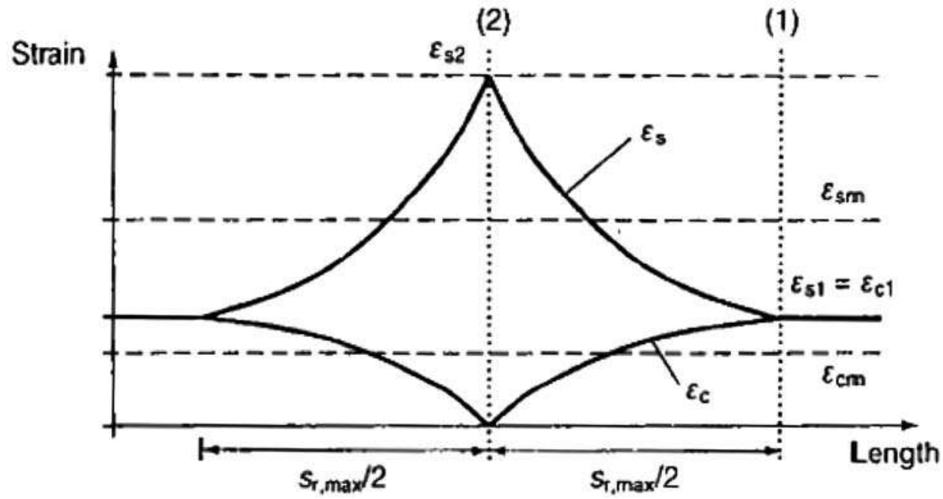
= 3.4c + 0.17 Ø/ρ. for pure bending.

= 3.4c + 0.34 Ø/ρ for axial tension.

Formula in earlier IRC code used to be 3.3c in place of 3.4c

Difference in strain between steel and concrete = $\varepsilon_{srm} - \varepsilon_{cm}$

Average strain in concrete and steel between cracks



Axial tension = N = E_s $s_{s\varepsilon}$ A_s

The average steel and concrete forces are given by

$N_s = E_s\ \varepsilon_{sm} A_s$ and $N_c = E_c\ \varepsilon_{cm}\ A_c$

$N = N_s + N_c = Es\ S_{r2}\ As = E_s\ \varepsilon_{sm} A_s + E_c\ \varepsilon_{cm} A_c$

Dividing by E_s A_s

$$\varepsilon_{s2} = \varepsilon_{sm} + (E_c/E_s)(A_c/A_s)\,\varepsilon_{cm}$$

Substitute E_s / E_c = α_e; As / A_c = ρ

$$\varepsilon_{s2} = \varepsilon_{sm} + \varepsilon_{cm}/\alpha_e\rho$$

$$\varepsilon_{sm} = \varepsilon_{s2} - \varepsilon_{cm}/\alpha_e\rho$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \varepsilon_{s2} - \frac{\varepsilon_{cm}}{\alpha_e\rho} - \varepsilon_{cm}$$

$$\varepsilon_{s2} = \varepsilon_{cm}\left(1 + \frac{1}{\alpha_e \rho}\right)$$

$$\varepsilon_{s2} = \frac{\varepsilon_{cm}}{\alpha_e \rho}(1 + \alpha_e \rho)$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \varepsilon_{s2} - \varepsilon_{cm}\frac{(1 + \alpha_e \rho)}{\alpha_e \rho}$$

$$\text{At section 1 } \varepsilon_{s1} = \varepsilon_{c1} = \frac{f_{ctm}}{E_c}$$

The Average Concrete Strain $\varepsilon_{cm} = \frac{K_t f_{ctm}}{E_c}$

$$\varepsilon_{sm} - \varepsilon_{cm} = \varepsilon_{s2} - \frac{K_t f_{ctm}(1 + \alpha_e \rho)}{E_c \alpha_e \rho}$$

$$\varepsilon_{s2} - \frac{K_t f_{ctm}(1 + \alpha_e \rho)}{E_s \rho}$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - \frac{K_t f_{ctm}(1 + \alpha_e \rho)}{\rho}}{E_s}$$

Further simplification is possible

Take K_t = 0.5 (Euro code recommends 0.6 and 0.4)

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - 0.5 f_{ctm}\left(\frac{1}{\rho} + \alpha_e\right)}{E_s}$$

Neglect when compared to $1/\rho$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - \frac{0.5 f_{ctm}}{\rho}}{E_s} \geq \frac{0.6\sigma_s}{E_s}$$

$\frac{0.5f}{\rho}ctm$ is tension stiffening.

$\left[3.4c + \frac{0.17\phi}{\rho}\right]\left[\frac{\sigma_s - \left(\frac{0.5 f_{ctm}}{\rho}\right)}{E_s}\right]$ for bending

$\left[3.4c + \frac{0.34\phi}{\rho}\right]\left[\frac{\sigma_s - \left(\frac{0.5 f_{ctm}}{\rho}\right)}{E_s}\right]$ for axial tension cases

Taking f_{ctm} as 2.8 for M35 Grade

$$\left[3.4c + \frac{0.17Ø}{\rho}\right]\left[\frac{\sigma_s - \left(\frac{1.45}{\rho}\right)}{E_s}\right] \text{ for bending}$$

And $$\left[3.4c + \frac{0.34Ø}{\rho}\right]\left[\frac{\sigma_s - \left(\frac{1.45}{\rho}\right)}{E_s}\right] \text{ for axial tension}$$

Provided

$$\left[\frac{\sigma_s - \frac{1.45}{\rho}}{E_s}\right] \geq \frac{0.6\sigma_s}{E_s}$$

Earlier IRC Code used to give $\frac{3.4c\,\sigma_s}{E_s}$

10.3.5 *Control of Shear Cracks within Webs* Cl. 12.3.5

In the prestressed concrete as well as in RCC members where shear cracking has to be checked, the code specfies the criteria for deciding reinforcement required for crack control.

10.3.6 *Control of Cracking without Direct Calculation* Cl. 12.3.6

On the basis of calculations under **Clause 12.3.4** above, Code includes a tables for different steel stresses and corresponding spacing or the specified bar diameters for crack widths of 0.2 mm and 0.3 mm tas a simplification as in the **Tables 12.2 and 12.3** for direct use in practice. When these values are ensured in design, subject to fulfilling the limitations as in 12.3.6 (3), no specific calculation for crack width is necessary.

The tabulation is based on the reinforcement stress determined from a cracked section analysis under the relevant combination of actions. The relevant effective concrete moduli for long-term and short-term loading are used. It is assumed that minimum reinforcement clauses in the code are satisfied. An advantage of this simplified approach is that many of the difficulties of interpretation of parameters and definitions involved in calculations to 12.3.4 for non-rectangular cross-sections (such as for circular sections) can be avoided.

For cracks caused mainly by direct actions (i.e. imposed forces and moments), cracks may be controlled by limiting reinforcement stresses to the values in either **Table 12.2 or Table 12.3** and corresponding bar diameters or suggested spacing. It is not necessary to satisfy bar size and spacing both. The first table sets limits on reinforcement stress based on bar diameter and the second table based on bar spacing. For cracks caused mainly by restraint (i.e., due to shrinkage or temperature), only **Table 12.2** can be used; cracks have to be controlled by limiting the bar size to match the calculated reinforcement stress immediately after cracking.

Tables 12.2 and 12.3 are produced from parametric studies carried out using the crack width calculation formulae in **Clause 12.3.4**. They are based on reinforced concrete rectangular sections in pure bending for various parameters as stated in **Clause 12.3.6 (3)** of the Code.

The **Clauses 12.3.6 (5) and (6)** address the treatment for combination of prestressing steel and un-tensioned reinforcement. The prestress can conservatively be treated as an external force applied to the cross-section (ignoring the stress increase in the tendons after cracking) and the resulting stress in the reinforcement is determined, ignoring concrete in tension as usual. The reinforcement stress derived can then be compared against the tabulated limits. For pre-tensioned beams with relatively little untensioned reinforcement, where crack control is to be provided mainly by the bonded tendons themselves, the clause permits use of any of the **Tables 12.2 and 12.3** with the steel stress taken as the total stress in the tendons after cracking, minus the initial prestress after losses. This is approximately equal to the stress increase in the tendons after decompression at the level of the tendons.

The **Clause 12.3.6 (7)** cautions about large cracks occurring in sections where there are sudden changes of stress such as at changes of section, near concentrated loads, where bars are curtailed or at areas of high bond stresses such as at the end of laps. The sudden changes of sections should generally be avoided (by tapers). However, when the same cannot be avoided, the Code requires the check by calculations. Compliance with recommended reinforcement detailing clauses normally is expected to provide satisfactory performance and will be a reasonable first assumption for check.

10.4 Limit state of Deflection **Cl. 12.4**

10.4.1 *General* Cl. 12.4.1

Excessive sagging deflections under permanent actions can generally be overcome by pre-cambering. Dynamic considerations are to be given for live load and wind-induced oscillations respectively. Resonance of bridges needs to be checked, especially for pedestrian bridges, where frequency of walking personnel can cause high amplitudes of vibrations.

The Code expects acceptable limit values for deflections should be established by owner or by agreement between the stake holders. In the absence of the same, the following deflection limits under Live Load may be considered for concrete bridges

- Vehicular : Span/800,
- Vehicular and pedestrian or pedestrian alone : Span/1000,
- Vehicular on cantilever : Cantilever Span/300,
- Vehicular & pedestrian and pedestrian only on cantilever arms : Cantilever Span/375

10.4.2 *Calculation of Deflection due to Sustained Loads* Cl. 12.4.2

In order to ensure a satisfactory behaviour in the serviceability limit state, deformations should be calculated as follows:

- The long-term deformations are calculated for the quasi permanent load combinations,
- The instantaneous deformations are calculated for the rare load combinations.

For the calculation of camber, only the quasi-permanent load combinations are considered. In order to calculate camber, the mean values of the material properties may be used.

The actual deformations may differ considerably from the calculated values; in particular if the values of the applied moments are close to the cracking moment. The difference will depend on the dispersion of the material properties, the ambient conditions, the loading conditions, the previous loading conditions, the restraints at the supports, etc.

Attention must be paid to cases where the basic assumptions of plane sections and uniformly distributed stresses across the section may not be adequate, such as in the case of shear lag effects in large prestressed structure.

For prestressed concrete members it may be necessary to control deflections assuming unfavourable deviations of the prestressing force and the dead load.

In case of cracked sections appropriate moment of inertia should be used. If the accurate determination of MI is not possible, 70 percent of uncracked MI should be used. For Prestressed Concrete members, generally uncracked MI should be used as the section is under compression.

In a cracked section under constant bending moment, changes in the stresses, strains and position of the neutral axis occur due to creep and shrinkage.

For loads with a long duration causing creep, the total deformation including creep may be calculated by using an effective modulus of elasticity for concrete $E_{c,eff}$ as below:

$$E_{c,eff} = \frac{E_{cm}}{1+\phi(\infty,t_0)}$$

where,

ϕ (∞, t_0) is the creep coefficient relevant for the load and time interval (as per **Clause 6.4.2.7**).

For Shrinkage deformations shrinkage coefficient to be used in a form of curvature is as below:

$$\frac{1}{r_{cs}} = \varepsilon_{cs}\alpha_e \frac{S}{I}$$

where,

$1/_{rcs}$ is the curvature due to shrinkage

ε_{cs} is the free shrinkage strain (as per **Clause 6.4.2.6**)

S is the first moment of area of the reinforcement about the centroid of the section

I is the second moment of area of the section

αe is the effective modular ratio = E_s / $E_{c,eff}$

The time dependent deflections are influenced by environmental and curing conditions, the age at time of loading, amount of compression reinforcement, magnitude of the stresses due to sustained loading and prestressing as well as strength gain of concrete after release of prestress. In particular, camber is especially sensitive to the concrete properties at the age of release of prestress, level of stresses, storage method, time of erection, placement of superimposed loads and environmental conditions.

Worked Example 1

Stress Control:

RCC deck slab, 350 mm thick and with M50 grade concrete, is subjected to a transverse hogging moment of 144 kNm/m under the combination of actions at SLS. This moment comprises 22.5 percent from self-weight and super-imposed dead load and 77.5 percent Live Load from traffic. The ultimate design (ULS) requires a reinforcement area of 2513 mm^2/m (20 mm diameter bars at 125 mm c/c) at an effective depth of 290 mm. Carry out the serviceability limit state checks.

Solutions:

Check for cracking:

Depth to neutral axis h/2 = 350/2 = 175 mm

Second moment of area, $I = bh^3/12 = 1000 \times 350^3/12 = 3.573 \times 10^9\ mm^4/m$.

If the section is un-cracked, compressive and tensile stress at the top and bottom of the section respectively, would be:

$\sigma_{top} = \sigma bot = M*y/I = 144 \times 10^6 \times 175/3.573 \times 10^9 = 7.05$ MPa From **Table 6.5** for M50 grade of concrete,

$f_{ctm} = 3.5 MPa < \sigma_{bot}$;

Therefore the section is cracked. Stresses therefore will need to be calculated ignoring concrete in tension. The relevant modular ratio, αeff, depends on the proportion of long-term and short-term loading.

(a) Check for stresses assuming short term creep and elastic modulus:

$E_s = 200$ GPa, and from **Table 6.5**, $E_{cm} = 35$ GPa

Thus

$E_{c,eff} = E_{cm} = 35$ GPa

The depth of concrete in compression from the following equation is:

$$dc = \frac{-A_sE_s + \sqrt{(A_sE_s)^2 + 2bA_sE_sE_{c,eff}d}}{bE_{c,eff}}$$

$$= \frac{-2513 \times 200 \times 10^9 + \sqrt{(2513 \times 200 \times 10^9)^2 + 2 \times 1000 \times 2513 \times 200 \times 10^9 \times 35 \times 10^9 \times 290}}{1000 \times 35 \times 10^9}$$

= 78.02 mm

The cracked second moment of area in steel units from following equation is:

$$I = A_s(d - d_c)^2 + \frac{1}{3}\frac{E_{c,\,eff}}{E_s} b d_c^3$$

$$= 2513 \times (290 - 78.02)^2 + \frac{1}{3} \times \frac{35}{200} \times 1000 \times 78.02^3 = 140.63 \times 10^6 \,\text{mm}^4$$

The concrete stress at the top of the section from following equation is:

$$\sigma_c = \frac{M_{Ed}}{z_c}\frac{E_{c,\,eff}}{E_s} = \frac{144 \times 10^6}{140.63 \times 10^6 / 78.02} \times \frac{35}{200} = 13.98\text{MPa}$$

From 12.2/**Clause 12.2 (2)**, the compression limit = $0.36 f_{ck}$ = 0.36 x 50 = 18 MPa > 13.98 MPa, hence OK.

The reinforcement stress from following equation is:

$$\sigma_s = \frac{M_{Ed}}{z_s} = 144 \times 10^6 \times (290 - 78.02)/(140.63 \times 10^6) = 217.06\text{MPa}$$

From Clause 12.2.2, the tensile limit = $k_3 f_{yk}$ = 0.8 x 500 = 400 MPa >217.06 MPa, hence OK.

(b) Stress checks after, all creep effect has taken place:

The creep factor is determined for the long-term loading using **Table 6.9** and is found to be ϕ = 2.2. This is used to calculate an effective modulus of elasticity for the concrete under the specific proportion of long-term and short-term actions defined using equation

$$E_{c,\text{eff}} = \frac{(M_{qp} + M_{st})E_{cm}}{M_{st} + (1+\phi)M_{qp}} = \frac{(0.225 + 0.775) \times 35}{0.775 + (1 + 2.2) \times 0.225} = 23.41\text{GPa}$$

Repeating the calculation process in (a) above, the depth of concrete in compression is 92.17 mm and the cracked second moment of area in steel units is 128.9 x 10^6 mm^4.

This concrete stress at the top of the section from following equation is:

$$\sigma_c = \frac{M_{Ed}}{z_c}\frac{E_{c,\,eff}}{E_s} = \frac{144 \times 10^6}{128.9 \times 10^6 / 92.17} \times \frac{23.41}{200} = 12.05\text{MPa}$$

< 18 MPa , hence OK.

The reinforcement stress from following equation is:

$$\sigma_s = \frac{M_{Ed}}{z_s} = 144 \times 10^6 \times (290 - 92.17)/(128.9 \times 10^6) = 221.0\text{MPa}$$

< 400 MPa, hence OK

The effect of creep here is to reduce the concrete stress and slightly increase the reinforcement stress.

Example 2

Crack checks by simplified method without direct calculation method & minimum reinforcement in RCC deck:

In this problem, 350 mm thick RCC deck slab analysed in Worked Example above is again considered, assuming the same reinforcement (20 mm diameter bars at 125 mm centres with 50 mm cover) and concrete grade M50. The exposure class is moderate. The method of **Clause 12.3.6** (without direct calculation) is used to check crack control and minimum reinforcement is checked in accordance with **Clause 12.3.3**.

Table 12.1 requires crack widths to be limited to 0.3 mm under the quasi-permanent load combination for a moderate exposure class. Thermal actions have Ψ2 = 0.5 and so should be considered. Only the secondary effects of temperature difference, however, need to be considered; the primary self equilibrating stresses may be ignored.

For convenience here, the same moments as in Worked Example 1 of 144 kNm/m, comprising 22.5 percent (DL + SDL) and 77.55 percent LL actions will be taken. The make-up would, however, be very different as discussed above; it is unlikely that temperature difference would produce effects anywhere near as severe as those from characteristic traffic actions.

From the above example, the serviceability stress in the reinforcement has already been calculated as 220.35 MPa. To comply with **Clause 12.3.6**, either:

1) The maximum bar size must be limited to 12 mm (from **Table 12.2**); or
2) The maximum bar spacing must be limited to 225 mm (interpolating within **Table 12.3**).

The provision of 20 mm diameter bars at 125 mm centres complies with the limit on bar spacing in (2) above (which permits a reinforcement stress of 300 MPa for bars at 125 mm centres) and the design is therefore acceptable. It does not matter that it does not comply with the limit in (1) as well.

Additionally, it is necessary to check that the reinforcement complies with the minimum reinforcement area required in accordance with **Clause 12.3.3**. This will rarely govern at peak moment positions, but may do so near points of contra flexure if reinforcement is curtailed:

From Clause **12.3.3**: $A_{s,min}\sigma_s = k_c k f_{ct,eff} A_{ct}$

A_{ct} is the area of concrete within the tensile zone just before the first crack forms. The section behaves elastically until thetensile fibre stress reaches f_{ctm}, therefore, for a rectangular section, the area in tension is half the slab depth, thus:

A_{ct} = 350 / 2 X 1000 = 175 X 10^3 mm^2

$f_{ct,eff} = f_{ctm}$ but not less than 2.8 MPa – clause 12.3.3. From **Table 6.5** for M50 concrete, f_{ctm} = 3.5 MPa so $f_{ct,eff}$ = 3.5 MPa.

For rectangular sections of less than 300 mm depth, k should be taken as 1.0 and can in general be taken as 1.0 conservatively. For sections with no axial load, i.e. σ_c = 0 MPa, reduces to k_c = 0.4 x (1 – 0) = 0.4.

σ_s may in general be based on the maximum allowable value from either **Table 12.2** (222 MPa for 20 mm diameter bars) or **Table 12.3** (300 MPa for 125 mm bar centres). However, for minimum reinforcement calculation, it is possible that cracking may arise mainly from restraint, rather than load and, therefore, the value from **Table 12.2** is used here in accordance with the Note to **Clause 12.3.2**. Therefore σ_s = 220 MPa assuming 20 mm bars and so:

$$A_{s,min} = 0.4 \times 1.0 \times 3.5 \times 175 \times 10^3/220 = 1280.7 \text{ mm}^2/\text{m}$$

The 20 mm bars at 125 mm centres provide A_s = 2513 mm^2/m, which exceeds this minimum value, so the design is adequate. From minimum reinforcement considerations alone, the bar centres could be increased or the bar diameter reduced in zones of low moment, but further crack control and ultimate limit state checks would then be required at these curtailment locations.

Example 3

Crack width calculation using direct method :

The above worked out example is repeated using direct calculation of the crack width:

By **Clause 12.3.4(1)**/Eq. (12.5): $W_k = s_{r,max}\,(\varepsilon_{sm} - \varepsilon_{cm})$

By **Clause 12.3.4(3)**/Eq. (12.11): $s_{r,max} = k_3c + k_1k_2k_4Ø / \rho_{p,eff} = 3.4c + 0.425k_1k_2Ø/\rho_{p,eff}$

c = 50 mm and Ø = 20 mm; therefore d = 250 – 50 – 20/2 = 290 mm and the depth to the neutral axis, x = 92.17 mm (from worked example above). By **Clause 12.3.4(3)**, this equation is valid provided the actual bar spacing is less than 5(c + Ø/2) = 5 x (50 + 20/2) = 300 mm, which is OK.

By **Clause 12.3.**4 **(2)**/Eq (12.7): $\rho_{p,eff}$ = $\rho_{p,eff} = \dfrac{A_s + \xi_1^2 A_p}{A_{c,eff}}$

where A_s = area of reinforcement = π x 10^2/0.125 = 2513 mm^2/m. Ap = 0 since no prestress. $A_{c,eff}$ = effective tension area = $bh_{c,ef}$ with $h_{c,ef}$ taken as the lesser of:

2.5(h – d) = 2.5 x (350 – 290) = 150 mm

(h – x) / 3 = (350 – 92.17) / 3 = 85.9 mm

h/2 = 350/2 = 175 mm

Thus $h_{c,ef}$ = 85.9 mm and $A_{c,eff}$ = 1000 x 85.9 = 85.9 x 10^3 mm^2/m

Therefore $\rho_{p,eff} = \dfrac{2513}{85.9\times10^3} = 0.029$

k_1 = 0.8 for high bond bars and k_2 = 0.5 for bending, therefore: $s_{r,max}$ = 3.4 x 50 + 0.425 x 0.8 x 0.5 x 20 / 0.029 = 287.2 mm

(It should be noted that the concrete cover term, 3.4c, contributes 170 mm of the total 287 mm crack spacing here, so is very significant.)

By **Clause 12.3.4 (2)**/Eq. (12.6):

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \dfrac{f_{ct,\,eff}}{\rho_{p,\,eff}}(1+\alpha_e \rho_{p,\,eff})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s}$$

From Worked example above, the reinforcement stress assuming a fully cracked section is 221.0 MPa, so the minimum

value of 0.6 $\frac{\sigma_s}{E_s}$ is 0.6 x 221.0 / (200 x 10^3) = 0.663 x 10^{-3}

Kt = 0.5 from **Table 6.5** for M50 concrete, $f_{ct,eff} = f_{ctm}$ = 3.5 MPa, α_e = 200/35 = 5.714. Therefore:

$\varepsilon_{sm} - \varepsilon_{cm} =$

$$\frac{221.0 - 0.50 \times \dfrac{3.5}{0.029}(1+5.714\times 0.029)}{200\times 10^3} = \frac{221.0-70.34}{200\times 10^3} = 0.753\times 10^{-3}$$

which is greater than the minimum value of 0.663 x 10^{-3}.

Therefore, maximum crack width, w_k = 287.2 x 0.711 x 10^{-3} = 0.20 mm < 0.3 mm limit, hence OK.

Repeating the above calculation, the reinforcement stress could be increased to 288 MPa until a crack width of 0.3 mm is reached, assuming the same ratio of short-term and long-term moments as used in this example. This compares with an allowable stress of 300 MPa from **Table 12.3** for bars at 125 mm centres as used in Worked 2nd example above. The method without direct calculation therefore gives a more economic answer here.

Example 4

Check for compression:

Problem –

A beam has width of 150 mm and depth of 300 mm. it is partially prestressed with area of steel as 100 mm^2 located at a depth of 230 mm from the top of section. Grade of concrete is M40.

a) If the stress in steel is 815 MPa, evaluate the depth of neutral axis and maximum stress and strain values in the beam.

b) Find the force in steel corresponding to decompression limit. Also evaluate the stresses in concrete.

c) Evaluate the stresses when concrete reaches permissible tensile limit in service.

d) When the beam is subjected to a moment of 19.94 kN-m, analyze the beam as partially prestressed.

Notes :

1. For the purpose of this example, the beam may be treated as weightless, i.e. no moments due to dead load.
2. The given stress value (815 MPa) is the effective prestress (after accounting for all the losses) in the cable at the given section.

Solution –

Depth of beam, $D_b := 0.30$ m

Width of beam, $B_b := 0.150$ m

Gross sectional area, $A_b := D_b \cdot B_b = 0.045$ m^2

Second Moment of intertia, $I_o := \frac{B_b \cdot D_b{}^3}{12} = 3.375 \times 10^{-4} \cdot m^4$

Distance of CG from top, $y_t := \frac{D_b}{2} = 0.15\,m$ $\qquad y_b := y_t$

Cross sectional area of steel, $A_s := 100\ mm^2$

Distance of CG of steel from CG of section, $e_s := 0.23\,m - \frac{D_b}{2} = 80 \cdot mm$

Stress in prestressing steel, $\sigma_s := 815MPa$

Hence force in prestressing steel, $P_s := \sigma_s . A_s = 81.5 \cdot kN$

Stress due to axial load, $\sigma_{csP} :- \frac{P_s}{A_b}\ 1.811 \cdot MPa$

Stress due to bending, $\sigma_{csM} := \frac{P_s \cdot e_s}{I_b} \cdot \frac{D_b}{2} = 2 \cdot 898 \cdot MPa$

Hence stress at top, $\sigma_{tc} := \sigma_{csP} - \sigma_{csM} = -1.087 \cdot MPa$(tension)

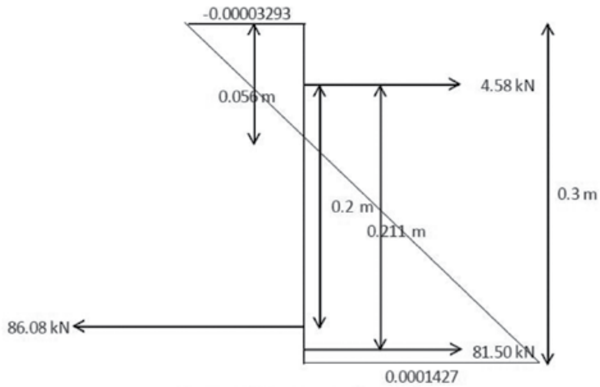
Stress at bottom, $\sigma_{bc} := \sigma_{csP} + \sigma_{csM} - 4.709 \cdot MPa$ (compression)

Modulous of elasticity of concrete, E_c := 33000MPa (for M40 grade, as per Table 6.5, IRC:112)

Modulous of elasticity of steel, $E_s := 195000MPa$

Strain at top, $\varepsilon_t := \frac{\sigma_{tc}}{E_c} = -3.293 \times 10^{-5}$

Strain at bottom, $\varepsilon_b := \frac{\sigma_{bc}}{E_c} = 1.427 \times 10^{-4}$



Strain profile along depth of girder

Neutral axis lies at a distance from top, $d_{NA} := \frac{|\varepsilon_t| \cdot D_b}{|\varepsilon_t| + |\varepsilon_b|} = 0.056\ m$

At steel level, compressive strain,
$\varepsilon_{sc} := -\frac{-\varepsilon_b}{D_b - d_{NA}} \cdot \left(e_s + \frac{D_b}{2} - d_{NA}\right) = -1.017 \times 10^{-4}$

Steel strain in prestressing cable, $\varepsilon_{s2} := \frac{P_s}{A_s \cdot E_s} = 4.179 \times 10^{-3}$

Compressive force, $F_{cb1} := -0.5\sigma_{bc}.B_b.(d_{NA}) = 4.584 \cdot kN$

Tensile force, $F_{tb1} := -0.5 \cdot \sigma_{bt}.B_b \cdot (d_{NA}) = 4.584 \cdot kN$

Force in prestressing steel, $P_s = 81.5 \cdot kN$

Net external axial force in section, $F_{net1} := F_{cb1} + F_{tb1} + P_s = 0 \cdot kN$

Net external moment in section,

$$M_{net1} := F_{cb1} \cdot \frac{2}{3} \cdot (D_b - d_{NA}) - F_{tb1} \cdot \frac{2}{3} \cdot d_{NA} + P_s \cdot \left(\frac{D_b}{2} + e_s - d_{NA}\right) = 0 \cdot kN \cdot m$$

Hence, forces are balanced

b) Load corresponding to zero strain in concrete at the level of steel

This stage is called decompression stage. Decompression stage is a stage in which the surrounding stress at the prestressing level is zero.

Note that as per IRC:112 **Clause 12.3.2 (1)**, the zero strain location should not be within 100 mm of the prestressing duct, but for the purpose of example we will consider it to be at the steel location)

In the previous example –

Strain in steel, $\varepsilon_{s2} = 4.179 \times 10^{-3}$

Strain in surrounding concrete, $\varepsilon_{sc} = -1.017 \times 10^{-4}$

Any external moment applied has to reduce the strain in surrounding concrete to 0 at the decompression limit.

Hence, for decompression state, total strain in steel, $\varepsilon_{sd} := \varepsilon_{s2} + |\varepsilon_{s0}| = 4.281 \times 10^{-3}$

Force in steel, $F_p := \varepsilon_{sd} \cdot E_s \cdot A_s = 83.483 \cdot kN$

If neutral axis is at level of steel –

Depth of NA from top, $d_{NA} := e_s + \frac{D_b}{2} = 0.23m$

let strain at bottom be ε_2 and strain at top be ε_1

As the strain diagram is linear –

$$\varepsilon_2(\varepsilon_1) := \frac{-(D_b - d_{NA})}{d_{NA}} \cdot \varepsilon_1$$

Here let $k_1 := \frac{-(D_b - d_{NA})}{d_{NA}} = -0.304$

By equating net compression with the prestressing force, we get –

$$F_p = \frac{B_b . d_{NA} \cdot E_c}{\varepsilon_1} \cdot \frac{[\varepsilon_1^2 - (k_1 \cdot \varepsilon_1)^2]}{2}$$

Implying

Top strain (compression positive), $\varepsilon_1 := \frac{2F_p}{B_b \cdot d_{NA} \cdot E_c \cdot (1 - k_1^2)} = 1.616 \times 10^{-4}$

Bottom strain, $\varepsilon_2 := \varepsilon_2(\varepsilon_1) = -4.919 \times 10^{-5}$

Hence, at decompression level –

Force in steel, $F_p = 83.483 \cdot kN$

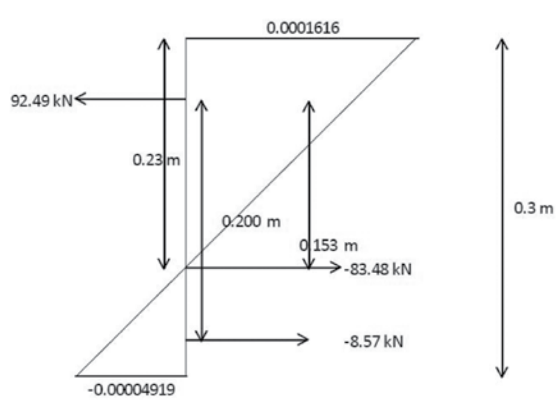
Stress in concrete at top, $\sigma_{t1} := \varepsilon_1 \cdot E_c = 5.334 \cdot MPa$

Stress in concrete at bottom, $\sigma_{b1} := \varepsilon_2 \cdot E_c = -1.623 \cdot MPa$

Compressive force in section, $F_{sc1} := 0.5 \cdot d_{NA} \cdot B_b \cdot \sigma_{t1} = 92.006 \cdot kN$

Tensile force in section, $F_{st1} := 0.5 \cdot (D_b - d_{NA}) \cdot B_b \cdot \sigma_{b1} = -8.522 \cdot kN$

Lever arm between center of compressive and tensile forces, $L_1 := \frac{2}{3} \cdot D_b = 0.2\,m$



Strain profile along depth of girder

Writing equation of moment equilibrium about center of compressive force

Net Moment at section, $M_{ulr1} := F_p \cdot d_{NA} \cdot \left(1 - \frac{1}{3}\right) + |F_{st1}| \cdot L_1 = 14.505 \cdot kN \cdot m$

Force balance –

Compressive force, $F_{cb} := 0.5 \cdot \sigma_{cb} \cdot B_g \cdot (d_{NA} - d_{cr}) = 101.94 \cdot kN$

Tensile force in concrete, $F_{tb} := -0.5 \cdot \sigma_{cp} \cdot B_g \cdot d_{cr} = -3.875 \cdot kN$

Force in steel, $F_s = 98.064 \cdot kN$

Hence, net external axial force, $F_{net} := F_{cb} + F_{tn} - F_s = 0 \cdot kN$

Distance of 0 stress line from CG of uncracked section,

$$d_{cgn} := d_{NA} - d_{cr} - d_{cg}(d_{NA}) = 29.161 \cdot mm$$

Total moment about CG of section,

$$M_{net} := F_{cb} \cdot \left[\frac{2}{3} \cdot (d_{NA} - d_{cr}) - d_{cgn}\right] - F_{tb} \cdot \left(\frac{2}{3} \cdot d_{cr} + d_{cgn}\right) + F_s \cdot \left(d_s - d_{cg}(d_{NA})\right)$$

i.e., $M_{net} = 19.94 \cdot kN \cdot m$

which is the external moment, hence forces are equilibrium

c) when concrete reaches maximum permissible tensile stress

Allowable tensile stress, $\sigma_{cy} := 3MPa$ $(f_{ctm}$ as per Table 6.5, IRC: 112 – 2011)

Corresponding strain, $\varepsilon_{cy} := \frac{\sigma_{cy}}{E_c} = 9.091 \times 10^{-5}$

Existing tensile strain in bottom fiber, $|\varepsilon_2| = 4.919 \times 10^{-5}$

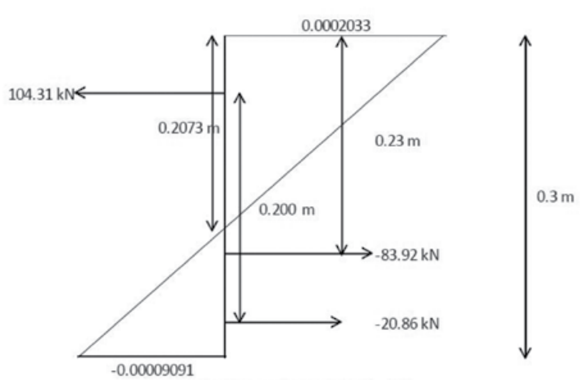
Additional strain till tensile failure, $\Delta\varepsilon_c := \varepsilon_{cy} - |\varepsilon_2| = 40172 \times 10^{-5}$

Additional moment in section, $\Delta M_{ult} := \Delta\varepsilon_c \cdot E_c \cdot \frac{I_g}{\left(\frac{2h}{3}\right)} = 3.098 \cdot kN \cdot m$

Hence, total moment at section, $M_{ult2} := M_{ult1} + \Delta M_{ult} = 17.603 \cdot kN \cdot m$

Total compression strain at top, $\varepsilon_{t2} := \varepsilon_1 + \Delta\varepsilon_c = 2.033 \times 10^{-4}$

Total tensile strain at bottom, $\varepsilon_{b2} := \varepsilon_2 - \Delta\varepsilon_c = -9.091 \times 10^{-5}$



Strain profile along depth of girder

Depth of neutral axis from top, $d_{NA} := \frac{|\varepsilon_{t2}|}{|\varepsilon_{b2}| + |\varepsilon_{t2}|} \cdot D_b = 0.2073m$

Distance of prestressing CG from top, $d_s := \frac{D_b}{2} + e_s = 0.23m$

Additional strain in steel (from 0 tension state),

$$\Delta\varepsilon_{s2} := \varepsilon_{b2} \cdot \frac{(d_s - d_{NA})}{D_b - d_{NA}} = -2.225 \times 10^{-3}$$

Hence total strain in steel, $\varepsilon_{s2} := \varepsilon_{sd} + |\Delta\varepsilon_{s2}| = 4.303 \times 10^{-3}$

Force in steel, $F_{s2} := \varepsilon_{s2} \cdot E_s \cdot A_s = 83.917 \cdot kN$

d) When the beam is subjected to a moment of 19.94 kN-m, analyze the beam as partially prestressed.

$$M_{ext} := 19.94kN \cdot m$$

In this type of problem assume depth of neutral axis, $d_{NA} := 92.25mm$

modular ration, $m_s := \frac{E_s}{E_c} = 5.090$

1) Area of cracked section, $A_{cr}(d) := B_b \cdot d + m_s \cdot A_s$

$$A_{cr}(d_{NA}) = 14428.409 \cdot mm^2$$

2) C.G. of cracked section from top, $d_{cg}(d) := \frac{B_b \cdot \frac{d^2}{2} + m_s \cdot A_s \cdot d_s}{A_{cr}(d)}$

3) Moment of inertia about CG,

$$I_{cg}(d) := B_b \cdot \frac{d^3}{12} + B_b \cdot d \cdot \left(\frac{d}{2} - d_{cg}(d)\right)^2 + m_s \cdot A_s \cdot \left(d_s - d_{cg}(d)\right)^2$$

$$I_{cg}(d_{NA}) = 2.897 \times 10^7 \cdot mm^4$$

Section modulus –

$$Z_t(d) := \frac{I_{cg}(d)}{d_{cg}(d)} \qquad Z_t(d_{NA}) = 5.4 \times 10^5 \cdot mm^3$$

$$Z_b(d) := \frac{I_{cg}(d)}{D_b - d_{cg}(d)} \qquad Z_b(d_{NA}) = 1.176 \times 10^5 \cdot mm^3$$

$$Z_{steel}(d) := \frac{I_{cg}(d)}{d_s - d_{cg}(d)} \qquad Z_{steel}(d_{NA}) = 1.643 \times 10^5 \cdot mm^3$$

$$Z_{NA}(d) := \frac{I_{cg}(d)}{d - d_{cg}(d)} \qquad Z_{NA}(d_{NA}) = 7.507 \times 10^5 \cdot mm^3$$

Prestressing force at decompression, $F_p = 83.483 \cdot kN$

Moment due to prestress at decompression, $M_p(d) := F_p \cdot \left(d_s - d_{cg}(d)\right)$

$$M_p(d_{NA}) = 14.722 \cdot kN \cdot m$$

Stress at Neutral Axis, $\sigma_{cNA}(d) := \frac{F_p}{A_{cr}(d)} - \frac{(M_{ext} - M_p(d))}{Z_{NA}(d)}$

As this value is tensile, we have to raise the neutral axis to make it 0.

By solving iteratively, the depth of neutral axis can be obtained.

Below we solve for depth of uncracked section, taking into account tensile capacity of concrete.

Maximum tensile capacity of concrete,

$$\sigma_{cp} = 3 \cdot MPa \;\; (f_{ctm} \; as\, per\, Table\, 6.5, IRC:112-2011)$$

Corrected depth of uncracked concrete,

$$d_{NA} := root(\sigma_{cNA}(d_{NA}) + \sigma_{cp}, d_{NA}) = 105.559 \cdot mm$$

Top stress, $\sigma_{t3} := \frac{P_p}{A_{cr}(d_{NA})} + \frac{(M_{next} - M_p(d_{NA}))}{Z_t(d_{NA})} = 15.387 \cdot MPa$

Hence, depth below 0 stress where section starts cracking,

$$d_{cr} := \frac{\sigma_{cp}}{\sigma_{cp} + \sigma_{t3}} \cdot d_{NA} = 17.223\ mm$$

Top strain, $\varepsilon_{t3} := \frac{\sigma_{t3}}{E_c} = 4.663 \times 10^{-4}$

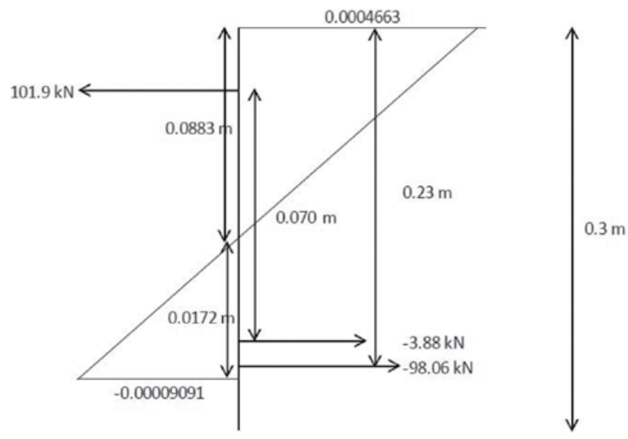
Strain in steel, $\varepsilon_{s3} := \frac{\varepsilon_{t3}}{(d_{NA} - d_{cr})} \cdot [d_s - (d_{NA} - d_{cr})] = 7.477 \times 10^{-4}$

Total strain in steel, $\varepsilon_s := \varepsilon_{s00} + \varepsilon_{s3} = 5.029 \times 10^{-3}$

Raise in strain from 0 level, $\varepsilon_{s3} = 7.477 \times 10^{-4}$

Force in steel, $F_s := \varepsilon_s \cdot E_s \cdot A_s = 98.064 \cdot kN$

Raise in stress in steel, $\Delta\sigma_s := E_s \cdot \varepsilon_{s3} = 145.81 \cdot MPa$



Strain profile along depth of girder

Force balance –

Compressive force, $F_{ch} := 0.5 \cdot \sigma_{t3} \cdot B_h \cdot (d_{NA} - d_{cr}) = 101.94 \cdot kN$

Tensile force in concrete, $F_{cb} := -0.5 \cdot \sigma_{cp} \cdot b_b \cdot d_{cr} = -3.875 \cdot kN$

Force in steel, $F_s = 98.064 \cdot kN$

Hence, next external axial force, $F_{net} := F_{cb} + F_{tb} - F_s = 0 \cdot kN$

Distance of 0 stress line from CG of uncracked section,

$$d_{cgn} := d_{NA} - d_{cr} - d_{cg}(d_{NA}) = 29.181 \cdot mm$$

Total moment about CG of section,

$$M_{net} := F_{cb} \cdot \left[\frac{2}{3} \cdot (d_{NA} - d_{cr}) - d_{cgn}\right] - F_{tb} \cdot \left(\frac{2}{3} \cdot d_{cr} + d_{cgn}\right) + F_s \cdot \left(d_s - d_{cg}(d_{NA})\right)$$

i.e., $M_{net} = 19.94 \cdot kN \cdot m$

Which is the external moment, hence forces are in equilibrium

CHAPTER 11

SECTION 13 : PRESTRESSING SYSTEM

11.1 General — Cl. 13.1

This section covers requirements of the parts of prestressing systems which are incorporated in the structure, which include anchoring devices and coupling devices for application in post tensioned construction. It also covers provision of reinforcement in anchorage zones in concrete.

11.2 Anchorages for Post Tensioning Systems — Cl. 13.2

11.2.1 *Type of Anchorages* — Cl. 13.2.1

Both, external and embedded types of anchorage systems are covered in this sub-clause. Partially or fully embedded anchorages transfer the prestressing force to the members by combination of bearing, friction and wedge action while the externally mounted anchorages transfer prestressing force of the tendons to concrete through an externally mounted bearing plate. **Figs. 13.2.1 & 13.2.2** show the typical details of two types of anchorages.

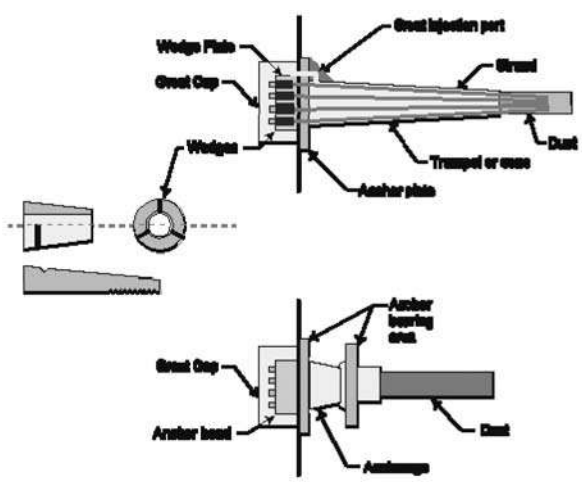


Fig. C13.2.1 Showing Partially or Fully Embedded Anchorages

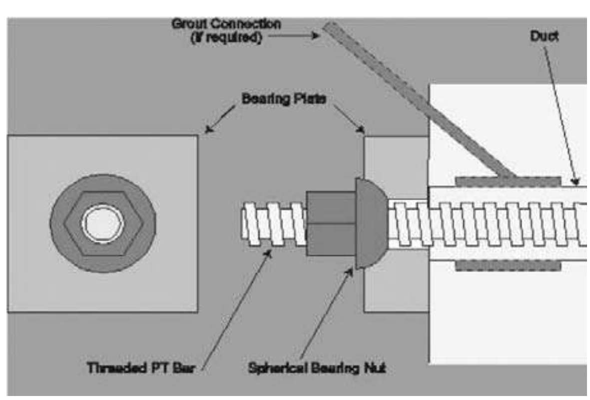


Fig. C13.2.2 Showing Externally Mounted Anchorages

C11.2.2 *Requirements of Anchorage Capacity* Cl. 13.2.2

The requirements spelt out in this clause are meant to ensure that the anchorage and coupler assemblies have sufficient strength, proper force transfer mechanism to concrete and fatigue characteristics to meet the design requirements.

C11.2.3 *Load Transfer to Concrete Element through End Block* Cl. 13.2.3

The end zone (or end block) of a post-tensioned member is a region which is subjected to high and complex state of stresses from the anchorage system This zone is termed as the anchorage block.

Minimum concrete strength required at the time when tensioning takes place and load is transferred to concrete member depends mainly on the design of anchorage, the edge distance of the anchorage and the spacing between adjacent anchorages. The anchorage zone is provided with reinforcement to take care of tensile stresses in this zone and maintain its integrity, Strictly a check of crack width is required as per this clause. However, determination of the tensile forces from the dispersal of prestress in local zone creates a multi axial state of stresses which gets modified as the concrete cracks and transfers load to the reinforcement. Even with the advent of powerful computers designers will generally find it difficult to carry out such analysis, where combined bearing, friction and wedge action is mobilized. In lieu of such check, Eurocode EC2 suggests that the reinforcement stress shall be limited to 250 Mpa to control the width of the cracks. This is similar to controlling crack widths in water retaining structures by limiting the steel stresses, Alternatively, the design of this zone can be proved by testing of typical assembly and varying the load carrying capacity and inspection/ control of crack widths as specified in the acceptance tests.

C11.2.4 *Acceptance Tests for Anchorage-Tendon Assembly* Cl. 13.2.4

The tendon and the anchorage together is called the Anchorage-Tendon assembly. The three tests recommended in the FIP document define the performance, testing procedures and quality assurance requirements necessary to make the post tensioning system safe and acceptable. These tests have also been adopted by IS Code on Prestressed Concrete (IS:1343: 2010).

(1) Anchorage efficiency Test:

The aim of static load test carried on the tendon anchorage assembly, is to assess the performance of the same in holding tendons effectively upto a minimum specified fraction of the UTS of tendons with required minimum extension and without causing failure of more than a specified % of tendons. IS:1343 specifies these limits as 95 percent of UTS and not less than 2.0 percent elongation of a 3 m long specimen. Failure limit of tendons is not over 5 percebt area of cross-section. This reduction in holding capacity is acceptable due to the low values of tensions in actual use.

Fig. C13.2.3 Anchor Efficiency Test Using Specially Cast Beam (on 19 Strands of 13 mm Dia - UTS 350 T)

(2) Dynamic load test:

The aim of dynamic load test is to determine the capacity of the tendon-anchorage assembly under load fluctuation between specified variation of stress of tendons in terms of minimum specified cycles of loading as an indication of the quality and reliability of the assembly. IS:1343 specifies these limits as loading range of 80 Mpa for steel with upper limit of 65 percent of the UTS, and number of cycles as >2.5 million. Not more than 5 percent of cross sectional area of tendons are allowed to fail. This test is carried out without using the trumpets, which bring the tendon assembly to a closer configuration for passing through the duct. If tested with the trumpet, the tendons (strands) fail at much lesser number of cycles (Thousands, rather than millions) by fretting at the points of contact between tendon and the trumpet. In this sense, the test does not replicate actual loading condition, but is purely a test of the mechanical efficiency of the anchorage block, wedges and tendon assembly in a stressed condition. This is acceptable for grouted tendons where the relative slip between the tendon and trumpet is avoided by grouting and ultimate reliance for holding tendons in the stressed condition is placed on the grout.

Fig. C13.2.4 (a) Dynamic Load Test with Specimen Mounted in Actuator of 100 T Capacity (9 Strands of 15 mm dia)

Fig. C13.2.4 (b) Dynamic Load Test : Test Specimen (Partial Tendon) with 9 Strands of 15 mm dia

(3) The load transfer test (Static or Cyclic):

This test is carried out to ascertain that the transfer of prestressing forces from the mechanical anchorage and its components to the concrete takes place without causing any excessive cracks in concrete or excessive deformation of the anchorage components.

Fig. C13.2.5 Load Transfer Test - Using High Capacity Testing Machine in Lab. for 19 Strands of 15 mm : 500T UTS Block

11.3 Mechanical Couplers — Cl. 13.3

Couplers are used for extending tendons which have been already installed, stressed and grouted in a previous section. Typical details of Fixed & Moveable coupler is shown below in **Fig. C13.3.1A and C13.3.1B** respectively.

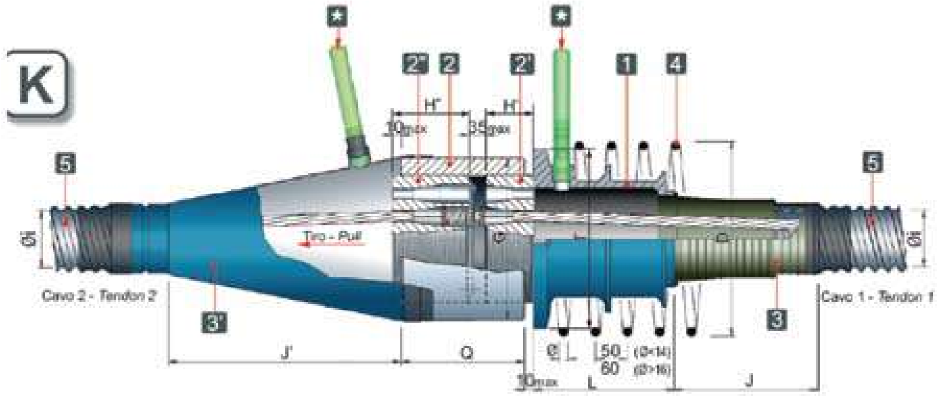


Fig. C13.3.1 A Fixed Couplers

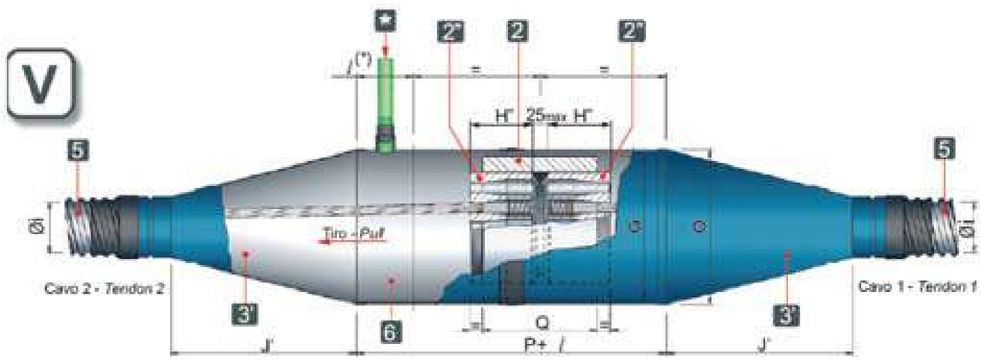


Fig. C13.3.1 B Moveable Couplers

Notations Used :

1 - Casting Unit ; 2 - Coupling Sleeve ; 2' – Stressing Head; 2" – Coupling Head ; 3 & 3' – Cone; 4 – Spiral; 5 – Sheath; 6 - Tube

11.4 End Block Design & Detailing **Cl. 13.5**

In design, attention needs to be given to highly stressed concrete compression zone in the immediate vicinity of the anchorages; bursting stresses generated in the localized area of the anchorage as complimentary transverse tensile force arise from spread of load in the mass of concrete, The design of Anchorage zones is an area of dual responsibility between the supplier of post tensioning system & the design consultant. The local anchorage zone reinforcement, which may be either spiral or an orthogonal reinforcement or combination thereof, shall generally be specified by the prestressing system supplier which has been tested for satisfactory performance in the load transfer test. The recommendations of the prestressing supplier normally suffice for satisfactory performance of the local zone behind the anchorage. This however may have to be modified for a group of anchorages in their specific configuration for a specific design situation by the design consultant. This is referred as the end block design.

11.4.1 *Bursting Reinforcement for Post-Tensioned Sections* Cl. 13.5.1

The local zone behind the bearing plate for an external anchorage or around anchorage for embedded anchorage is subjected to high bearing stress and internal stresses. The stress is 'compressive' for a distance $0.2Y_0$ from the end. Beyond that it is 'tensile' upto $2.0Y_0$. The resultant of the tensile stress in a transverse direction is known as the bursting force (F_{bst}).

The design bursting reinforcement for an individual externally mounted anchorage with individual square end block or rectangular end block is given in **Clause 13.5.1.1**. It can be observed that with the increase in size of the bearing plate the bursting force (F_{bst}) reduces. For internal (embedded) anchorages **Clause 13.5.1.2** allows use manufacturer's recommendation instead of complex detailed design.

11.4.2 *Spalling Reinforcement for Post-Tensioned Tendons* Cl. 13.5.2

Local tensile stress concentration exists along the loaded edge (surface) of the member, which require reinforcement to control compatibility induced cracking. This sub-clause of the code provides specification for check of the section as a whole containing the anchorages.

Proportioning of spalling reinforcement in the these zones may be done with strut-and-tie models Alternatively, the spalling zone reinforcement may be designed for a force equal to $0.03 * P_{max}/(f_{yd} * \gamma_{p,unfav})$ as per Euro Code EC-2. For more details and for refined analysis, Specialist literature, such as CIRIA-1 guide (Ref. 3) may be referred on use of strut and tie model as well as for distribution of this reinforcement.

11.5 Blister Blocks & Deviator Blocks

No guidance is given in the code regarding design and detailing of reinforcement for 'Intermediate Anchorage Blocks' (or Blister Blocks) and 'Deviators', which are frequently used in segmenteal construction and in PSC Box Girder structures. For the benefit of designers this document gives some guidelines which are based on specialist literatures (such as AASHTO LRFD Code, CIRIA guidelines/EC2 …etc.).

a) Blister Blocks

Blister Blocks are projections for anchoring a tendons at intermediate points from within the body of a member. Such anchorages exert compressive forces to the slab behind the anchorage. Tensile force is generated in the slab in front of the anchorage, away from tendon. Apart from the normal bursting and spalling forces, treatment of which is similar to any normal anchorages, blisters and slabs are also subjected to local shear, bending and direct force effects. The slabs at blister blocks are also subjected to high tensile stresses in the prestressing direction away from the tendon, for which tie-back reinforcement is required, to prevent cracks occuring at the blister block. Intermediate anchorages shall not be used, as far as possible, in regions where significant tension is generated at the anchor from other loads. Wherever practical, blisters should be located in the corner between flange and web where the compressive stresses at high or form a continuous rib, suitably reinforced, extended over the full flange width or web height, along which the anchorages can be spresd, CIRIA guide recommends that for the intermediately anchored tendons tie-back reinforcement into the concrete behind the anchorage point should be provided to cater for at least 50 percent of the prestressing force. This is based on the simplified assumption that in the elastic analysis the in plane force is resisted equally on both front and back side of anchorage. The contribution of residual compression at the location to resist the said tension is allowed to contribute to get net reinforcement required. This amount of reinforcement can be reduced, with increasing compressive stress in the region of the anchorage, produced by other prestressing tendons, for which detailed local analysis, using FEM may have to be carried out.

Tendons anchored in blister block usually will have curved profile within the blister zone, which will give rise to radial component of the load, which must be resisted by additional radial reinforcement anchoring back into the main body. In addition, high local bending & shear stresses develop in the slabs at the location of blister blocks. **Fig. C13.5** shows typical reinforcement detail for blister blocks.

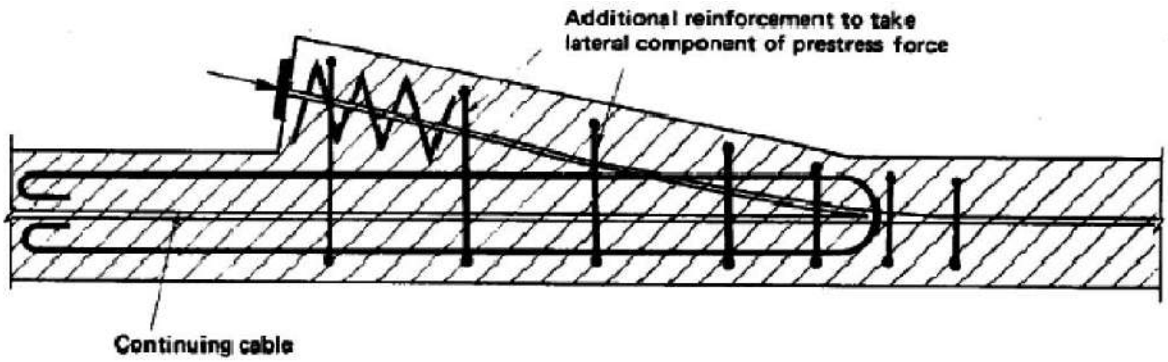


Fig. C13.5 Typical Reinforcement Detail of Blister Block

b) Deviator Blocks

The term 'deviator' refers to discrete structural element which allows change in profile of an external post tensioning tendon to be deviated from an otherwise straight alignment. The protective duct surrounding the tendon strands is normally continuous through a deviator without direct connection to it. The structural unit providing deviation may be concrete or structural steel.

The deviator block must withstand the longitudinal and transverse forces that the tendon applies to it and transmit these forces to the structure. The deviators must also ensure that the radius of curvature of the prestressing tendon does not cause any overstressing or damage to it.

A sample worked example for Design & Detailing of End Block is given at the end of this chapter for the benefit of designers.

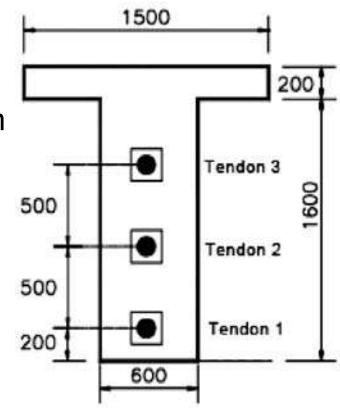
REFERENCES

1. fib Bulletin 7 : Corrugated Plastic Ducts for Internal Bonded Post Tensioning.
2. FIP Recommendations : Recommendations for the Acceptance of Post Tensioning System (June 1993).
3. CIRIA GUIDE 1 (June 1976) : A Guide to the Design of Anchor Blocks for Post-Tensioned Prestressed Concrete Members.
4. C R Hendy and D A Smith, Designer's Guide to EN 1992-2, Thomas Telford, 2006.
5. CEB-FIP Model Code 1990 : Design Code (For Concrete Structures).
6. Eurocode 2 : Design of Concrete Structures : 1992-1-1:2004.

Design Example 1

This example of a Post Tensioned PSC Girder with internally embedded anchorage is provided to understand the design process by which reinforcement of an end block is claculated.

1. Salient Details of Beam & Cabies



VIEW AT END BLOCK

a)	Number of Tendons	= 3
b)	Nos. of Strands per Tendon	= 12
c)	Area of strands	= 140 Sq.mm
d)	Breaking strength of Strands	= 1860 Mpa
e)	Dimension of Anchorage, Ypo	= 190 mm
f)	Overall Depth, hw	= 1.8 m
g)	Width of Flange, b	= 1.5 m
h)	Depth of Flange, hf	= 0.2 m
i)	Width of Web, D	= 0.6 m
j)	Area of Beam, A	= 1.26 Sq.m
k)	Cg of Beam from Bottom, y	= 1.014 m
l)	Moment of Inertia of Beam, I	= 0.3909 m^4
m)	Zt, Top Sectional Modulus of Beam	= 0.4973 m^3
n)	Zb Bottom Sectional Modulus of B	= 0.3855 m^3
o)	Cg of Tendon 1 from bottom	= 0.2 m
p)	Cg of Tendon 2 from bottom	= 0.7 m
q)	Cg of Tendon 3 from bottom	= 1.2 m
r)	Concrete Grade, f_{ck}	= M45 (Cube Strength)
s)	Min. strength at the time of stressing	= 35 Mpa

The design of Anchorage Zone of Post Tensioned members includes consideration of the following:

a) The highly stressed compression in concrete in the immediate vicinity of the anchorage.

b) Bursting stress generated in the localised area of the Anchorage.

c) Spalling at the loaded face.

d) Transverse tensile force arising from any further spread of load outside this localised area.

2. Design of Local Zone behind anchorage

As per **Clause 13.5.1.2** of IRC:112 Recommendation of the Anchorage System Supplier shall be followed in case of internal Anchorages for Anchorage Dimension, Minimum Spacing, Minimum Concrete Grade and Reinforcement for Bursting. There is no need for any additional calculation by the Designer. The anchorage dimensions, spacings, edge distances, minimum concrete grade and the bursting reinforcement details shall be obtained from the system supplier.

2.1 Calculation of Stress Behind Anchorage as per Annex.-J of EC2-2

Since IRC:112 do not give any guideline on method of checking of concrete bearing stresses behind anchorage, reference amy be made to Euro Code EC2- Annex.-J for checking of concrete stresses. The relevant part of the code is reproduced below:

The reinforcement required to prevent bursting and spalling in anchorage zones is determined in relation to a rectangular prism of concrete, known as the primary requlansation prism, located behind each anchorage. The corss-section of the prism associated with each anchorage is known as the associate rectangle. The associate rectangle has the same centre and the same axes of symmetry as the anchorage plate (which should have two axes of symmetry) and should satisfy.

$$\frac{P_{max}}{c \cdot c'} \le 0{,}6 \cdot f_{ck}(t)$$

where,

P_{max} = Maximum Force applied to the tendon (J.101)

C & C = are the dimensions of he associated rectangle

$f_{ck\,(t)}$ = Concrete Strength at the time of Tensioning (Cylinder Strength)

The prism dimension for various Tendons are as given in the Table below:

The dimensions are based on rectangular prism associated with each anchorage having the same centre.

Tendon No.	Prism Dimensions (2Yo)	
	Vertically	Laterally
1	400	600
2	500	600
3	500	600

C x C' = 400 x 600 = 240000 mm^2 for Tendon 1

= 500 x 600 = 300000 mm^2 for Tendon 2 & 3

$f_{ck\,(t)}$ = 35 Mpa (cube) = 28 Mpa (Cylinder)

P_{max} = Max. Force applied to the Tendon = 77% of the UTS (say)

= 12 x 140 x 0.77 x 1860 x 10–3 = 2406 KN

σ_1 = P_{max}/CxC' = 2406 x 10^3/240000 = 10.03 Mpa for Tendon1,

σ_2 = P_{max}/CxC' = 2406 x 10^3/300000 = 8.02 Mpa for Tendon 2 & 3

Permissible compressive stress behind anchorage = 0.6 x 2 = 16.8 Mpa < 10.03 Mps,

2.2 Calculation of Bursting Reinforcement as per Annex.-J of EC2-2

Since IRC:112 do not give any guideline on method of checking of bursting reinforcement, reference may be made to Euro Code EC2 - **Annex.-J** for checking of bursting reinforcement. The relevant part of the code is reproduced below:

(103) reinforcement to prevent bursting and spalling of he concrete, in each regularisation prism (as defined in rule (102) above should not be less than.

$$A_s = 0.15\ \frac{P_{max}}{f_{yd}}\ \gamma_{p.unfav} \text{ with } \gamma_{p.unfav} \geq 1{,}20 \qquad \text{(J.102)}$$

P_{max} = 2406 KN, f_{yk} = 500 Mpa, f_{yd} = 0.87 x 500 = 435 Mpa, $\gamma_{p.unfav}$ = 1.2

A_s = 0.15 x 2406 x 10^3/435 x 1.2 = 995.6 mm^2

Strictly, a check of crack width would be necessary. To avoid such checks, stress in reinforcement can be restricted to 250 Mpa

$A_{s,\ revised}$ = 995.6 x 435/250 = 1732 mm^2

Spiral Reinf. provided as per Catalogue of M/s. Dynamic Prestress = 2 x 16ϕ – 8 legs = 3216 mm^2

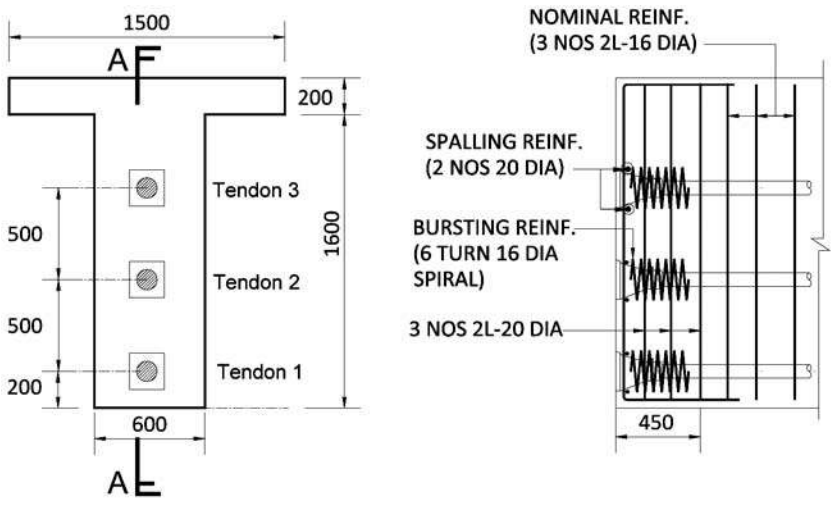
Hence OK

2.3 Calculation of Spalling Reinforcement (As per Annex.-J of EC2-2)

Clause 13.5.2 describes the need for spalling reinforcement. However the quantum of reinforcement required is not specified in the code for post tensioned beams. In absence of specific provision in IRC:112, reference may be made to Euro Code EC 2-2. As per Annex.-J of EC 2-2 Area of such reinforcement shall not be less than 0.03 x P_{max}/f_{yd} x $\gamma_{p,urfav}$ in each direction. This reinforcement needs to be distributedin each direction over the length of the rectangular prism. Toavoid crack width calculations, f_{yd} may be considered as 250 Mpa.

$A_{s,\ reqd}$ = 0.03 x 2406 x 10^3/250 x 1.2 = 346.5 mm^2

Provide 2 Nos. ϕ16 bars in each direction.



CHAPTER 12

SECTION 14 : DURABILITY

C12.1 General Cl. 14.1

In addition to having required load-carrying capacity to withstand various design combinations of ultimate loads and combinations of serviceability design actions, concrete structures are required to have adequate durability so as to provide satisfactory service life.

Durability of concrete is its resistance to various deteriorating elements that may reside inside the concrete itself or be present in the service environment to which the concrete structure is exposed. In so far as deteriorating elements residing inside the concrete are concerned, these originate from the ingredients of concrete e.g. water, aggregate, and cement, mineral and chemical admixtures. **Section 18** of this Code requires that all the ingredients used should conform to the requirements of the respective BIS Specifications.

The various actions that can affect the durability of concrete can be mechanical, physical or chemical. Impact, abrasion, erosion and cavitation are examples of the mechanical causes. Physical causes of deterioration include high temperature effects; effects of thermal gradients inside concrete, especially in mass concrete; alternate freezing and thawing and incompatibility between coefficients of thermal expansion of the aggregate and the matrix. However, it is the attack by chemical agencies like chloride, sulphate, CO_2, and chemical causes of alkali-silica and alkali-carbonate reactions that are more prominent. Deteriorating elements present in the service environment include gases like oxygen, carbon dioxide, liquids like water, chloride and/or sulphate ions in solutions and other potentially deleterious substances.

Most of the harmful reactions are expansion-producing and are activated with presence of water or moisture.. For example;

- Atmospheric CO_2 is converted to carbonic acid (H_2CO_3) in the presence of moisture, which attacks hydrated cement paste. This is called carbonation. Carbonation lowers the pH value of concrete and reduces the protection to steel by the alkalinity of the surrounding medium.
- Corrosion of steel is controlled by the rate of penetration of chloride ions. It requires the presence of oxygen and water and is aided by carbonation.
- Sulphate attack depends on the penetration of sulphate ions into the concrete. The reaction takes place in the presence of moisture.
- Frost attack concrete has to be above a critical range of water saturation for the damage due to freezing and thawing to take place.
- For alkali silica reaction (ASR) water is required to produce the expansive gel and therefore, depends upon the rate of water penetration into the concrete.

It is quite apparent that transport of fluids – both liquids and gases – into concrete is important consideration. The various processes can be summarized as follows;

- Flow of water due to application of a hydrostatic head, characterized by water permeability coefficient.
- Water absorption and uptake of water resulting from capillary forces, characterized by a sorptivity coefficient.
- Ion diffusion: movement of ions as a result of concentration gradient, characterized by ion diffusion coefficient.
- Other processes like gas diffusion, water vapour diffusion, pressure induced gas flow etc.

In view of so many possible modes, one should really be concerned with a notion of collective ‘penetrability’ of fluids.; Nevertheless, the commonly accepted term is ‘permeability’, which is mostly adopted to describe transport of fluids through concrete (Neville, 2000).

Fluid transport depends mainly upon the structure of hydrated cement paste. The micro-structure that forms upon hydration of cement consists of solids having pores of various sizes in addition to the spaces originally occupied by the water. The porosity depends upon the age, the degree of hydration, the water/cement ratio and the type of binders. The primary influence is of water/cement ratio, as depicted in **Fig. C14.1**.

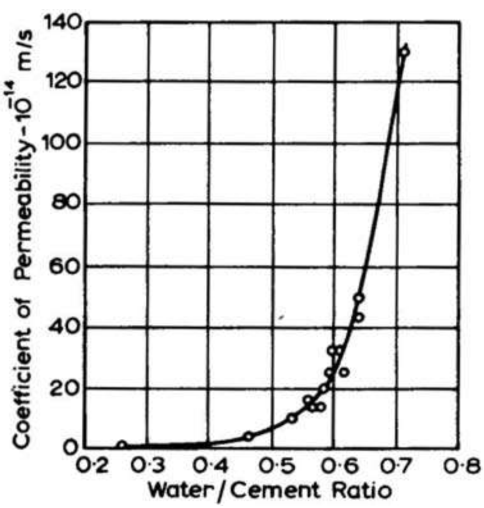


Fig. C14.1 Dependence of Permeability on the Water Cement Ratio

According to **Fig. C14.1**, the permeability at water/cement ratio of about 0.4 or below is quite less; on the other hand, permeability increases asymptotically above water/cement ratio of 0.6 or more. For a given water/cement ratio, the permeability is lower when blended cements or mineral admixtures are used than in case of OPC alone. This is due to blocking of pores due to formation of secondary hydration products, e.g. pozzolanic reactions of fly ash or silica fume with calcium hydroxide formed due to hydration of cement, or hydration of granulated slag activated and promoted by the calcium hydroxide.

The foregoing describes the intrinsic permeability of the concrete mix. Ingress of harmful elements is also facilitated by the cracking that may be caused by load effects or internal deformations like shrinkage or temperature effects. In order to limit ill effects of cracking the Code limits the crack width **Table 12.1** under **Clause 12.3.2**, permissible for different exposure conditions.

Various other factors influencing durability are mentioned in this clause. Workmanship to obtain full compaction and efficient curing are important parameters. A suitably low permeability is achieved by ensuring thorough compaction of concrete and by adequate curing. The shape or design details of exposed structures should be such as to promote good drainage of water. Member profiles and their intersection with other members should facilitate easy flow of concrete and proper compaction. Chamfering the corners or using circular cross sections reduces the ingress of fluids. Use of half joints may be avoided, unless there are adequate provisions kept in detailing for drainage and for proper inspection and maintenance.

Regular maintenance provides the opportunity to intervene if deterioration is taking place at a rate greater than expected.

C12.2 Common Mechanisms leading to Deterioration of Concrete Structures Cl. 14.2

Salient description of the mechanisms of deteriorations is included in **Annexure B-2**.

The basic mechanism of corrosion of steel, as an electro-chemical phenomenon, can be summarized in terms of an anode process and a cathode process;

- Anode: $Fe \rightarrow 2\,e^{-} + Fe^{2+}$
  (Metallic iron)
- Cathode: $\frac{1}{2}\,O_2 + H_2O + 2\,e^{-} \rightarrow 2(OH)^{-}$

In addition, the corrosion undergone by steel is due to combination of iron and (OH-) ions;

- $Fe + \frac{1}{2}\,O_2 + H_2O \rightarrow Fe^{2}+ + 2(OH) \rightarrow$ iron hydroxide (rust).

The process is schematically depicted in the **Fig. C14.2.1**. It shows that ingress of chloride ions is facilitated by presence of cracks.

For transformation of metallic iron to rust, the requirements are;

1. Iron must be available in a metallic (Fe) state at the surface of the reinforcing steel;
2. Oxygen and moisture must be available for the cathode process;
3. The electrical resistivity of concrete must be low to facilitate the electron flow in the metal from anodic to cathodic areas.

From engineering stand point; two types of situations are necessary to be considered. The steel reinforcement, being well protected in the alkaline medium in concrete, can withstand a certain amount of chloride ions to be present before corrosion can take place. On the other hand, if the passivity is destroyed because of one reason or the other and the pH of concrete is below a certain threshold value, only oxygen and water are needed for corrosion to take place. Presence of chlorides is not necessary.

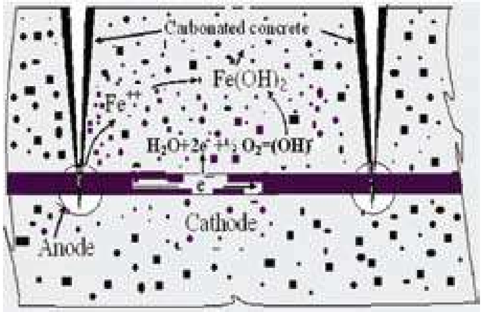


Fig. C14.2.1 Mechanism of Corrosion of Steel in Concrete – Schematic

Concrete constructions, in order to be durable, have to ensure that neither the limiting amount of chloride ions is exceeded in concrete nor the pH value of concrete is lowered below the threshold value. Practical limits of tolerable chloride ion concentration and limiting pH value are best arrived at by in-service record of concrete. The interaction shown in **Fig. C14.2.2** is based on data of a large number of concrete structures in India, which have undergone distress due to corrosion of steel (Mullick, 2000).

C12.3 Designs for Durability **Cl. 14.3**

The provisions for durability in this Section are essentially against corrosion of steel. Provisions for other mechanisms are either in terms of choice of the binder system (cements and mineral admixtures) or others measures described in **Clause 14.4**.

C12.3.1 *Classification of Exposure Conditions* Cl. 14.3.1

The first step is to establish the aggressiveness of the service environment (exposure conditions). In deciding the appropriate class of service environment, the following factors should be taken into account (fib, 2009);

- The general environmental conditions of the area in which the structure is situated,
- The specific location and orientation of the concrete surface being considered and its exposure to prevailing winds, rainfall etc.,
- Localized conditions such as surface ponding, exposure to surface run-off and spray, aggressive agents, regular wetting, condensation etc. These aspects include factors such as cladding to structure, or ponding due to poor detailing etc.

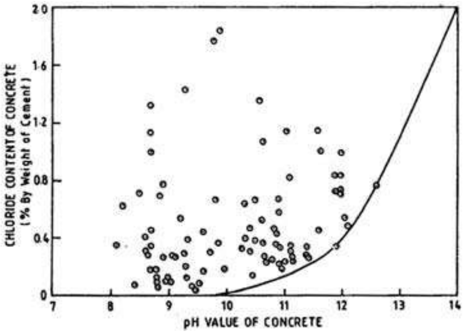


Fig. C.14.2.2 Interaction of Chloride Content and pH Value of Corrosion-Damaged Concrete Structures in India

Carbonation is not significant when the pores of concrete are saturated, because of slow rate of diffusion of CO_2 in water compared to that in air. On the other hand, if there is insufficient water in the pores, CO_2 remains in gaseous form and does not react with the hydrated cement. The highest rate of carbonation occurs at a relative humidity of 50 to 70 percent (Neville, 2000).

'Moderate' category is for situations where the chances of carbonation are insignificant because the pores of concrete are either saturated or dry. No ingress of chloride from external sources is anticipated. Inadequate workmanship can lead to corrosion of steel. Provision is also made against attack by other deleterious chemical agents, which are facilitated by the presence of moisture.

'Severe' category is for situations, where presence of moisture (wet, rarely dry) and some carbonation under humid conditions can lead to corrosion of steel. Wet, rarely dry condition includes concrete surfaces subject to long term water contact and many foundations. Concrete exposed to coastal environment can have access to chloride ions increasing the risk of chloride-induced corrosion. Concrete components exposed to industrial waters containing chloride will be included in this category. Inspite of presence of significant amount of chloride ions in sea water, risk of corrosion of concrete completely submerged in sea water below mid-tide level is comparatively less because of paucity of oxygen.

'Very severe' category is for situations where exposure to air-borne chloride ions in marine environment adds significantly to the risk of chloride-induced corrosion. Saturated concrete subjected to cyclic freezing and thawing is prone to effects of expansion due to formation of ice, leading to spalling.

'Extreme' category is for conditions, where the risk of corrosion of steel and sulphate attack are the highest such as concrete exposed to tidal variations, splash and spray zones in sea, because of accumulation of salts in the pores and accompanied by damage due to wave action. Concrete in direct contact with aggressive sub-soil/ground water can lead to severe attack on concrete in foundations, without being accessible to periodic inspection and maintenance. If harmful effluents from nearby chemical industries are discharged into the water body, where the bridge is situated, it poses serious threat to the durability of concrete. Cyclic wet and dry conditions allow accumulation and build-up of deleterious elements. Example demonstrating classification of environment for a bridge structure is given at the end.

C12.3.2 *Durability Provisions* Cl. 14.3.2

In exposure to chloride-bearing environments, build-up of chloride ions inside concrete in the early ages is due to sorption, and due to diffusion in the longer term. Models for prediction of service life of concrete adopt the concept of age-dependent 'effective diffusion coefficient'. Its value is initially high, reflecting the sorptive component, and reduces with time. The effective diffusion coefficient depends upon the type of cement, use of mineral admixtures and the water – binder ratio, and the degree of hydration of cement. Using values of effective diffusion coefficient, error function solution of Fick's second law of diffusion has been adopted to predict rate of chloride ingress (Buenfeld, 1997).

The strategy to guard against on-set of corrosion is to ensure that the amount of chloride penetrated after the design service life is less than the threshold level of chloride permitted, at the cover thickness.

Since the ingress of chloride is controlled by both the chloride diffusion coefficient and the surface chloride level, the defense against corrosion of steel in a particular service environment, as adopted in the Code, is integral of cover depth and water/cement ratio, the latter governing the chloride diffusion coefficient.

The cover requirements in **Table 14.2** are from durability considerations related to corrosion of reinforcement alone. The requirement may have to be increased from other design/detailing considerations, for which reference shall be made to **Sections 15, 16 and 17** of the Code. For Example, **Clause 15.3.1.2** specifies the requirement of cover to post tensioned ducts. In case of other mechanisms of deterioration (e.g. sulphate attack, alkali silica reaction or frost attack) depth of cover is not important consideration.

To emphasize the need of adequate impermeability, 'Acceptance Criteria' for concrete in **Clause 18.6.7** prescribes Rapid Chloride Ion Permeability test (ASTM C1202). In this test, the total electrical charge in Coulombs (ampere-seconds) passed during a specified time interval (6 hours) through a concrete disc specimen placed between solutions of sodium chloride (NaCl) and sodium hydroxide (NaOH) is measured, when a potential difference of 60 V d.c. is maintained. The charge passed is related to the penetrability of concrete to chloride ions, charge being greater, the larger the amount of chloride ions penetrated. Guidelines relating chloride permeability of concrete to the charge passed (Coulombs) during the test are given in ASTM C1202.

The following upper limits for RCPT values at 56 days are suggested for different exposure conditions:

- Severe – 1500 Coulombs
- Very severe – 1200 Coulombs and
- Extreme – 800 Coulombs

Apart from water - cement ratio and cover, **Table 14.2** of the Code also lists two other parameters – minimum cement content and minimum grade of concrete. Minimum cement content specified is to ensure adequate workability of concrete. For a given water-cement ratio, a given cement content corresponds to a particular water content, which may result in high, medium or low workability. An appropriate value has to be chosen keeping in view the placing conditions, cover thickness, and concentration of reinforcement. For the values of water-cement ratio and cement content shown in the **Table 14.2**, the water content in the concrete mix works out to 140 to 160 liters/m^3, which will generally result in low workability (0 – 50 mm slump). For higher workability, higher cement content (and higher water content, maintaining the water-cement ratio) will have to be adopted or chemical admixtures used. Minimum cement content, along with the water-cement ratio, is also required to result in sufficient volume of cement paste to overfill the voids in compacted aggregates. For crushed aggregate of 20 mm size, on which the **Table 14.2** is based, the voids content is about 25-27 percent. The values of water-cement ratio and cement content specified correspond to paste volume of about 27-28 percent, equaling the voids content of the aggregate. A fuller description is available in IS:SP-23.

The Note(5) below **Table 14.2** clarifies that minimum cement content should include all cementitious materials inclusive of additions mentioned in **Clause 18.4**, as all these binders comprise the paste volume with the water. Similarly, the water-cement ratio is water-binder ratio when mineral admixtures are added, which control the chloride diffusion coefficient.

The strength grade of concrete will be chosen from structural design considerations. Concrete mix design should be based on that strength grade (see **Clause 18.5.3**). Water-cement ratio and cement content arrived at the mix design for that grade should be checked with the provisions of **Table 14.2**. Lower water-cement ratio and higher cement content between the two should be adopted. Compressive strength of concrete alone does not guarantee durability under service conditions. The values of minimum strength grade in **Table 14.2** are those which can be generally expected with the corresponding water cement ratio and with the cements or binders available in India. So, the minimum strength grade specified is an indirect control on the durability parameters.

Stainless steel reinforcement is now permitted, in line with practice in some countries for concrete structures exposed to very severe service conditions. Since there is no Indian Standard (IS) specification for stainless steel as concrete reinforcement, provisions of British Standard BS:6744:2001 shall apply (see **Clause 18.2.3.3**).

Conventional steel or timber formwork is essentially impermeable and traps the entrapped air and water that migrate towards the formwork during compaction. As a result, water/cement ratio in the cover zone is higher than in the bulk, and forms a weak link; having lower resistance to the ingress of air, water and CO_2 etc. In comparison, controlled permeability formwork (CPF) liners act as a filter through which air and bleed water can pass and cement is retained. The passage of water and entrapped air from the concrete through the permeable formwork lining fabric results in a 'local' reduction in water/cement ratio at the formed concrete surface. This is schematically explained in **Fig. C.14.3.2.2**.

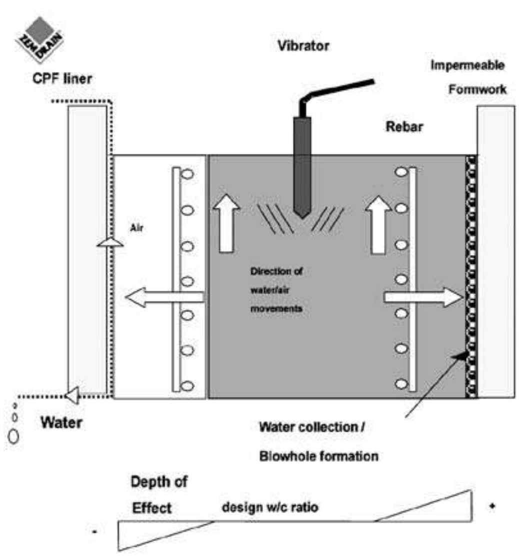


Fig. C.14.3.2.2 How CPF Liners Help in Improving Cover Concrete

It has been reported that chloride diffusion coefficient of cover concrete can be reduced upto 50 percent and service life prolonged with use of controlled permeability formwork (Mullick, 2008).

For plain cement concrete, the chances of deterioration due to corrosion of steel are not significant, and as such the maximum water-cement ratio can be exceeded by 0.05 for each category and concomitantly the strength grade lowered by 5 MPa.

C12.3.2.1 Adjustments for other aggregate sizes Cl. 14.3.2.2

If aggregate of larger size than 20 mm, is used, the requirement of water in the mix for a particular workability is reduced; so the cement content can also be reduced (to maintain the water-cement ratio).

C12.3.2.2 Chloride content Cl. 14.3.2.3

The strategy to guard against onset of corrosion is to ensure that the amount of chloride at the level of reinforcement after the design service life is less than the threshold level of chloride permitted.

The threshold is best considered in terms of corrosion risk associated with different amounts of chloride ions present in the structures. The following risk classification has been proposed (Browne, 1982) and commonly accepted:

Chloride (% by wt. of Cement)	Risk of Corrosion
< 0.4	Negligible
0.4 to 1.0	Possible
1.0 to 2.0	Probable
> 2.0	Certain

The values in **Fig. C.14.2.2** incorporating data of corrosion damaged concrete structures in India are of similar magnitude. Since the bridges will be designed for a service life of 100 years, a conservative value of 0.30 percent for RCC in 'moderate' exposure condition is specified as against 0.40 percent in many other Codes. For RCC in other exposure conditions which are more stringent and for prestressed concrete, still lower values are specified. It may be noted that the amount of chloride ion specified is on 'acid soluble' basis, indicating the total chloride ion content in the concrete. Part of the chloride ions get bound in the cement hydration products; this is called 'chloride binding'. Only the reminder is the free chloride, which is available for causing corrosion. This is expressed as 'water soluble' chloride and, as a very general guide, can be half of the total chloride content. Total acid soluble chloride is specified for ease of measurement.

C12.3.2.3 Sulphate content Cl. 14.3.2.4

Cements contain sulphate added during manufacture. More sulphate ions may come from soil, sub-soil water and ground water, sea water and effluents from industrial sources. Excessive sulphate ions will lead to sulphate attack in concrete. Code therefore specifies the limit in terms of water soluble sulphate content.

C12.3.2.4 Maximum cement content Cl. 14.3.2.5

The limit of maximum cement content is essentially from considerations of heat of hydration and thermal cracks.

C12.4 Additional Provisions for Specific Mechanisms of Deterioration Cl. 14.4

C12.4.1 *Corrosion of Reinforcement*

Other than the measures as specified in Code, which may be adopted for preventing reinforcement corrosion, following additional measures are also practiced:

a) Bright metal sheathing duct (**Clause 13.4.2**) should be used only under 'moderate' environmental conditions.

b) Use of cable pockets at the top of the deck shall be avoided from durability considerations.

C12.4.2 *Sulphate Attack* Cl. 14.4.2

The main reactions of sulphate attack on concrete are;

- Formation of sulpho-aluminates (ettringite) on reaction of sulphates with the calcium aluminate (C_3A) phase of cement,
- Formation of calcium sulphate with reaction of sulphates with the calcium hydroxide released on hydration of cement.

  Volumes of reaction products in both cases are greater than the volume of the reactants, for which there is no space in the hardened concrete. They result in expansion and spalling.
- Magnesium sulphate reacts; in addition, with the main hydration product calcium silicate hydrate (C-S-H) phase, leading to its decalcification i.e. substitution of Mg^+ for Ca^+ and formation of magnesium silicate hydrates in place of C-S-H, and other expansive salts identified above. Magnesium sulphate thus formed is more dangerous than sodium or calcium salts.

The essential solution is in having cement with lower C_3A content – as in sulphate resistant Portland cement (IS:12330). Use of blended cements (PPC or PSC) or mineral admixtures reduces the OPC component and thereby the amount of C_3A available. Consumption of calcium hydroxide by pozzolanic reaction also helps.

C12.4.3 *Alkali-Silica Reaction* Cl. 14.4.3

Annexure B-2 lists the types of siliceous rocks in aggregate which may prove to be reactive. In India, siliceous rocks like granite, granite gneiss and schist, quartzite and sandstone, when containing 'strained quartz' have been found to be reactive (Mullick, 1992). Need of appropriate methods of evaluation of potential reactivity have been emphasized.

C12.4.4 *Frost Attack* Cl. 14.4.4

Freezing of water inside concrete due to low temperature is accompanied by 9 percent increase in volume (ice has specific gravity of 0.91). Repeated cycles of freezing and thawing have a cumulative effect. In hardened concrete, there has to be space to accommodate this increase in volume, otherwise cracking will occur. Air entraining admixtures create a system of small, discrete, nearly spherical air bubbles inside concrete, typically about 50 µ in diameter, i.e. much smaller than accidentally entrapped air due to inadequate compaction. These air bubbles provide the extra space needed. Obviously, if concrete is relatively dry, the problem of freezing of water is minimized; therefore, concrete is required to be protected from saturation. Lower water-cement ratio minimizes the volume of capillary pores inside concrete and ensures strength of concrete such that it can better resist the damaging forces induced by freezing (Neville, 2000). In case of severe freezing, restriction of water-cement ratio to about 0.45 and minimum strength of 45 MPa is recommended.

C12.4.5 *Abrasion*

The compressive strength of concrete is the principal factor controlling the resistance to abrasion (Neville, 2000). The minimum strength required depends on the severity of abrasion expected. For members in contact with water having high velocity of flow and carrying abrasive bed material, concrete of higher grade may be necessary.

C12.5 Example for Environmental Classification

To decide the exposure condition (**Table 14.1**) applicable to a bridge structure or its components, the chances of carbonation and availability of chloride ions should be taken into account. A bridge structure in dry climate or its parts constantly immersed in water will have less probability of carbonation corrosion. If no ingress of chloride ions or other harmful chemicals are anticipated, such components can be classified in ‘moderate’ category. If the ambient conditions are likely to cause carbonation and ingress of chloride ions or other harmful chemicals are expected, the classification can be ‘severe’ to ‘very severe’, depending upon the relative humidity and level of chloride ions, as described in **Table 14.1**. Classification under ‘extreme’ category is self-explanatory from **Table 14.1**.

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CHAPTER 13

SECTION 15 : DETAILING : GENERAL REQUIREMENTS

C13.1 General Cl. 15.1

This Section pertains to general rules for detailing of reinforcement and prestressing, in particular. Modifications for coated steels are included in a separate clause. These rules are to be read along with the provisions of **Section 16 and 17**, which cover detailing for specific structural members and ductile detailing for bridges under Seismic Zone III, IV & V, respectively. The rules are for normal weight concrete only.

C13.2 Reinforcing Steel Cl. 15.2

C13.2.1 *Spacing of Bars* Cl. 15.2.1

The minimum limits are specified to permit concrete to flow readily into the spaces between bars and between bars and forms, without honey-comb and enabling adequate bond strength to be developed along the full length of the bar.

C13.2.1 *Permissible Mandrel Diameter for Bending of Rebars* Cl. 15.2.2

The recommended diameters for bars are given in **Table 15.1 and 15.2** for avoidance of bending cracks and of splitting/crushing the concrete inside the bend. The diameters are higher for smaller values of concrete cover as well as higher grade of reinforcing bars. **Table 15.1** is applicable for bent-up and curved bars - as applicable say for portal frames continuous beams or bent-up shear reinforcement in beams. This Table is not applicable for stirrups, link bars, loop bars etc., for which **Table 15.2** is applicable.

C13.2.3 *Bond* Cl. 15.2.3

C13.2.3.1 Bond conditions Cl. 15.2.3.1

The code describes the bond condition as “favorable” or “unfavorable” depending upon the depth of the member & inclination of the reinforcement with respect to direction of concreting. For “unfavorable” bond condition, the allowable bond stress is taken as 70 percent of allowable stress under “favorable” condition. The bond condition is to be always considered as ‘favorable’ for bars having an inclination of 45º to 90º to the horizontal. For bars which are horizontal or have inclination up to 45º to the horizontal, the bond condition depends upon depth of the member as indicated in the **Fig. 15.1.b, 15.1.c and 15.1.d**.

C13.2.3.2 Ultimate bond stress Cl. 15.2.3.2

The ultimate bond stress is a function of type and condition of reinforcing bar, tensile strength of concrete ($f_{ctk,0.05}$), concrete cover, bar spacing & transverse reinforcement. The design value is to be taken as per **Table 15.3** of the code for favorable bond condition. The bond stress increases uniformly till concrete grade M40 and almost at half the rate from M40 to M55 for plain bars. The increasing brittleness may be the reason for lesser rate of increase between M40 to M55. A comparatively sharp increase is however specified for grade M60 than for M55. The design value of bond stress is however restricted beyond M60 concrete to account for the increased brittleness.

C13.2.3.3 Basic anchorage length Cl. 15.2.3.3

The basic anchorage length of a bar is obtained by assuming an average bond stress, equal to the ultimate bond stress, which acts over the full perimeter of the bar and uniformly along its length. The multiplication factor, 'k' for the anchorage length is tabulated for various concrete grades and for various grades of reinforcement bars, up to Fe 600.(**Table 15.4**).The basic anchorage length is based on the principle that it must avoid longitudinal cracking or spalling of the concrete.

C13.2.4 *Anchorage of Longitudinal Reinforcement* Cl. 15.2.4

C13.2.4.1 General Cl. 15.2.4.1

The anchorage of bar shall be done in such a way that:

a) the bar is capable of developing the required stress and

b) the force in the bar is safely transmitted to the surrounding concrete without causing longitudinal cracks or spalling.

Generally, transverse reinforcement will be required in the anchorage zone to resist secondary forces induced locally. The same is usually available or provided in form of bars in perpendicular direction. Special attention to detailing is required where mechanical devices are used. It may be necessary to check their capacity to transmit the concentrated force by test.

C13.2.4.2 Anchorage methods Cl. 15.2.4.2

Fig. 15.2 of the Code illustrates the methods of achieving the anchorage of reinforcing bar. Anchorage of HYSD bars in tension can be provided using any of the methods the type "b" & "c" is not allowed for plain bars. Also for plain bars straight anchor is permitted for bar up to 8 mm diameter only. The Code also does not permit the use of length in bends or hooks for anchorage of bar in compression. For plain bars anchorage is traditionally with 180 hooks but these are probably missed in methods as the bars are seldom used now days.

C13.2.4.3 Design anchorage length Cl. 15.2.4.3

The design anchorage length is obtained by allowing for the beneficial effects of actual cover, confinement by transverse reinforcement, transverse clamping pressure, shape of bent up bars besides the actual force in bar, expressed as ratio of required area to actual provided (normally less than 1.0). Anchorages of shear reinforcements or links are normally to be achieved by using bends, hooks or by welded transverse reinforcements as in **Fig. 15.3** of the Code.

Use of welded bar for stirrup is a new addition in the Code. Also there is a change in the dimension of bends and hooks compared to previous practice.

C13.2.5 *Laps or Splices* Cl. 15.2.5

As the length of reinforcement bars is restricted and in many situations, likely to be less than the required length, splicing of bars will be necessary in most structural elements. Code recommends splicing by lapping of bars, welding and mechanical devices. At laps, forces are transmitted from one bar to another through the concrete surrounding the lapped bars while others have a direct force transfer.

C13.2.5.1 Splices of bars by laps Cl. 15.2.5.1

The rules are splicing of bars by lapping are detailed in this sub clause. Lapping of 100 percent of bars is permitted with restrictions specified therein. The increased lap lengths are specified for various quantum of the bars to be lapped at a section, including 100 percent lapping. Reinforcement requirement across the lap is specified for various situations.

C13.2.5.2 Splicing by welding Cl. 15.2.5.2

For HYSD bars, welding should be proposed only in special cases, when other alternative methods of splicing are not feasible. Bars of diameter greater than or equal to 20 mm must be butt welded. Welded splices have to comply with the rigorous requirements of the Code.

C13.2.5.3 Splicing by mechanical devices Cl. 15.2.5.3

A mechanical splice including its connecting element shall develop at least 125 percent of the characteristic strength, 'f_y'. This has been regarded as a minimum safety requirement for safety to prevent brittle failures. Field tests of samples from actual site supply are recommended. Reduced cover to concrete at the location of mechanical splice is permitted subject to a minimum cover of 30 mm.

C13.2.6 *Additional Rules for HYSD Bars Exceeding 32 mm in Diameter* Cl. 15.2.6

This clause specifies the additional rules for bar exceeding 32 mm which are complementary to those given in **Clause 15.2.3**. Such bars should only be used in elements where the member thickness is not less than 15ϕ. They should be anchored either as straight bars with links provided as confining reinforcements or using mechanical devices.

C13.2.7 *Bundled High Strength Deformed Bars*

C13.2.7.1 General Cl. 15.2.7.1

This clause specifies the special rules for bundled high strength deformed bars. The code allows bundling of a maximum of 4 bars in compression zone including laps and 3 bars in all other cases. All bars in a bundle must be of the same type and grade. This will ensure uniform deformation for the developed strain. Use of bars of same diameter is preferred but not mandatory.

C13.2.7.2 Anchorage of bundled bars Cl. 15.2.7.2

Bundled bars need elaborate detailing at anchorage points. These are detailed in this sub clause. Bar with equivalent diameter of 32 mm and more shall be staggered at the anchorage point in case the bar is in tension. For compression bars staggering is not required.

C13.2.7.3 Lapping of bundled bars Cl. 15.2.7.3

Special considerations are required for lapping of bundled bars. Bars in a bundle shall generally be lapped one by one with a stagger, unless the number of bars in a bundle is restricted to two with equivalent diameter of less than 32 mm. For bundle with more number of bars or equivalent diameter more than 32, specific provisions of the sub-clause are to be followed.

C13.3 Prestressing Units

C13.3.1 *Arrangement of Prestressing Tendon/Cable Duct* Cl. 15.3.1

This clause permits grouping of cables horizontally or vertically touching each other in the straight portion of the cables, subject to limitations given in **Fig. 15.9** of the Code. The Code includes the provisions for pre-tensioning tendons.

C13.3.1.1 Post-tensioning systems Cl. 15.3.2.1

Rules for use of couplers are now available in this sub-clause besides requirements of at end anchorages, intermediate anchorages and blister blocks. The requirements of residual compression at location of intermediate anchorages or additional reinforcement in lieu of it and cover to anchorages of 200 mm at local thickening or blister blocks are some major requirements. The guidelines for external tendons are not included in the Code. For this specialist literature may be referred.

C13.3.1.2 Pre-tensioning systems Cl. 15.3.2.2

The code draws distinction between the transmission length (over which the prestressing force is fully transmitted to the concrete), the dispersion length (over which the concrete stress gradually disperse to a distribution which is compatible with assumption of plane sections remaining plane) and the anchorage length (over which the tendon force at the ultimate limit state is fully transmitted to the concrete). The design rules for these are included in the Code. Complete detailing of the anchorage zone is possible with these. Basic requirements for deviators for pre-tensioning systems are now available in the Code.

C13.4 Coated Steels Cl. 15.4

In case of fusion bonded epoxy coated bars the permissible bond stress shall be considered as 80 percent of the value given in the Code for uncoated bars while anchorage and lap lengths are 1.25 times. The galvanized and stainless steel bars treated same as normal reinforcement bars.

CHAPTER 14

SECTION 16 : DETAILING REQUIREMENTS OF STRUCTURAL MEMBERS

C14.1 General — Cl. 16.1

This section gives additional rules for specific members including beams, slabs, columns, walls and some special elements such as corbels, deep beams, etc. Foundation detailing is not directly covered in this code, except for ductile detailing covered in **Section 17**. The foundation elements can be designed with corresponding clauses for other elements or reference shall be made to provisions of IRC:78 for foundation detailing. Minimum reinforcement specified in this section, for various elements ensures that when the moment of resistance of the un-cracked section is exceeded, the available reinforcement is at least able to provide a minimum moment of resistance which is at least as that of the gross concrete, so that sudden (brittle) failure is not initiated on cracking.

This section also recommends the minimum dimensions to be kept for various elements of bridge from practical considerations of constructability and workmanship.

C14.2 Columns of Solid Section — Cl. 16.2

C14.2.1 *Sectional Dimensions* — Cl. 16.2.1

Section with larger cross sectional dimension ‘b’ less than or equal to 4 times the smaller dimension ‘h’, is classified as column or pier. If the cross section is solid, it is termed as ‘solid column/Pier. In case the cross-section is hollow, it is termed as ‘hollow column/pier’. The columns are further classified as “pedestal columns” and “other columns” depending upon the le/i ratio (Refer **Clause 11.2 for details**). ‘Other columns’ include ‘long’ as well as ‘short’ columns.

For cross-section with b > 4h, the section is classified as ‘wall’.

C14.2.2 *Longitudinal Reinforcement* — Cl. 16.2.2

The minimum percentage of longitudinal reinforcement, as specified for columns is to cater for un-intended eccentricities and to control creep deformations. Under sustained loads, the load is transferred from concrete to the reinforcement because the concrete creeps and shrinks. In case the area of reinforcement in a column is lesser than minimum specified percentage, the reinforcement may yield. The minimum percentage reinforcement therefore depends upon gross area of concrete and the design axial compressive force in column.

The maximum percentage of reinforcement in columns (4 percent outside lap portion and 8 percent at laps) is chosen partly from practical considerations of placing and compaction the concrete and partly to prevent cracking from excessive internal restraints to concrete shrinkage caused by the reinforcement.

Salient detailing features of longitudinal reinforcement in “other columns” are as under:

- Minimum diameter, $\phi_{min} \geq 12$ mm

- Minimum area, $A_{s,min} = 0.10.N_{Ed}/f_{yd}$, but $\geq 0.002\ A_c$
- Maximum area, $A_{s,max} = 0.04\ A_C$ (= $0.08\ A_C$ at laps)
- Minimum number of bars in a Circular Section is 6
- For regular polygons, at least one bar is to be placed at each junction of two surfaces

C 14.2.3 *Transverse Reinforcement* Cl. 16.2.3

All longitudinal corner bars in compression should be enclosed within lateral ties to hold them in place and avoid its buckling. No longitudinal bar in a compression zone should be further than 150 mm away from a restrained bar. Transverse reinforcement is also required in columns to provide adequate shear resistance. Combination of various forms of ties/links, loops or spiral is allowed, as per choice of designer. Salient detailing features of transverse reinforcement in column areas under:

- All transverse reinforcement must be adequately anchored
- Minimum Diameter, $\phi_{min} \geq$ max [8 mm ; $\phi_{long}/4$]
- Maximum Spacing, $S_{cl,max}$ = min [$12.\phi_{long,min}$; h ; 200 mm]

C14.3 R.C. Walls and Wall Type Piers Cl. 16.3

Wall is defined as vertical load bearing member whose larger lateral dimension is more than 4 times its least lateral dimension (**Clause 7.6.4.1**). In case a wall is subjected to predominantly out of plane bending (e.g. retaining wall, solid abutment or vertical of box structure), the provisions for slab are applicable for the said wall. In situations where a wall is also subjected to high concentrated load (e.g. plate type pier), the design and detailing may be based on strut-and-tie model or an appropriate FEM model.

C14.3.1 *Vertical Reinforcement* Cl. 16.3.1

Salient detailing features of vertical reinforcement in wall are as under:

- Minimum diameter, $\phi_{min} \geq 12$ mm
- Minimum area, $A_{s,vmin} = 0.0024\ A_C$ (minimum half at each face)
- Maximum area, $A_{s,vmax} = 0.04\ A_C$ (At lap location specific percentage is not indicated in the Code but $0.08\ A_C$ may be permitted at laps with a cue from provision for column)
- Maximum spacing, $S_{v,max}$ = 200 mm

C14.3.2 *Horizontal Reinforcement* Cl. 16.3.2

Salient detailing features of transverse reinforcement in wall is as under:

- Horizontal reinforcement shall be placed between the vertical reinforcement and the face of wall.
- Minimum diameter, $\phi_{min} \geq$ max [8 mm ; $\phi_{long}/4$]
- Minimum area, $A_{s,hmin}$ = max [$0.25\ A_{s,v}$; $0.001A_C$]

- Maximum Spacing, $S_{h,max}$ = 300 mm

C14.3.3 *Transverse Reinforcement* Cl. 16.3.3

Where vertical reinforcement exceeds 0.02 A_C, stirrups are required as for columns.

C14.4 Hollow Piers/Columns Cl. 16.4

In case of hollow piers/columns, following conditions should be satisfied:

- Largest overall dimension ≤ 4 times smallest overall dimension
- Ratio of effective length/radius of gyration ≥ 12
- Wall thickness ≥ 300 mm
- Two ends of the hollow section shall be capped by thick solid RCC slab having thickness ≥ 1/3rd the clear inside dimension of hollow section in the direction of spanning of slab and integrally connected to walls.

C14.5 Beams Cl. 16.5

C14.5.1 *Longitudinal Reinforcement* Cl. 16.5.1

C14.5.1.1 Minimum and maximum reinforcement percentage Cl. 16.5.1.1

- Minimum long. tensile reinforcement :
- $A_{sl,min}$ = max. [0.26 (f_{ctm}/f_{yk}) b_t.d ; 0.0013 b_t.d]
- Maximum long. tensile reinforcement :
- $A_{sl,max}$ = 0.025 A_C other than at Laps
- Maximum total long. reinforcement, $A_{st,max}$ = 0.04 A_C

C14.5.1.2 Tensile steel in flanged section Cl. 16.5.1.2

The total amount of tensile reinforcement A_{sl} of a flanged cross-section (e.g. at intermediate supports of continuous 'T' beam) need not be within web but may be spread over the effective flange width of the beam section.

C14.5.1.3 Curtailment rules for longitudinal reinforcement Cl. 16.5.1.3

This clause provides altogether different shift rules for curtailment compared to earlier practice for the length of the longitudinal tension reinforcement and anchorage in tension. Salient features of shift rule are as under:

- Sufficient reinforcement should be provided at all sections to resist the envelope of the acting tensile forces, including the effect of inclined cracks in webs and flanges.
- For members with shear reinforcement, the additional tensile force, ΔF_{td}, should be calculated according to **Clause 16.5.1.3 (2)**.
- For members without shear reinforcement ΔF_{td}, may be estimated by shifting the moment curve by a distance a_l = d according to **Clause 16.5.1.3 (3)**. This shift rule may also be applied as an alternative for members with shear

reinforcement, where $a_l = z (\cot\theta - \cot\alpha)/2 = 0.5\, z \cot\theta$ for vertical shear links.

- Depending upon the angle of strut considered in design, the value of 'al' can vary from 0.45 d (for θ = 45º) to 1.125 d (for θ = 21.8º).
- For reinforcement in the flange, placed outside the web, 'a_l' should be increased by a distance equal to the distance of bar from web face.
- The curtailed reinforcement should be provided with an anchorage length $l_{b,net}$, but not less than 'd', effective depth from the point where it is no longer needed. The diagram of the resisting tensile force should always lie outside the envelop line of the acting tensile force, displaced as described above.

  Fig. 16.2 of the code should be referred, which clearly explains the shift rule philosophy.

C14.5.1.4 Anchorage of span reinforcement at an end support Cl. 16.5.1.4

- $A_{sb,sup}$ bottom steel at support ≥ 0.25 times $A_{sb,span}$, steel provided in the span. The code recommends that the bottom reinforcement should be anchored to resist a force of V_{Ed} x $(a_l/d) + N_{Ed}$, as defined in **Clause 16.5.1.4 (2)**.
- The anchorage length is required to be measured from the line of contact of the direct support. Minimum length should be = 2/3rd/$_{bnet}$.
- For indirect support, it is measured from a distance of1/3rd width of support from the face of support, beyond which a minimum length of &$_{bnet}$ should be provided.

C14.5.1.5 Anchorage of span reinforcement at intermediate supports Cl. 16.5.1.5

- $A_{sb,sup}$ bottom steel at intermediate support ≥ 0.25 times $A_{sb,span}$, steel provided in the span
- Anchorage length, l, ≥ 10ϕ for straight bars
- Anchorage length, l, ≥ d_m for hooks and bends
- Continuous reinforcement is recommended at intermediate supports to resist accidental loads. However this does not mean that the intermediate support must have width greater than 20ϕ, as the bars from each side can be made continuous or lapped

C14.5.2 *Shear Reinforcement*

- Shear reinforcement should form an angle of 45º to 90º w.r.t longitudinal member axis. It can be in the form of links or a combination of links, bent-up bars or assembly in the form of cage. At least 50 percent of the bars should be in the form of links.
- Though these recommendations are practicable for prismatic beams, it would be difficult to implement this clause in case of haunched beams, in

which case the reinforcement shall be provided perpendicular surface of the element.

- Minimum shear reinforcement, $\rho_{w,min}$ = 0.072 (f_{ck})0.5/f_{yk}
- Maximum longitudinal spacing of :
  - i) Links, $S_{l,max}$ = 0.75d x (1 + cot α)
  - ii) Bent-up bars, $S_{b,max}$ = 0.6d x (1 + cot α)
- Maximum transverse spacing, $S_{t,max}$ = 0.75d ≤ 600 mm

C14.5.3 *Torsional Reinforcement* Cl. 16.5.3

- Torsion links need to be in form of closed loops.
- Provision of shear reinforcement in **Clause 16.5.2** is generally sufficient to provide the minimum torsion links required.
- There should be at least one longitudinal bar at each corner of the torsion link. Others longitudinal bars need to be distributed uniformly along the inner periphery.
- Longitudinal bars within loop for torsion shall have spacing ≤ 350 mm.
- The limiting spacing of torsion links is u/8, where 'u' is the outer perimeter of the member and shall satisfy **Clause 16.5.2 (7)** as well.

C14.5.4 *Surface Reinforcement* Cl. 16.5.4

Skin reinforcement (or surface reinforcement) may be provided to control cracking and to ensure adequate resistance to spalling of the cover as in members with depth more than 1 m or in situations where cover to reinforcement provided is more than the minimum cover required as per **Section 14** of the code. This is also required in situations where bundled bars or bars of size greater than 32 mm have been used (refer 15.2.6 (10). In case of deep beams this reinforcement generally comprises of smaller diameter bars placed in the tension zone within the links. Other requirements are:

- Maximum spacing of the bars = 200 mm
- Minimum area of surface reinforcement, $A_{s,surf}$ ≥ 0.01 $A_{ct,ext}$, where $A_{ct,ext}$ is the area of cover portion outside the stirrups/links.
- The surface reinforcement may be taken into consideration as a part of the longitudinal bending steel or as link. Then it shall satisfy for the same saw well.

C14.6 Solid Slabs Cl. 16.6

The clause applies for members defined as slab i.e. span to thickness ration is equal to more than 5.

C14.6.1 *Flexural Reinforcement* Cl. 16.6.1

- Primary reinforcement – minimum and maximum areas as per provisions for beam.

- Curtailment, shear, torsion – as per provisions for beam, except that for the shift rule, a_l = d may be used.
- Secondary transverse reinforcement ≥ 20 percent of the main reinforcement.
- Maximum spacing of primary reinforcement - minimum of [2h, 250 mm], where ‘h’ is the overall depth of slab.
- Maximum spacing of secondary reinforcement: minimum of [3h, 400 mm].
- For slab that is part of flange of a beam, if primary slab reinforcement is parallel to the beam, 60 percent of span reinforcement in slab shall be provided at top over continuous support, unless quantum is specifically calculated. The detailing shall be as in **Fig. 16.5**.

C14.6.1.2 Anchorage of bottom main steel at intermediate support Cl. 16.6.1.2

Provisions for beam are applicable for slab also.

C14.6.1.3 Reinforcement in slabs near end-support Cl. 16.6.1.3

- $A_{ss,sup}$ bottom steel at support ≥ 0.5 $A_{ss,span}$ provided in the span. The code recommends that the bottom reinforcement should be anchored to resist a force of V_{Ed} x (a_l/d) + N_{Sd}, as defined in **Clause 16.5.1.4** applicable for beams.
- The top reinforcement shall be capable of resisting 25 percent of the span moment in situations where partial fixity can occur but is ignored in analysis.

C14.6.1.4 Reinforcement at the free edges Cl. 16.6.1.4

Along a free (unsupported) edge, a slab should normally be stiffened. Code specifies the requirements for edges along the traffic and across the traffic directions. For the former it is either in terms of specific resistance capacity or in form of the reinforced kerb of crash barrier. For the latter it shall be by ensuring the resistance capacity.

The edges not requiring carrying traffic not covered by earlier described conditions are to be detailed with suitable closing reinforcement comprising of transverse U-bars enclosing the longitudinal bars, as per **Fig.16.6** of the code. The available slab bars shall be considered and detailed accordingly.

C14.6.2 *Shear Reinforcement* Cl. 16.6.2

Slabs requiring shear reinforcement should have a depth of at least 200 mm in order for the links to contribute to shear resistance. The general detailing rules for shear reinforcement are as for beams apply, except:

- In slabs if V_{Ed} ≤ 1/3 x $V_{Rd,max}$, all of the shear reinforcement may be provided either by bent-up bars or of shear assemblies.
- Maximum longitudinal spacing of bent-up bars can be increased to S_{max} = d
- Maximum longitudinal spacing of successive series of links is given by : s_{max} = 0.75d (1 + cot α)

C14.7 Corbels — Cl. 16.7

Two cases are covered for design of corbel using strut & tie method:

a) $a_c \leq 0.5 \times h_c$

In addition to the main reinforcement provided at the top of corbel (with total area of $A_{s,main}$), closed horizontal or inclined links (secondary tie bars) to be provided distributed within the depth of the corbel, where $A_{s,link} > 0.25\ A_{s,main}$. Reinforcement shall be detailed as in **Fig. 17**. (a) or (c) of the Code.

b) $a_c > 0.5 \times h_c$

In addition to the main tension reinforcement, vertical links/stirrups are required where the shear force exceeds the concrete shear. Same shall be such that $A_{s.stirrup} \geq 0.5 \times f_{cd}/f_{yd}$. Reinforcement shall be detailed as in **Fig. 17**. (b) of the Code.

C14.8 Articulations (Half Joints) — Cl. 16.8

Articulation (or half Joint) is usually provided in bridges at a connection where the construction depth is limited or in case of suspended span resting on tip of a cantilever. The treatment of articulation shall be similar to a corbel or a nib. The Code cautions to avoid edge stress concentration and to ensure rotation capacity.

C14.9 Deep Beams — Cl. 16.9

A deep beam is a member whose span is less than 3 times the overall section depth. In bridge design, this will most frequently apply to diaphragms in box girder, cross girders between bridge beams etc. Specific detailing rules are included in the Code.

C14.10 Members with Unbonded Tendons — Cl. 16.10

For the purpose of detailing, members with only unbonded tendons shall be treated the same way as reinforced concrete members. In case members are with a combination of bonded and unbonded tendons, requirements of bonded tendons will apply.

C14.11 Concentrated Forces — Cl. 16.11

C14.11.1 *General* — Cl. 16.11.1

The rules in this sub clause apply to zones where concentrated load acts in superstructures or substructures. The Eqn. 16.13 is derived from the confinement provided to the core by the surrounding concrete & supplementary reinforcement, whose perimeter is defined by 'b_2' and 'd_2' in **Fig. 16.9**. The surrounding area resists transverse expansion of the core by acting in ring tension prior to spalling. The distribution of load should be such that adjacent areas do not overlap and the slope should not exceed 1H:2V.

The value of $F_{R,du}$ should be reduced if the load is not uniformly distributed on the loaded area A_{co}. Though no guidance is given in the code, the bearing pressure check could be based on the peak pressure in case it is not uniform. Also in case shear force is less it can be ignored. For higher shear, the code recommends adoption of 3D – FEM analysis.

C14.11.2 *Zones Below Bearings* Cl. 16.11.2

- The minimum distance between the edge of the loaded area and edge of the section should not be less than 50 mm or less than 1/6th of the corresponding dimension of the loaded area.
- Additional sliding wedge mechanism needs to be checked to avoid edge sliding. The reinforcement needs to be provided parallel to the loaded face for a depth as indicated in **Fig. 16.10** of the code. The amount of reinforcement is given by $A_t f_{yd} \geq F_{Rdu}/2$, which should be uniformly distributed over height 'h' (Ref.1). The provided reinforcement needs to be suitably anchored, necessitating closed links.

C14.12 Indirect Supports Cl. 16.13

Suspension reinforcement needs to be provided where the load from the primary (supported) beam is transferred to the support indirectly through the cross (supporting) beam. Such provisions arise in situations e.g. where the bearings are not provided directly under the girder. In general suspension reinforcement will add to reinforcement for other effects.

C14.13 Anchorage Zones for Post Tensioning Forces Cl. 16.14

Anchorage zone is defined as a zone within which the concentrated forces of post-tensioned anchorages disperse and spread over the full section of the prestressed structural element. Usually this zone in length is taken as equal to the larger of depth/width of the section (Ref.1).

REFERENCE

1. Designers' Guide To EN 1992-2; Eurocode 2 : Design of Concrete Structures ; Part 2 : Concrete Bridges by C.R. Hendy & D. A Smith.

CHAPTER 15

SECTION 17 : DUCTILE DETAILING FOR SEISMIC RESISTANCE

C15.1 GENERAL Cl. 17.1

These provisions are based on the current national and international practices. Seismic design and detailing are still evolving globally, making its codification an ongoing process. The bridge designers are therefore encouraged to refer to specialist literatures wherever required to augment the design and detailing practices.

The purpose of these design requirements for bridges in Seismic Zone III, IV & V are to ensure that the bridge substructures are provided with adequate ductility to ensure that the required overall global ductility of the structure is met and the plastic hinge formation is preferably within the substructure rather than at foundations, which is difficult to inspect and repair.

In order to ensure that plastic hinge is formed at pre-determined location in pier (generally at pier base), the Code allows curtailment of longitudinal reinforcement only in case of tall piers only after the designer ensures that the potential of formation of plastic hinge just beyond the point of curtailment is avoided. The Code however is silent on the definition of 'tall pier' for the purpose of this clause and reference may be made to other national/international standards/literature for the definition of tall pier in this regard.

C15.2 Concrete Piers/Columns Cl. 17.2

C15.2.1 *Confinement* Cl. 17.2.1

Requirements of this section are concerned with confining the concrete and providing increased lateral support to the longitudinal reinforcement in plastic hinge zone. The main function of the transverse reinforcement for confinement is to ensure that the axial load carried by the bridge pier after spalling of the concrete cover will at least be equal to the load carried before spalling & to ensure that buckling of the longitudinal reinforcement is prevented. Thus, the spacing of the confining reinforcement is also important.

C15.2.1.1 General requirements Cl. 17.2.1.1

- Eq.17.1 gives the minimum axial stress in piers beyond which the confinement of the compression zone is required. The lightly loaded bridge piers having normalized axial force less than 0.08 times the capacity of concrete section (calculated without reinforcement) will not require confinement reinforcement.
- Eq.17.2 gives the required quantity of confining reinforcement, with the intent that spalling of cover concrete will not result in a loss of axial load strength of the column. This is defined in the form of mechanical reinforcement ratio. ω_{wd} in Eq. 17.2 is also referred to in Eq. 17.5 & 17.7.

C15.2.1.2 Minimum confining reinforcement Cl. 17.2.1.2

- For rectangular stirrups and cross-ties, the minimum design confining reinforcement (ω_{wd}) is greater of two values given in Eq.17.5. The second term in the Eq. 17.6 is applicable in case the % vertical reinforcement in pier is more than 1 percent. In case the percent reinforcement is less than 1 percent, this term can be ignored in the calculation. The minimum reinforcement condition is to be satisfied in both directions. For circular sections, the minimum confining reinforcement provided by hoops/spirals determined as higher of two values given in Eq. 17.7.
- The value of ω_{wd} is larger for circular sections since the circular sections do not have lateral ties. In rectangular sections, there are numbers of lateral ties which are anchored in central core concrete.

C15.2.1.3 Spacing of ties/hoops/spirals Cl. 17.2.1.3

The requirement that spacing of hoops or ties shall not exceed one-fifth of the minimum member dimension for rectangular sections or one-fifth the diameter of concrete core for circular sections is prescribed to obtain adequate concrete confinement.

The requirement that spacing of hoops or ties not to exceed five times the smallest longitudinal bar diameter is intended to prevent buckling of longitudinal reinforcement after spalling.

Worked Examples:

Example 15.1 Confinement reinforcement for Rectangular Piers

Width of Pier, B	2 m		
Depth of the Pier, D	2.5		
f_{ck}	35 Mpa	f_{cd}	15.63 Mpa
f_{tk}	500 Mpa	f_{yd}	434.78 Mpa
Long. Reinforcement ratio	0.02		
Clear cover	50 mm		
Designed axial load, N_{ED}	1600 Tonnes		
Dia of tie, d	12 mm	*Asw, B*	1356.48 sq. mm
No. of legs along width	12	*Asw, D*	1808.64 sq. mm
No. of legs along depth	16		
Spacing of ties	150 mm		
Gross area of concrete section *Ac*	5 sq.m		
Confined concrete area, *Acc*	4.56 sq.m		
Normalized axial force, η_k	0.0914	(Confinement is required)	
Volumetric ratio, ρ_w (B)	0.0045		
Volumetric ratio, ρ_w(D)	0.0048		
$\omega_{wd,}$B	0.1258	(O.K.)	

$\omega_{wd,}$ D	0.1341	(O.K.)
$\omega_{w,req}$	0.0732	
$\omega_{w,min}$	0.12	
Designed ω_{wd}	0.12	

Example 15.2 Confinement Reinforcement for Circular Piers

Dia of Pier, D	2.2 m		
f_{ck}	35 Mpa	f_{cd}	15.63 Mpa
f_{tk}	500 Mpa	f_{yd}	434.78 Mpa
Long. Reinforcement ratio	0.02		
Clear cover	50 mm		
Designed axial load, N_{ED}	1200 t		
Dia of hoop/spiral, d	20 mm	Asp	314 sq. mm
Spacing of hoop/spiral	90 mm		
Gross area of concrete section Ac	3.7994 sq.m		
Confined concrete area, Acc	3.46185 sq.m		
Normalized axial force, η_k	0.0902	(Confinement is required)	
Diameter of hoop/spiral, D_{sp}	2.14 m		
Volumetric ratio, ρ_w	0.0065		
ω_{wd}	0.1814	(O.K.)	
$\omega_{w,req}$	0.102		
$\omega_{w,min}$	0.18		
For ω_{wd} = 0.18, ρ_w =		0.006472	
Required Asp		311.64 sq. mm	
Required d		19.92 mm	
Designed ω_{wd}	0.18		

C15.2.1.4 Length of potential plastic hingeso Cl. 17.2.1.4

This section stipulates the minimum length of potential plastic hinge zone and minimum length beyond the plastic hinge zone over which closely-spaced transverse reinforcement is required to be provided within the member. This is based on normalized axial force, η_k.

C15.2.2 *Buckling of Longitudinal Compression Reinforcement* Cl. 17.2.2

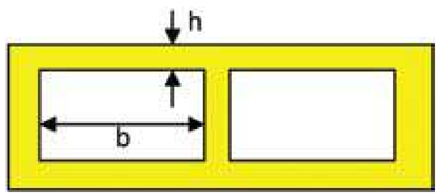
Once the cover concrete in the plastic hinge zone spalls due to several hysterics of the seismic action, the longitudinal bars are prone to buckling. The transverse reinforcement shall be adequate to prevent this buckling by providing transverse reinforcement at spacing not exceeding 5 times the smallest diameter of the longitudinal bars.

C15.2.3 *Other Rules* Cl. 17.2.3

Due to loss of concrete cover in the plastic hinge zone as a result of spalling, proper anchorage and careful detailing of the confining steel is required for its effectiveness.

Lapping of longitudinal reinforcement with dowels is also strictly prohibited within plastic hinge region, generally at the column base, where plastic hinge is likely to form. This is because the splice occurs at the location where requirement of bond is critical. Further, lapping at the base is likely to stiffen the base and shift up the plastic hinge beyond the lapping region, thereby increasing the seismic demand.

C15.2.4 *Hollow Piers* Cl. 17.2.4



- Dimension b/h as shown above for rectangular hollow pier shall not exceed 8 in the plastic hinge region. In case of hollow circular piers, the ratio d_i/h shall not exceed 8, where 'd_i' is the inner diameter of the hollow pier.
- No confinement is necessary in case the normalized axial force, $\eta_k \le 0.2$. However the requirement of controlling buckling as per **Clause 17.2.2** is required to be met.

C15.3 Foundations Cl. 17.3

C15.3.1 *General* Cl. 17.3.1

It is desirable that inelastic response in strong ground shaking occurs above the foundations, as repairs to foundations can be extremely difficult and expensive. Code does not allow foundations to enter within plastic range under design seismic action, except for pile foundation in unavoidable circumstances. This is generally ensured by:

a) Designing the foundation for 25 percent additional base shear on account of seismic action (as specified in IRC:6-2010), so that the weakest link is within the pier or at the pier base (capacity protection).

b) Ensuring adequate ductility at the plastic hinge location by proper detailing and confinement for pile foundations as hinge formation is allowed if unavoidable.

C15.3.2 *Pile Foundation* Cl. 17.3.2

In situations where it is not possible to ensure hinge formation at the pier base in any direction (e.g. in case of plate type piers), it may not be possible to avoid localized hinge formation in the pile foundation due to seismic loads. In such cases, ductile behavior of the piles shall be ensured by following measures:

- Treat the following locations in pile for potential plastic hinge :
  - ➢ Top of pile
  - ➢ Location of maximum bending moment
  - ➢ Interface of soil layers with marked difference in shear deformability

It is to be understood that for a pile which is fixed at top with pile cap, there can be two locations where plastic hinge may form at the time of collapse and therefore will require confining reinforcement at all such locations to ensure large deformation without failure. These locations are generally the top of pile and location of second peak bending moment at intermediate level. The scour level or fixity point below scour level can be likely locations. In situations, where the sub-strata is such that there is an interface of soil layers with marked difference in shear deformability which shifts the location of second peak bending moment at the intermediate level, the plastic hinge is likely to form at this interface in addition to top of pile.

- Provide confinement reinforcement at pile top, along a vertical length equal to 3 times the pile diameter if a large pile group prevents the rotation of pile cap.
- In case the pile is analyzed by approximate method (e.g. equivalent cantilever method as per IS:2911), confinement reinforcement is also required to be provided for a length of two times the pile diameter on either side of the point of second peak maximum moment in pile at intermediate level (other than at pile head).
- In case more accurate analysis using soil-structure interaction is adopted for pile foundation design (e.g. using soil springs), confinement reinforcement needs to be provided at the top of pile where bending moment is maximum as well as any other locations where peak moments occur.

CHAPTER 16

SECTION 18 : MATERIALS, QUALITY CONTROL AND WORKMANSHIP

C16.1 General — Cl. 18.1

This section deals with the material properties and specifications for procurement of Reinforcement, Prestressing Steel and Concrete.

It is important to note that although, primarily these are specified in terms of the Indian Standards governing these materials, they are not limited to the same. If the materials used in the design conform to other countries' codes, they can be used as long as they meet the minimum specifications of the BIS Codes. For materials where BIS Codes are not available, other Codes are specified.

The Code makes it clear that for assessment of properties of materials in existing bridges, the standards and specifications prevailing at the time of construction of the bridge only are to be considered.

In the following explanatory part only the noteworthy points are discussed, since the Code is very clear on the aspects of specifications for procurement. For items which involve site fabrication or design and making and placing concrete, detailed guidelines are given.

C16.2 Untensioned Steel — Cl. 18.2

C16.2.1 *Products with Improved Corrosion Resistance* — Cl. 18.2.3

There are many methods in use for providing additional protection against corrosion. The acceptable methods to the code are:

a) Using galvanized steel

b) Using stainless steel

c) Using epoxy-coated re-bars

Each method has its own advantages and disadvantages. Suitability or otherwise of these methods in a particular case is not commented upon by the Code. Discretion of the user is implied for adopting.

Other available techniques such as Cathodic Protection are outside the scope of the Code.

C16.3 Prestressing Steel — Cl. 18.3

C16.3.1 *Coated Wires/Strands* — Cl. 18.3.1

Prestressing steel is susceptible to stress corrosion due to high level of stress as compared to the reinforcing steels. The prestressing steel is also susceptible to hydrogen embrittlement in aggressive environments. Hence, it requires to be adequately protected. For bonded tendons, the alkaline environment of the grout provides adequate protection. For unbonded tendons, corrosion protection is provided by one or more of the following methods.

1) Duct fillers like cement grout, wax, and nuclear grade grease (low sulphates contents) or similar options
2) Factory coated HTS with polyethylene ducts, filled with suitable fills
3) Factory made Epoxy coating
4) Mastic wrap (grease impregnated tape)
5) Galvanized bars
6) Encasing in tubes

The use of specialist materials and techniques shall be as per manufacturer's specification and as approved by the owner/bridge authority.

C16.4 Material Ingredients of Concrete — Cl. 18.4

C16.4.1 *Chemical Admixtures* — Cl. 18.4.2

Natural or manufactured chemicals are often added to the concrete in the form of admixture, before or during mixing. The most used admixtures are air-entraining agents, water reducers, retarders and accelerators. It is a general knowledge, not specifically required by the Code, that Admixtures shall be evaluated for compatibility with the cementitious materials, construction methods and job specifications before being used.

C16.4.2 *Mineral Admixtures* — Cl. 18.4.3

Mineral admixtures are finely divided siliceous materials which are added to concrete in relatively large amounts, generally in the range of 20 to 70 percent by mass of the total cementitious material. These materials are generally by products from other processes or natural materials. They are also sometimes referred to as "supplementary cementitious material". Though there are several types of mineral admixtures which are available for use, the code permits use of fly ash (by product of coal fired furnaces), ground granulated blast furnace slag (non-metallic manufactured by product from a blast furnace) and silica fume (by product from the manufacture of silicon or ferro-silicon metal) only.

C16.5 Grouting — Cl. 18.7

C16.5.1 *Properties of the Grout* — Cl. 18.7.4

The main properties considered relevant for the performance of grouts in post-tensioned concrete structures are :

a) **The flowability of grout:** This is considered important to ensure complete filling of the tendon duct.

b) **Volume change of grout:** This is considered important to be maintained within a specified range around zero to completely fill the tendon duct.

c) **Bleed of grout:** It is considered important to limit free water inside the tendon duct, and any bleed water to be reabsorbed by the grout within a specified time.

d) **Strength of grout:** This is considered to provide an indication of the grout quality with respect to its bond and shear strength.

e) **Resistance of grout to freezing:** This is considered important for applications in cold climates. Since there is no mention of this in the Code, specialist literature may be followed in situations where such condition exists.

C16.5.2 *Load Tests of Structures*

The situations under which load testing of flexural members of abridge structure would be resorted to are -

a) Structures which are sub-standard due to quality of design or construction.

b) Deterioration of structures, due to material degradation or physical damage.

c) Non-standard design methods which may cause the designer, authorities or other stake holders to have proof of the concept used.

For members other than flexural members (e,g. Corbels, Deep Beams …etc.), where stress – strain relationship is not linear, the acceptance should preferably be based on analytical investigation.

CHAPTER 17

ANNEXURE A1 : ACTIONS, DESIGN SITUATIONS AND COMBINATIONS OF ACTIONS

CA1-1 General A1-1

This annexure gives details of the nature of actions, basis of their classification and the philosophy adopted for deciding the design values of actions. Various values of actions like characteristic values, design values and combinational values are explained with special reference to design of concrete bridges, where they differ from other types of bridges. The explanations are along with some of the clauses of **Sections 3 & 5**, as these also deal on same subject matter. Hence corresponding clause numbers are referred to, whenever necessary.

CA1-2 Classification of Actions A1-2

Actions are of two types viz. direct and indirect actions. The actions (loads) such as self-weight, superimposed dead load, carriageway live load, footpath live load and wind load etc. are directly applied to the structure and hence they are termed as direct actions. Indirect actions are those which are generated in the structure due to imposed deformation by settlement, temperature or seismic accelerations. Thus indirect actions are generated actions. Actions can be represented in a diagram as given below:

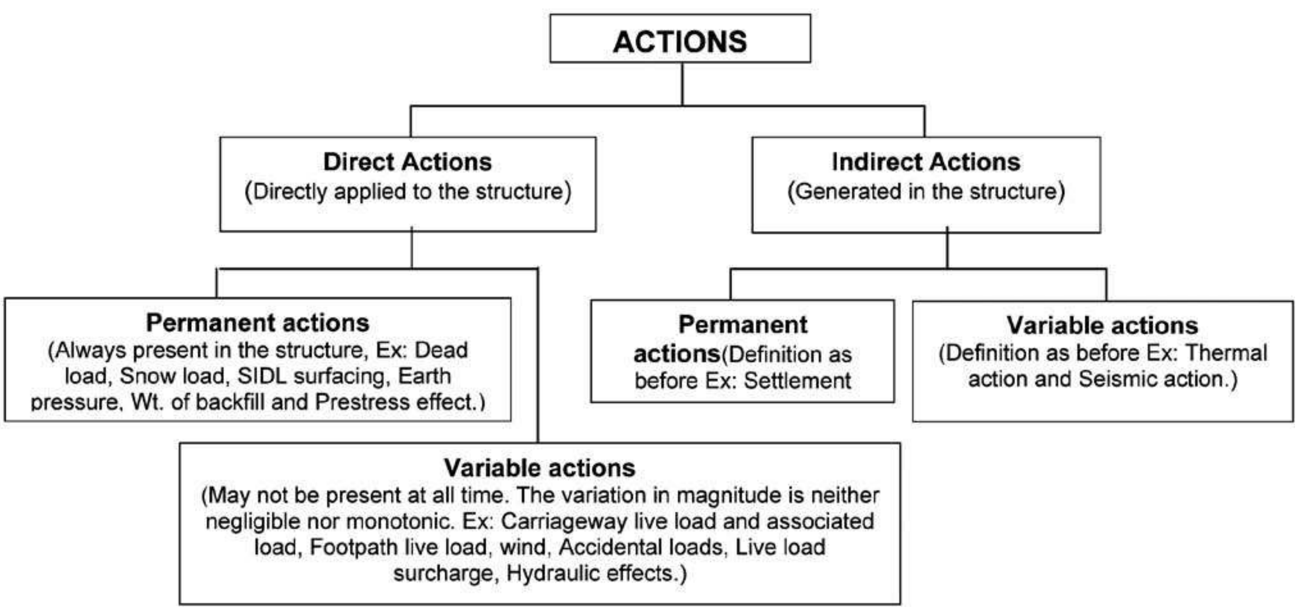


Fig. 1-A1-1

Direct and indirect actions can be further classified as permanent and variable actions. Direct actions which are always present in the structure are termed as permanent action, e.g. Self-weight, Superimposed dead load, Back fill weight, Earth pressure, Prestress effects etc. Actions, which vary with respect to time are treated as variable actions e.g. Carriageway live load, Footpath live load, Wind, Thermal action etc. Snow load, even though it varies with respect to time, is treated as direct permanent action. Settlement effect, which falls under the category of indirect action, is termed as permanent action as the settlement effect becomes permanent, once the settlement takes place. Thermal effect which also falls under the category of indirect action will be termed as variable action as it can vary with respect to time. Thus it can be seen that some indirect actions fall under the category of permanent action and some under the category of variable action. Semi-permanent (Quasi permanent) actions are certain fractions of variable actions which are likely to be present in the structure at all the time, e.g. some fraction of thermal effect.

In addition, there are two more variable actions viz. Accidental action and Seismic action. Accidental actions are those actions, of which the occurrences themselves and their periodicity cannot be predicted. These are actions which the structure may or may not be subjected to during its life time, exemplified by barge impact and vehicle collision with the parts of the bridge. Accidental action is direct variable action. Accidental actions caused by explosions are special cases, which is not covered by IRC:6, hence not included in the Code.

The seismic action shall be considered as an indirect variable action as the forces are generated in the structure due to ground acceleration.

Representative Values of Actions Cl. 3.1.3

Values of actions used within a particular combination of actions take account of low probability of more than one action having their characteristic values at the same time for determining the representative value. It also takes account of reduction in likely maximum value of an action, if considered over periods of less than the design life.

Characteristic Value of Actions Cl. 5.4.2

Characteristic value of an action is based on nature of load and statistical distribution of magnitudes of action. It could be mean value, upper fractional or lower fractional value or a nominal value. Upper and lower characteristic values commonly correspond to 95^{th} and 5^{th} percentiles. When statistical distribution is not adhered to, a nominal value is specified which is treated as characteristic value. For permanent action which varies very little about its mean value, the characteristic value corresponds to the mean value, which is a single value. In cases where the design is expected to be sensitive with respect to variations in the density or thickness or time dependent losses in case of prestressed concrete, then the values are defined as lower case and upper case values, referred as 'inferior' and 'superior' values. The 'inferior' and 'superior' values of actions are generally given as a multiple of characteristic value. For variable actions, the characteristic value shall correspond to:

- an upper value with an intended probably of not being exceeded or
- a lower value with an intended probably of being achieved during a reference period or
- a nominal value

The values given in the code corresponds to nominal value since these are not fixed on statistical basis. It has been fixed based on acquired experience.

CA1-3 Design Situation & Load Combination Cl. 15.4.2.2

1) A structure, during its construction and service life is acted upon by various actions at different times. These actions, which can act simultaneously, need to be combined in order to verify the safety of structure. The design situations & combinations specified in the Code will give rise to 9 primary combinations. The same are explained in detail under sub clause, "Limit states to be considered" subsequently.

2) All variable actions such as live load, wind, temperature etc. do not act at their peak values simultaneously at the same time. Hence, when these actions are to be combined, a reduction factor needs to be applied to scale down their peak values. While combining several variable actions, one variable action shall be treated as the leading variable action and all other variable actions shall be treated as accompanying variable actions. It will be for the designer to choose the leading variable action. As an example, if the carriageway live load is taken as leading variable action in a combination, then, thermal action or wind action shall be considered as accompanying variable action and the combination factors shall be taken accordingly. For the next trial, the thermal action can be taken as leading variable action and the carriageway live load shall be considered as accompanying variable action. While combining, various variable actions, the leading action shall be assumed to act at its peak and all other actions are to be scaled down in all combinations, except in frequent combination where the combination factor has also to be applied even to the leading action, in order to convert the characteristic action to frequently occurring action. The reduction factors which are used to scale down the peak values while combining are called combination factors. The product of combination factor and characteristic value of action is called combination value of an action, Q_k, $\psi_0 Q_k$ or $\psi_1 Q_k$ or $\psi_2 Q_k$. It is made clearer that combination factors are to be applied only on variable actions.

Partial Factor for Actions A1-3

The partial factor is used for enhancing the combination value of actions (loads) for verifying the ultimate limit state. The partial factor consists of two factors γ_s and γ_D. γ_s is the partial factor for taking into account the uncertainty in modeling the effect of action, which is considered as 1.15 for permanent actions and 1.1 for variable actions. γ_D is the partial factor for taking in to account the possibility of unfavorable deviation of action, which is considered as 1.17 for permanent actions (load) and 1.36 for variable actions (load). Hence the partial factor for a permanent action (load) will work out to 1.15 x 1.17 = 1.35 and for a variable action 1.1 x 1.36 = 1.50.

A word of caution, that enhancing the permanent loads is to be done only when it causes unfavorable effect (adding to the effects of variable action). In case it causes favourable effect (opposing the effects of variable action) then the permanent action shall not be enhanced by partial factor. For example, this situation will happen in continuous structure or while establishing factor of safety.

For the verification of serviceability limit state and accidental combination, the partial factor on actions shall be taken as 1.0 only i.e. the actions shall not be enhanced. For prestressing action, the partial factor will vary under different conditions, for which **Clause No. 7.9.5** of the Code shall be referred to. The factors γ_s and γ_D are from other International Codes as there are no data available for these factors.

Combinations of Actions and Combination Factors

As explained earlier, the variable actions and permanent actions acting simultaneously are to be combined in to order to carry out the verification of Limit States. When actions are to be combined, the reduced values of variable actions are to be used in order to arrive at the probable severity of the effects as all the variable actions do not act at their peak values at the same time. Combination factors are to be used on the variable actions in order to arrive at the characteristic combination (basic combination) value or infrequent combination value (rare combination) or frequent combination value or quasi permanent combination value of variable actions from the nominal values.

Partial Factor for Actions Given in IRC:6 A1-1

The partial factor for actions given in **Annex.-B** of IRC:6-2010 includes both the combination factor and partial factor for variable actions. This means that partial factor given in the code for variable action = Partial Factor for Variable Actions X Combination Factor for Variable Actions i.e.γ, x (ψ_0 or ψ_1 or ψ_2). Hence, the partial factor as specified in the code only needs to be applied. The philosophy of combination factor is given herein only for the understanding of the engineers. As the combination factor is not applicable for permanent actions the partial factor for permanent action does not include the combination factor. The use of these partial factors given in the code is explained in the worked out examples where they are shown in the brackets.

CA1-4 Limit States to be Considered

There are two types of limit states which are to be satisfied i.e. Limit state of Strength (ULS) and Limit State of Serviceability (SLS).

Under ULS, strength and stability are to be ensured under three limit state combinations i.e. Basic Combination, Accidental Combination & Seismic Combination.

For checking the equilibrium of super-structure and sub-structure, and structural strength different load factors are specified for different set of load combinations, for which **Table 3.1 and 3.2 of Annex-B** of IRC:6 shall be referred. For checking of equilibrium of foundation and for base pressure check of foundation, reference shall be made to IRC:78. **Table 3.4 of Annex-B** of IRC:6 shall be referred for structural design of foundation under ULS.

Under SLS; stress, deformation, crack width, settlement, creep and shrinkage effects are to be checked. For checking the serviceability limit state 3 combinations will be made use of. Cl. 15.4.2.2

1) **Rare Combination**

   This combination is used to check the maximum stress levels in structure for a rare load combination and load values, which is expected to occur infrequently on the structure.

2) **Frequent Combination**

This combination of loads and load values is expected to occur frequently and therefore is used to check crack width in case of prestressed concrete structure and deformations in case of both R.C.C and Prestressed Concrete Structures.

3) **Quasi Permanent Combination**

This combination is used to evaluate the settlements, creep effects, permanent stresses in the structure and to check crack widths in case of reinforced concrete structure. This combination provides an estimate of sustained loads on the structure.

All permanent loads and a fraction of variable loads which will be present at all times will be combined to arrive at this combination.

Verification of Limit States: Cl. A1-4

The variable actions are to be combined with permanent actions keeping in mind that all variable actions do not attain their peak values simultaneously at the same time and are to be combined appropriately to verify.

1) The static equilibrium; overturning, sliding, uplift for Basic, Accidental and Seismic combinations.
2) The structural strength under ultimate state for Basic, Accidental and Seismic combinations.
3) The serviceability limit states for Rare combination, Frequent combination and Quasi permanent combination.

Thus 9 primary combinations of actions have to be checked to complete the design. There can be several sub combinations. The necessary checks required to be carried out under ultimate limit state and serviceability limit states are detailed under relevant chapters.

Worked Examples

Partial factors are taken from IRC:6-2010. The examples will illustrate the methods to carry out the stability checks and to work out the disturbing and stabilizing forces/moments for the various combinations.

8.1 Stability Check: To Check the Stability of the Structure: Example 1:

Overhang Beam - Stability Check

Self-weight of member g = 15 kN/m

Concentrated Dead Load G = 6 Kn

Live Load Q_1 = 9 kN/m

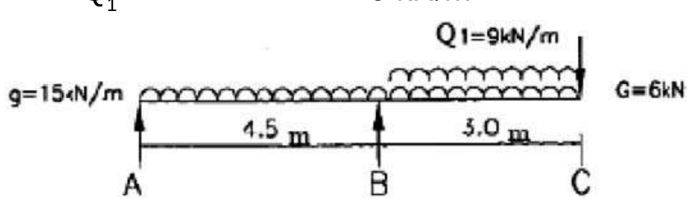


Fig. 1 Loading Diagram

The structure can overturn with respect to B

Overturning Moment = (1.05) x 15 x $\frac{3^2}{2}$ + (1.05) x 6 x 3 + (1.50) x 9 x $\frac{3^2}{2}$

= 70.87 + 18.9 + 60.75 = 150.52 kNm

Restoring Moment = (0.95) x $\frac{15x4.5^2}{2}$ = 144.28 kNm

144.28 kNm< 150.52 kNm

Structure overturns and hence unstable. To make it stable either reduce the cantilever length or reduce the concentrated load or provide anchoring at A.

Example 2:

Retaining Wall:

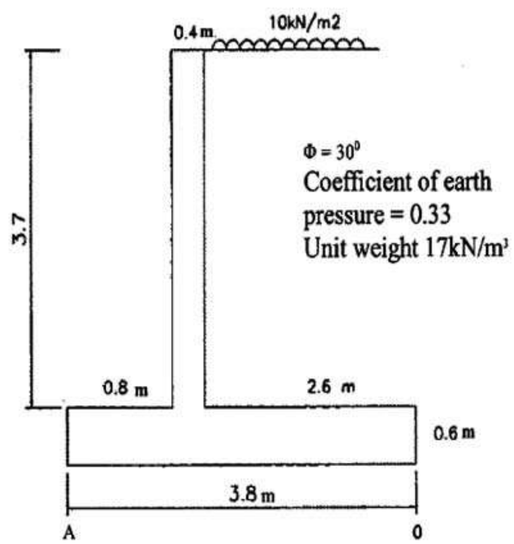


Fig. 2 Details of Retaining Wall

1) Overturning Check

Overturning moment (for unit length) about A

Earth pressure moment = (1.50) x $\frac{1}{2}$ x 17 x $(4.3)^3$ x 0.33 x 0.42 = 140.50 kNm

Moment of due to surcharge = (1.2) x 10 x 0.33 x $\frac{4.3^2}{2}$ = 36.61 kNm

Total overturning moment = 140.50 + 36.61 = 177.11 kNm

Restoring moment (for unit length) about A

Raft = 25 x 0.6 x 3.8 x 3.8/2 = 108.3 kNm

Wall = 25 x 3.7 x 0.4 (0.8 + 0.2) = 37 kNm

Fill = 17 x 3.7 x 2.6 x 2.5= 408.9 kNm

Total Restoring moment = 108.3 + 37 + 408.9 = 554.2 kNm

Reduced moment = (0.95) x 554.2 = 526.49 kNm

526.49 kNm > 177.11 kNm

Hence the structure does not overturn.

2) Sliding Check

Sliding check (for unit length)

Horizontal force due to Earth pressure = (1.50) x $\frac{1}{2}$ x 0.33 x 17 x 4.3^2 = 78.00 kN

Horizontal force due to Surcharge = (1.2) x 10 x 0.33 x 4.3 = 17.00 kN

Total Horizontal force = 78.00 + 17.00= 95.00 kN

Vertical Force:

Self-weight = 2.5 x 3.8 x 0.6 + 25 x 3.7 x 0.4 = 94 kN

Soil Load = 17 x 3.7 x 2.6 = 163.4 kN

Total Vertical Load Per Unit Length = 94 + 163.4 = 257.4 kN

Taking friction coefficient as 0.5

Resisting forces = (0.95) x 257.5 x 0.5 = 122.3 kN

As 122.3 kN> 95 kN

Hence the structure will not slide.

8.2 Calculation of Design Bending Moments:

Example 1:

1) Moments under Ultimate Limit State

Taking the same example of overhanging beam

a) **Moment due to concentrated Load**

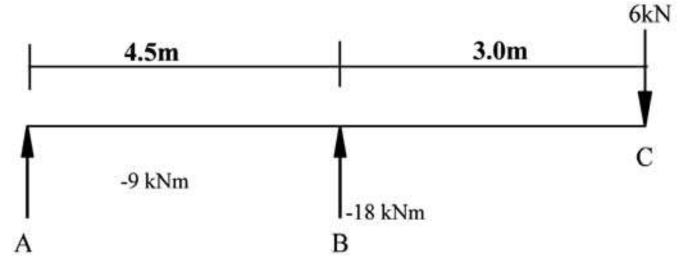


Fig. 3 Moment Due to Concentrated Dead Load

b) **Moment due to self-weight**

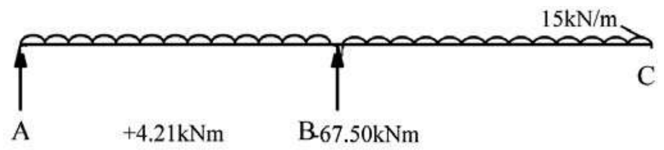


Fig. 4 Moment Due to Self Weight

c) Moment due to Variable Load (Live Load)

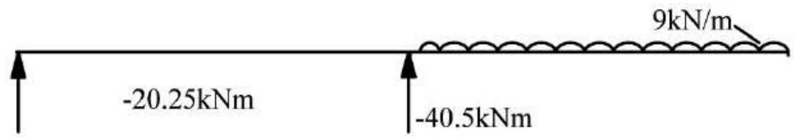


Fig. 5 Moment Due to Live Load on Overhang

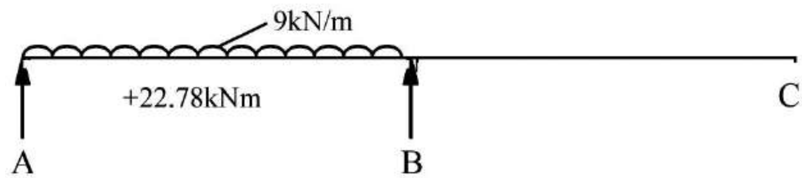


Fig. 6 Moment Due to Live Load within Span

d) **Design of Moments**

i) Max. (-) moment at B = -{1.35 (18 + 67.5) + 1.5 x 40.5 }= - 176.2 kNm

ii) Max. (+) moment at Mid-Point of AB = {1.0 (-9 + 4.21) + 1.5 x 22.78} = 29.38 kNm

It is to be noted as the dead load effects oppose the live load effect, the partial safely factor 1.0 has been used for dead load moment

iii) Maximum (-) moment of Mid-Point of AB = {1.35 (-9 + 4.21) – 1.5 x 20.25} = -36.84 kNm

As the dead load effects add to effect of live load effect the partial safely factor of 1.35 has been used for dead load moments.

2) Moments under Serviceability Limit State

a) **Rare Combination**

Maximum (-) moment at B = -{1.0 (18 + 67.5) + 1.0 x 40.5} = - 126 kNm

+ Moment at Mid-Point span of AB = {1.0 (-9 + 4.2) + 1.0 x 22.78} = 17.98 kNm

Maximum (-) moment at Mid-Point of span AB = -{1.0 (-9 + 4.21) – 1.0 x 20.25} = -25.04 kNm

b) **Frequent Combination**

Maximum moment at B = - {1.0 (18 + 67.5) + 0.75 x 40.5} = - 115.87 kNm

Max. (+) moment at Mid-Point of AB = 1.0 (-9 + 4.2) + 0.75 x 22.78 = 12.25 kNm

Max. (-) moment at Mid-Point of AB = 1.0 (-9 +4.2) – 0.75 x 20.25 = -19.98 kNm

c) **Quasi Permanent Combination**

Maximum moment at B = -{1.0 (18 + 67.5)} = - 85.5 kNm

Moment of Mid-Point of Span AB = 1.0(-9 +4.2) = -4.8 kNm

Thus it can be seen that at first the bending moment has to be calculated with the actions and then the partial factors to be chosen to arrive at the moment for the different combinations.

8.3 Calculation of Design Bending Moments in a Continuous Beam:

Example 1:

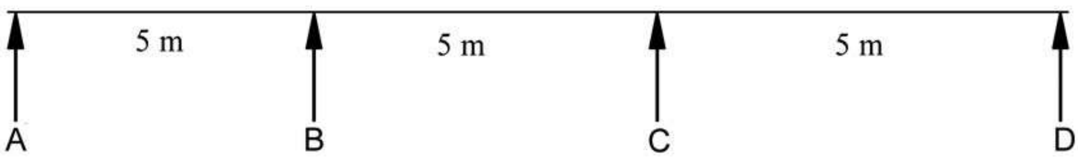


Fig. 7 Continuity Beam

Dead Load = 30 kN/m Live Load = 18 kN/m

For Basic Loads

1)	Dead Load moment in Mid span AB	=	0.08 x 5^2 x 30	=	+ 60 kNm
2)	Dead Load moment at support B	=	-{0.1 x 5^2 x 30}	=	- 75 kNm
3)	Dead Load moment at Mid span BC	=	0.025 x 5^2 x 30	=	+ 18.75 kNm
4)	Live Load + moment in Mid span AB	=	0.100 x 5^2 x 18	=	+ 45 kNm
5)	Live Load – moment at Mid span AB	=	- 0.025 x 5^2 x 18	=	- 11.25 kNm
6)	Live Load – moment at support B	=	- 0.117 x 5^2 x 18	=	- 52.65 kNm
7)	Live Load + moment at support B	=	0.015 x 5^2 x 18	=	+ 6.75 kNm
8)	Live Load + moment Mid span BC	=	0.075 x 5^2 x 18	=	+ 33.75 kNm
9)	Live Load – moment at Mid span BC	=	- 0.052 x 5^2 x 18	=	- 23.4 kNm

The moments have been calculated using coefficients to simplify the calculation.

1) **Calculation of Ultimate Moments**

a) For Mid Span of AB

+ Moment = (1.35) x 60 + (1.5) x 45 = 148.5 kNm Design Moment

- Moment = (1.0) x 60 – (1.5) x 11.25 = + 43.125 kNm

Note, as Dead load moment is opposing live load moment hence the partial factor on DL is 1.0 only.

b) **For support moment at B**

(-) Moment = -(1.35) x 75 – (1.5) x 52.65 = - 180.2 kNm Design Moment

+ Moment = -(1.0) x 75 + (1.5) x 6.75 = - 64.875 kNm

c) **For Mid Span moment of Span BC**

+ Moment = (1.35) x 18.75 + (1.5) x 33.75 = 76 kNm

(-) Moment = (1.0) x 18.75 – (1.5) x 23.4 = - 16.35 kNm

Note the section is undergoing Reversal.

2) **Calculation of Serviceability Limit state moments:**

a) **Rare Combination:**

i) Moment in Mid span AB

+ Moment = (1.0) x 60 + (1.0) x 45 = + 105 kNm

(-) Moment = (1.0) x 60 – (1.0) x 11.25 = + 48.75 kNm

ii) Moment at Support B

(-) Moment = -(1.0) x 75 – (1.0) x 52.65 = - 127.65 kNm

+ Moment = - (1.0) x 75 + (1.0) x 6.75 = - 68.25 kNm

iii) Moment at Mid Span BC

+ Moment = (1.0) x 18.75 + (1.0) x 33.75 = + 52.5 kNm

(-) Moment = (1.0) x 18.75 – (1.0) x 23.4 = - 4.65 kNm

b) **Frequent Combination:**

i) Moment at Mid Span AB

+ Moment = (1.0) x 60 + (0.75) x 45 = 94 kNm

(-) Moment = (1.0) x 60 – (0.75) x 11.25 = + 51.56 kNm

ii) Moment at Support B

(-) Moment at support = - (1.0) x 75 – (0.75) x 52.65 = - 114.48 kNm

iii) Moment at Mid Span BC

+ Moment = (1.0) x 18.75 + (0.75) x 33.75 = + 44.06 kNm

(-) Moment = (1.0) x 18.75 – (0.75) x 23.4 = + 1.2 kNm

c) **Quasi permanent combination:**

Only Dead Load will act

Moment at Mid span of AB (1.0) x 60 = + 60 kNm

Moment at Support B (1.0) x -75 = -75 kNm

Moment at Mid span BC (1.0) x 18.75 = +18.75 kNm

Ultimate Moment Diagram

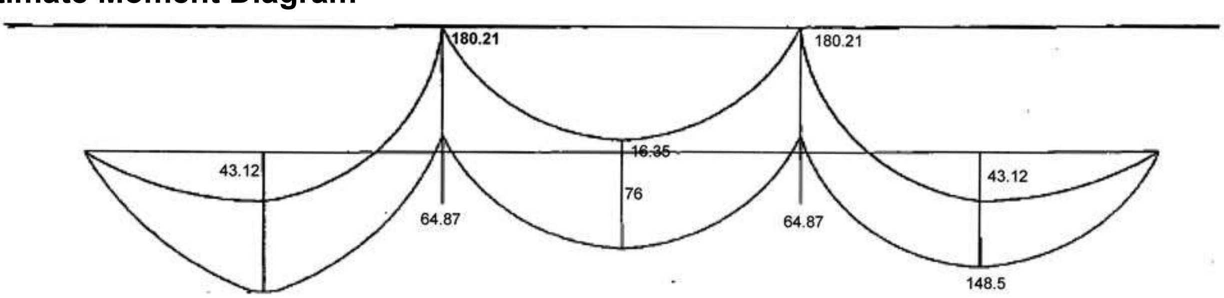


Fig. 8 Bending Moment Diagram

All Moments shown are in kNm

8.4 Design Moments for Super-structure and Loads and Moments for Substructure

Example 1:

Another Example of Designs of Bridge Super-structure and sub-structure is given here so that the designers can easily understand the concept of partial factors.

1) **Following are the moments to be resisted by super-structure**

a) BM due to Dead Load of Super Structure 3483.0 kNm

b) BM due to SIDL except surfacing 687.0 kNm

c) BM due to surfacing 190.0 kNm

d) BM due to FPLL and LL 913.0 kNm

2) **Design Moments**

a) **Design Moments for Ultimate Strength Check**

i) Design moment for strength check = 1.35 (3483.0 + 687) + 1.75 x 190 + 1.5 x 913 = 7331.5 kNm

b) **Design Moments for Serviceability Combinations**

i) Moment for Rare Combination check = 3483.0 + 687 + 190 +913 = 5273.0 kNm

Moment for Rare combination check, including temperature gradient effect = 5273.0 + 0.6 times the moment due to temperature gradient effect.

ii) Moment for Frequent Combination check = 3483.0 + 687.0 + 190+ 913 x 0.75 = 5044.7 kNm

Moment for frequent combination check, including temperature gradient effect = 5044.7 + 0.5 times the moment due to temperature gradient effect.

iii) Moment for Quasi permanent combination check = 3483 + 687 + 190 = 4360 kNm

Moment for Quasi permanent combination check, including temperature gradient effect = 4360 + 0.5 times the moment due to temperature gradient effect.

CHAPTER 18

INFORMATIVE ANNEXURE B-1 CONCRETE SHELL ELEMENTS

C-B-1 Introduction

The Annexure gives detailed method of design of shell element. As explained in **Section 9,** the shell element is subjected to eight number of force resultants. This method is generally applicable when the analysis is performed using finite element method. The internal actions in a shell Element at ULS are Sketched in **Fig. C-B-1.1**.

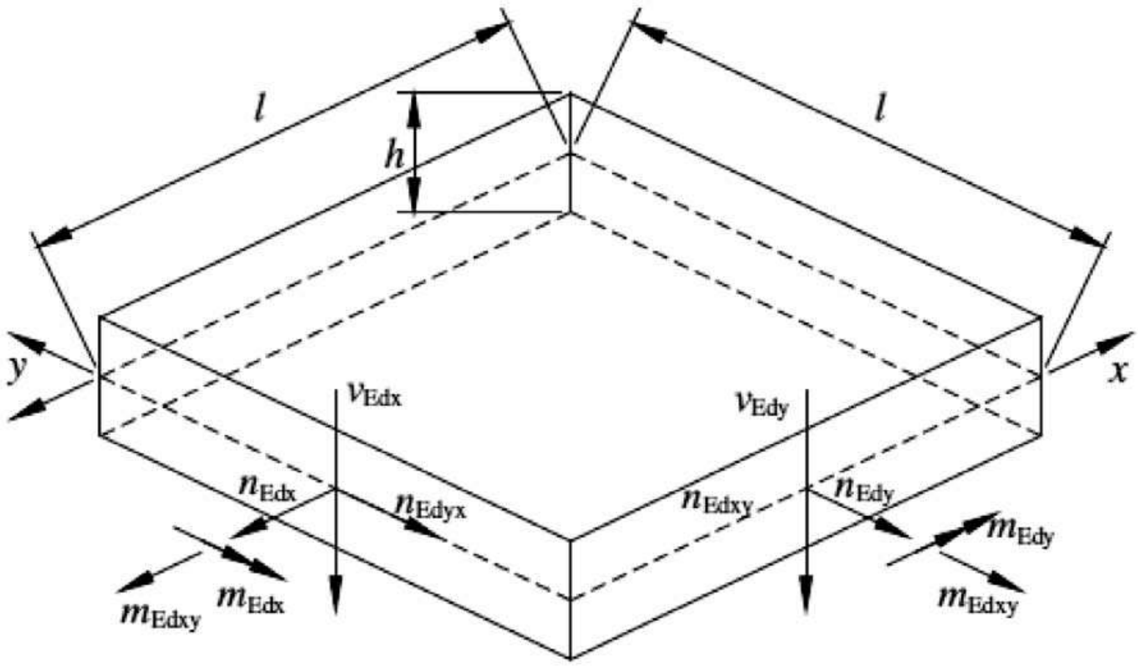


Fig. C-B-1.1 Shell Element

The method is similar to Wood-Armer's method popularly used method in the past. In this method, it is firstly verified whether the element is cracked using **Eqn. B-1-1.** If the element is uncracked, the only verification required is to check that minimum principal stress (i.e. maximum compressive stress) does not exceed the design compressive strength f_{cd}. If the element is cracked, the Annexure gives procedure to find out the required reinforced reinforcement.

A sandwich model is used to convert out-of-plane moments and twisting moments into stress resultants acting planes of the top and bottom layers. The core/middle layer is designed to carry the out of plane shear force.

After determining the in-plane forces in the outer layers as per the **Section-9**, verification at ULS is to be performed adopting the sandwich model for Shell Elements. The procedure is demonstrated with the help of worked example.

Worked Example C-B-1-1

Design of Shell Element as per Annexure B-1

Design Data:

Geometry		Material Data	
Thickness	: 300 mm	Concrete Grade	: M 30
Clear cover	: 40 mm	f_{cm}	: 40 MPa
		f_{ctm}	: 2.5 MPa
		f_{ck}	: 30 MPa
		f_{cd}	: 13.4 MPa
		Steel Grade	: Fe 500
		f_{yk}	: 500 MPa
		f_{yd}	: 435 MPa

Reinforcement details:

	Top Layer		Bottom Layer	
	Bar & spacing	Area /m	Bar & spacing	Area/m
X dir^n	16 @ 100	2009.6	16 @ 100	2009.6
Y dir^n	16 @ 100	2009.6	16 @ 100	2009.6

Summary of Forces on Shell Element

Load Case	n_{Edx}	n_{Edy}	n_{Edxy}	m_{Edx}	m_{Edy}	m_{Edxy}	v_{Edx}	v_{Edy}
	kN/m	kN/m	kN/m	kN-m/m	kN-m/m	kN-m/m	kN/m	kN/m
1	-177.1	3.46	26.23	34.981	2.047	-0.961	21.18	1.68
2	75.12	-36.53	5.32	-17.43	15.32	1.3	16.1	0.53

*Negative values of n_{Edx} & n_{Edx} are compressive forces

Check for cracked section

Summary of Stresses on Shell Element (MPa)

Load Case	σ_1	σ_2	σ_3	σ_m
1	2.55	1.28	-0.64	1.0633

* The above stresses have been obtained from finite element analysis for shell element subjected to forces as per Load case I

$$\phi = \alpha\frac{J_2}{f_{cm}2} + \lambda\frac{J_2}{f_{cm}} + \beta\frac{I_1}{f_{cm}} - 1 \leq 0$$

For Load case I

$$J_2 = \frac{1}{6}\left((\sigma_{1-}\sigma_2)^2 + (\sigma_{2-}\sigma_3)^2 + (\sigma_{3-}\sigma_1)^2\right)$$

$$J_2 = \frac{1}{6}\left[(2.55 - 1.28)\ ^2 + (1.28 - -0.64)^2 + (-0.64 - 2.55)^2\right]$$

= 2.5792

$J_3 = (\sigma_{1-}\sigma_n) \times (\sigma_{2-}\sigma_n) \times (\sigma_{3-}\sigma_n)$

J_3 = -0.549

$I_1 = \sigma_1 + \sigma_2 + \sigma_3$

= 2.55+1.28+-0.64

= 3.19

$\alpha = \frac{1}{9k^{1.4}}$

$\cos 3\theta = \frac{5.2}{2} \times \frac{-0.549}{4.142}$ = -0.344

$\lambda = C_1 \cos\left[\frac{1}{3} \text{ar} \cos(C_2 \cos 3\theta)\right]$ for $\cos 3\theta > 0$

$\lambda = C_1 \cos\left[\frac{\pi}{3} - \frac{1}{3} \text{ar} \cos(-C_2 \cos 3\theta)\right]$ for $\cos 3\theta < 0$

$C_1 = \frac{1}{0.7\, k^{0.9}} = 17$

$C_2 = 1\text{-}6.8(k - 0.07)^2 = 1.0$

$k = \frac{f_{ctm}}{f_{cm}} = \frac{2.5}{40} = 0.0625$

λ = 14 ;α = 5.4 ;

$\beta = \frac{1}{3.7\, k^{1.1}} = 5.7$

φ = 0.0213

Hence, The Section is cracked

Evaluation of forces within outer layers:

Since cover and reinforcement bar size on both the faces are equal and

ts	=	ti								
ys	=	150 – 40 -16	=	94 mm	ts	=	2 x 40 + 16 + 16	=	112 mm
yi	=	150 – 40 -16	=	94 mm	ti	=	2 x 40 + 16 + 16	=	112 mm
z	=	94 + 94	=	188 mm	tc	=	300 – 112 – 112	=	76 mm

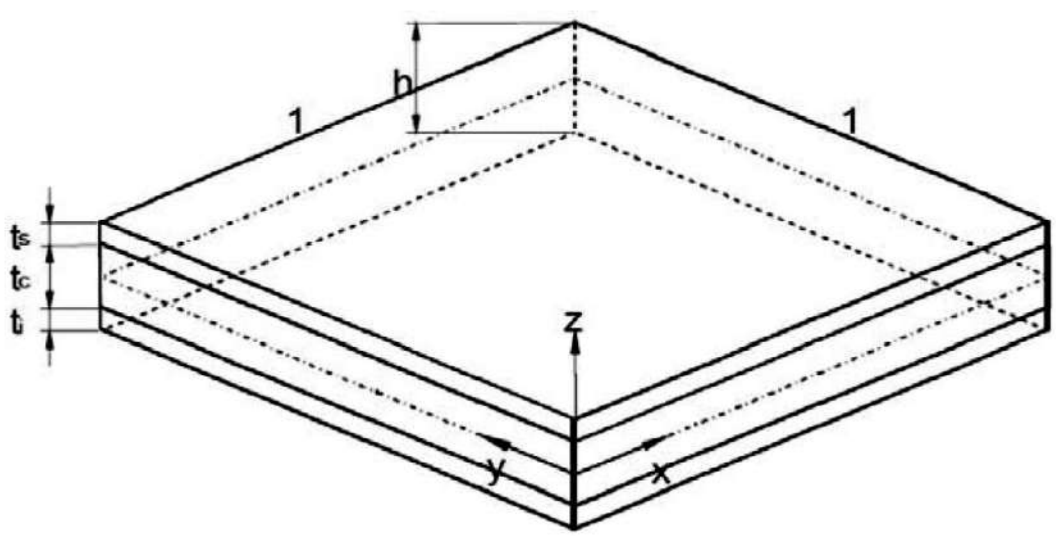


Fig. C-B. 1.2 Layers as per Sandwich Model

For Cases where shear reinforcement is not required:	For Cases where shear reinforcement is required :
$n_{Edxs} = n_{Edx}\dfrac{z-y_s}{z} + \dfrac{m_{Edx}}{z}$	$n_{Edxs} = n_{Edx}\dfrac{z-y_s}{z} + \dfrac{m_{Edx}}{z} + \dfrac{1}{2}\dfrac{v_{Edx}^2}{v_{Edo}}\cot\theta$
$n_{Edxi} = n_{Edx}\dfrac{z-y_i}{z} - \dfrac{m_{Edx}}{z}$	$n_{Edxi} = n_{Edx}\dfrac{z-y_i}{z} - \dfrac{m_{Edx}}{z} + \dfrac{1}{2}\dfrac{v_{Edx}^2}{v_{Edo}}\cot\theta$
$n_{Edys} = n_{Edy}\dfrac{z-y_s}{z} + \dfrac{m_{Edy}}{z}$	$n_{Edys} = n_{Edy}\dfrac{z-y_s}{z} + \dfrac{m_{Edy}}{z} + \dfrac{1}{2}\dfrac{v_{Edy}^2}{v_{Edo}}\cot\theta$
$n_{Edyi} = n_{Edy}\dfrac{z-y_i}{z} - \dfrac{m_{Edy}}{z}$	$n_{Edyi} = n_{Edy}\dfrac{z-y_i}{z} - \dfrac{m_{Edy}}{z} + \dfrac{1}{2}\dfrac{v_{Edy}^2}{v_{Edo}}\cot\theta$
$n_{Edxys} = n_{Edxy}\dfrac{z-y_s}{z} - \dfrac{m_{Edxy}}{z}$	$n_{Edxys} = n_{Edxy}\dfrac{z-y_s}{z} - \dfrac{m_{Edxy}}{z} + \dfrac{1}{2}\dfrac{v_{Edx}v_{Edy}}{v_{Edo}}\cot\theta$
$n_{Edxyi} = n_{Edxy}\dfrac{z-y_i}{z} + \dfrac{m_{Edxy}}{z}$	$n_{Edxyi} = n_{Edxy}\dfrac{z-y_i}{z} + \dfrac{m_{Edxy}}{z} + \dfrac{1}{2}\dfrac{v_{Edx}v_{Edy}}{v_{Edo}}\cot\theta$

Check for requirement of shear reinforcement

For Load Case I

$$v_{Edo} = \sqrt{v_{Edx}^2 + v_{Edy}^2} = \sqrt{21.18\text{^}2 + 1.68\text{^}2}$$

$$= 21.247 \text{ kN/m}$$

$$\tan\varphi_o = \frac{v_{Edy}}{v_{Edx}} = \frac{1.68}{21.18} = 0.0793$$

$\sin^2 \varphi_0 = 0.006$

$\cos^2 \varphi_0 = 0.994$

$p_1 = px\sin^2 \varphi_0 + py\cos^2 \varphi_0$

$= 0.007 \times 0.006 + 0.007 \times 0.994$

$= 0.007$

Calculation of $V_{Rd.c}$

$V_{Rd.c} = [0.12\, k\, (80 p_1 fck)^{0.33}]\, b.d$

where $k = 1 + \sqrt{\frac{200}{d}} = 2$

$V_{Rd.c} = 46.278 \text{ kN/m} > 21.247 \text{ kN/m}$

Hence, Shear reinforcement is not required

Using the equations as **Clause B1.14** sub-clause (a), planar forces are typically calculated for top layer for Load Case I as follows:

$$n_{Edxs} = \frac{177.14 \text{ x } (0.188-0.94)}{0.188} + \frac{34.9813}{0.188} = 274.6 \text{ kN/m}$$

$$n_{Edys} = \frac{-3.46 \text{ x } (0.188-0.94)}{0.188} + \frac{2.047}{0.188} = 9.2 \text{ kN/m}$$

$$n_{Edxys} = \frac{26.23 \text{ x } (0.188-0.94)}{0.188} + \frac{0.961}{0.188} = 18.2 \text{ kN/m}$$

Summary of Forces within outer layers of Shell Element

Load Case	n_{Edxs}	n_{Edxi}	n_{Edys}	n_{Edyi}	n_{Edxys}	n_{Edxyi}
	kN/m	kN/m	kN/m	kN/m	kN/m	kN/m
1	274.6	-97.5	9.2	-12.6	18.2	8.0
2	-130.3	55.2	99.8	-63.2	-4.3	9.6

* positive values of n_{Edx} & n_{Edx} are compressive forces

Design of Membrane Elements:

Case I: Stresses in X and Y direction are Tensile

Considering the membrane forces for the bottom layer for Load Case I

n_{Edxi} = -97.5 kN/m

n_{Edyl} = -12.62 kN/m

n_{Edxi} = 8.0033 kN/m

Corresponding stresses

σ_{Edx} = -0.871 MPa

σ_{Edy} = -0.113 MPa

T_{Edxy} = 0.0715 MPa

Since, both σ_{Edx} & σ_{Edy} are both tensile, reinforcement is required in both directions.

f_{Edx} = $\sigma_{Edx} + T_{Edxy} \cot\theta = -0.799$ MPa

px_{req} = $f_{tdx}/f_{yd} = 0.0018$

px_{pro} = 0.0067

Since, $px_{pro} > px_{req}$, Reinforcement provided in X-dirn is sufficient.

f_{tdy} = $\sigma_{Edy} + T_{Edxy} \tan\theta = -0.041$ MPa

py_{req} = $f_{tdy}/f_{yd} = 0.0001$

py_{pro} = 0.0067

Since, $py_{pro} > py_{req}$, Reinforcement provided in X-dirn is sufficient.

Check for compressive stress

$T_{Edxy} <= fcd \sin\theta \cos\theta <= 13.4 \times 0.707 \times 0.707 = 6.698$ MPa

Since, $T_{Edxy} < f_{cd} \sin\theta \cos\theta$, Section is safe in compression

Case II Stresses in X and Y direction are Compressive

Considering the membrane forces in the top layer for Load Case I

n_{Edxi} = 274.64 kN/m

n_{Edyi} = 9.1583 kN/m

n_{Edxyi} = 18.227 kN/m

Corresponding stressess

σ_{Edx} = 2.4521 MPa

σ_{Edy} = 0.0818 MPa

T_{Edxy} = 0.1627 MPa

Principal stresses

$$\sigma_1 = \frac{(\sigma_{Edx} + \sigma_{Edy})}{2} + \sqrt{\left(\frac{(\sigma_{Edx} + \sigma_{Edy})^2}{4} + T_{Edxy^2}\right)}$$

= 2.5443 MPa

$$\sigma_2 = \frac{(\sigma_{Edx} + \sigma_{Edy})}{2} - \sqrt{\left(\frac{(\sigma_{Edx} + \sigma_{Edy})^2}{4} + T_{Edxy^2}\right)}$$

= -0.01 MPa

Check for requirement of reinforcement as per **Clause 9.4.1** sub-clause (3)

$\sigma_{Edx}\,\sigma_{Edy} > T^2_{Edxy}$

$\sigma_{Edx}\,\sigma_{Edy} = 0.2005$

$T^2_{Edxy} = 0.0265$

Since, $\sigma_{Edx}\,\sigma_{Edy} > T^2_{Edxy}$, Reinforcement is not required

However, a check for maximum compressive stress shall be performed.

Max. Compressive Principal Stress = 2.544 MPa

f_{cd} = 13.40 MPa

Since, $\sigma_1 < f_{cd}$, Section is safe in compression

Case III Stress in X dir^n is Compressive and in Y dirn^n is Tensile

Considering the membrane forces for the top layer for Load Case II

n_{Edxs} = 55.153 kN/m

n_{Edys} = -63.22 kN/m

n_{Edxys} = 9.5749 kN/m

Corresponding stresses

σ_{Edx} = 0.4924 MPa

σ_{Edy} = -0.565 MPa

T_{Edxy} = 0.0855 MPa

f_{tdx} = $\sigma_{Edx} + T_{Edxy}$ Cot θ = 0.2158 MPa

Since in X-dirn the stress is compressive stress, minimum reinforcement shall be provided and

f_{tdx} = 0 MPa

The above condition changes the equation for stress in reinforcement in Y dirn as follows:

f_{tdy} = $\sigma_{Edy} + \frac{T^2\ Edxy}{|\sigma_{Edx}|}$ = -0.579 MPa

py_{req} = f_{tdy}/f_{yd} = 0.0013

py_{pro} = 0.0067

Since, $py_{pro} > py_{req}$, Reinforcement provided in Y-dirn is sufficient

The compressive stress in concrete is calculated as follows:

σ_{cd} = $\sigma_{Edx}\left(1 + \left[\frac{T_{Edxy}}{\sigma_{Edx}}\right]^{2}\right)$

= 0.5073 MPa

The above stress shall not exceed $v.f_{cd}$

$V.f_{cd}$ = $0.6\left[1 - \frac{f_{ck}}{310}\right] \times f_{cd}$ = 0.5419 x 13.4 = 7.261 MPa

Since, $\sigma_{cd} < v.f_{cd}$, Section is safe in compression

CHAPTER 19

ADDITIONAL EXPLANATIONS ON SECTION 7 : ANALYSIS

A.1 Introduction

This additional chapter giving Explanatory Notes and Guidelines for the application of '**Section 7**: Analysis' of the Code is arranged differently from the discussion of other sections. The need arises from the fact that the **Clauses** of **Section 7** assume substantial knowledge of the analytical methods on part of the designer. In order to explain the requirements of the Code Clauses it is useful to have general background of the classical as well as the modern methods of analysis.

This overview is necessary for another important reason. The users of the Code and the Explanatory Handbook are the practicing engineers belonging to different age groups. They have received their basic education at different times in which period, many new developments in the methods of analysis have taken place. An Element of 'Continued Education' for Engineers of senior and middle level of experience is thus unavoidable.

The present is a period of transition in which many design offices and individual designers are switching over to the automated and computerised, analytical and design tools. These tools allow use of more realistic models representing the behaviour of structures and materials than the models suitable for hand analysis.

The analysis of bridge components require application of the appropriate classical, modern or computerised analytical methods for calculating response of the components, when are subjected to different loading conditions. The methods for doing so are covered in the Code. These methods are further explained in an introductory manner in this Chapter. For the full treatment of any of these methods refer text books, advanced literature and instruction manuals of the computer programmes performing such analysis.

A.2 Classical Methods of Analysis

A.2.1 Essence of Classical Methods of Analysis

The classical methods are developed in last 150 years. Hallmarks of the classical methods are:

- Simple and idealised representation of structural elements, support conditions and loads.
- Equilibrium of external loads and support reactions as well as equilibrium of external load effects with internal forces developed in the structural elements due to elastic deformation of the material i.e. satisfying the requirements of equilibrium condition.
- Use of linear elastic constitutive laws describing the response of materials, i.e. providing constitutive laws of materials describing stress-strain characteristics.
- Assumption of consistent deformations of elements constituting the structure i.e. satisfying Kinematical relations, or compatibility of geometry in deformed shape.

- Use of continuum mechanics under these conditions to formulate general equations describing the response of the structure subjected to loads.
- Finding exact or approximate mathematical solutions of these general equations, which in turn allow computation of reactions, deformed shape and internal stresses and strains.

Many of the real life materials and structures do not fully comply with the simplified assumptions made in the classical solutions. However, till recently the limitations of the hand calculations hampered use of more realistic structural models.

A.2.2 Use of Mathematical Analysis

The application of mathematical methods to solve engineering problems is developed in the last 180 years. Some of the basic laws, such as the Hook's laws of linear elasticity, (AD 1660), were established earlier. However, the beam theory of Euler-Bernoulli, developed in AD 1750, can be considered as the beginning of the present day mathematical analysis of structures. Methods of the science of 'Strength of Materials', and 'Applied Mechanics', developed based on the growing knowledge about the laws of equilibrium of forces acting on bodies as a whole (rigid bodies) and elastic behaviour of linear members like columns and beams subjected to axial, bending and torsional forces, together with the laws describing behaviour of structural materials. These methods were suitable for hand analysis. Many of these are still being regularly used by bridge engineers, as exemplified by the techniques of 'equilibrium of joints' and 'method-of-section' used in design of trusses.

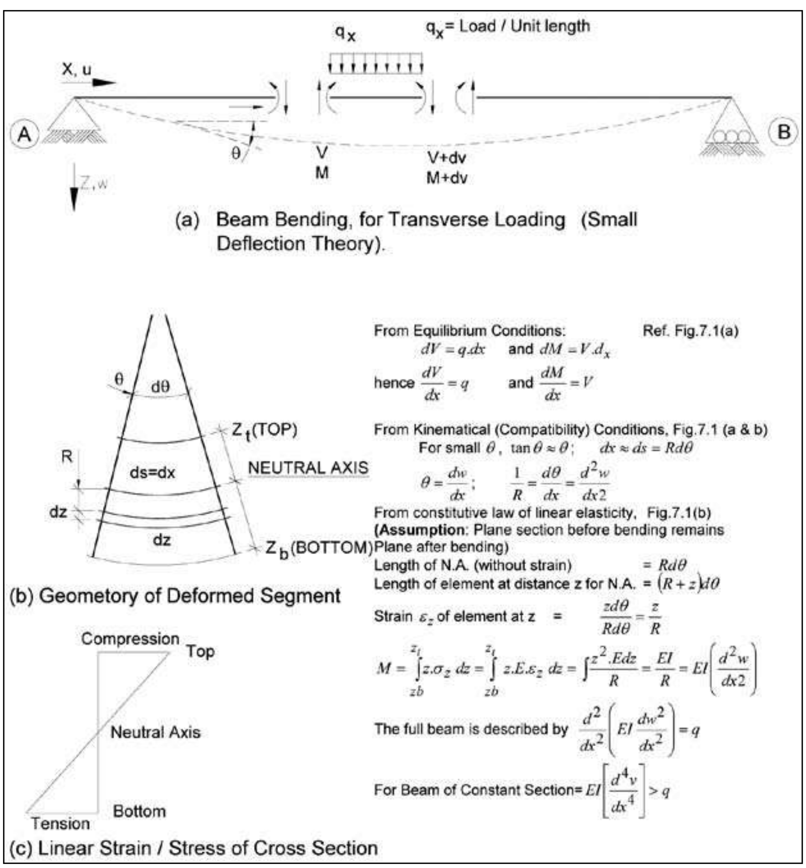


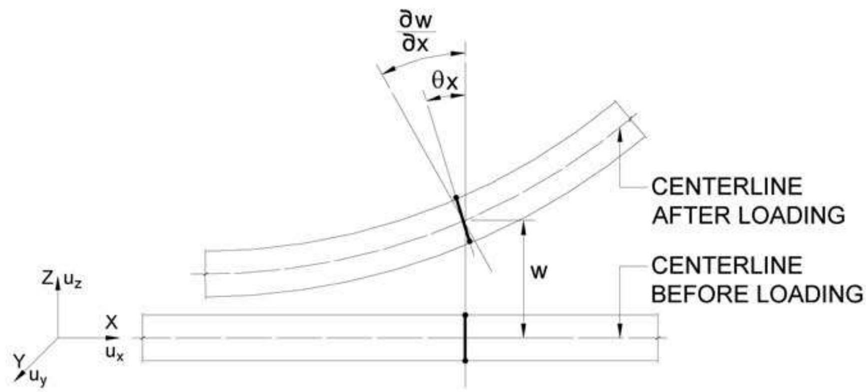
Fig. A.1 Euler Bernoulli's Beam Theory

The Beam Theory of Euler-Bernoulli expresses the behaviour of elastic beam having small deflection. **Fig. A.1** shows a rather simple mathematical representation (model) of the real life beam by its geometrical central axis, assuming that its deflected shape under action of transverse load is described by a continuous curve, shape of which is determined by the deformation of the cross-section of beam itself. It is further assumed that the cross-section deforms in such a way that the sections originally at right angle to the central axis remain at right angle to the deflected curve at all points. The internal strains developed by this deformed shape produce internal set of resisting forces, which provide equal and opposite internal couple to resist the bending moment generated by the external load. The internal stress-strain relationship is assumed to be linearly elastic following Hook's law. **Fig. A.1** shows this basic mathematical approach, which uses the method of equilibrating external forces and internal resistance forces acting on part of the beam taken as a free body. Use of 'free body diagram' is another classical technique still being commonly used for local analysis. The deformations induced in the cross-section by shear forces are dis-regarded in this analysis, resulting in acceptably small under estimation of deflections. The solution of the differential equations describing the deflected shape using mathematical methods provide the deflected shape, strains and stresses induced in the beam. Thus, the use of mathematical tools to solve engineering problem came into practice. This approach is described in some details here since it contains in essence all the elements of mathematical methods of analysis. Euler Bernoulli's beam theory is still the most commonly used method by engineers, even when more rigorous methods have been developed.

Advanced theories for linear members like beams/columns and for two and three dimensional structures like plates and shells, using principles of continuum mechanics and using simple constitutive laws for materials were developed by Timoshenko and others. Timoshenko's general beam theory is shown in **Fig. A.2** for comparison. These general methods also account for the effects of shear strains and are also applicable to short beams, in which the shear mechanism play significant role in load transfer. The generalised equations of the theory are valid for members exhibiting large deflections of the mid surface **(Article A.6.1)**. These methods assume homogenous material characteristics having linear relationship linking various types of stress and strains in the three directions which are described by different modulii of elasticity and Poisson's ratio. For details reference is made to the text books on this subject.

A.2.3 Saint-Venant's Principle

One of the most significant finding known as St. Venant's principle made it possible to apply the methods of continuum mechanics to practical structures. The assumptions about the distribution of internal strains and resulting stresses in the general portion of the three dimensional body where no local external forces act, are not valid in the near vicinity of the load, where they are affected by the way in which the loads are applied to the structure. However, the Saint-Venant's Principle states that (Quoted from Timoshenko and Goodier from Book 'Theory of Elasticity).



$$u_x(x,y,z) = -z\theta(x); u_y(x,y,z) = 0; u_z(x,y) = w(x)$$

For a point, (x,y,z) on the centreline of beam, u_x, u_y, u_z are the components of the displacement vector in the three co-ordinate directions, θ is the angle of rotation of the normal to the mid-surface of the beam, and w is the displacement of mix-surface in the z-direction and L is the length of the beam.

The governing equations are the following uncoupled differential equations:

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{d\theta}{dx}\right) = q(x,t)$$

$$\frac{\partial w}{\partial x} = \theta - \frac{1}{kAG}\frac{\partial}{dx}\left(EI\frac{d\theta}{dx}\right)$$

The Timoshenko beam theory is equivalent to the Euler-Bernoulli theory when the last term above is negligible. This is valid when $\frac{EI}{kL^2 AG} << 1$

Combining the two equations gives, for a homogeneous beam of constant cross-section, $EI\frac{d^4 w}{dx^4} = q(x) - \frac{EI}{kAG}\frac{d^2 q}{dx^2}$

Fig. A.2 Timoshenko's Beam Theory

'If the forces acting on a small surface of an elastic body are replaced by another statically equivalent system of forces acting on the same portion of the surface, this re-distribution of loading produces substantial changes in the stresses locally but has negligible effect on stresses at distances which are large in comparison with the linear dimension of the surface on which the forces are changed.'

This knowledge makes it possible to apply the overall general solutions of structures which include portions of locally applied loads without vitiating the overall analysis of the structure, except in the local zones in the near vicinity of the loads or supports. In other words, the overall evaluation of the internal stresses is substantially reliable and can be used in practical designs. However, the knowledge of the internal strain and stress distribution in the near vicinity of external forces is essential for the proper design of these local portions. This requirement is met by the methods of 'Local Analysis'.

A.2.4 Importance of Sign Convention

A.2.4.1 Sign Convention Guided by Understanding of Physical Effects

Following consistent sign conversion for describing physical quantities in various stages of analysis and interpreting results using the same signs is useful, but not vitally important as long as the direction and sense of the physical effect could be directly understood. The use of rigorous sign convention can even be avoided, as illustrated by different practices of assigning signs to bending moments. Treating bending moments as sagging or hogging for beams under gravity loads or treating moments causing sagging of members inwards of the closed section as positive for elements forming closed sections or the practice of plotting the bending moment on the tension side of the member are the methods based on the physical understanding and variously used in the same design without causing confusion or miss interpretation. Mathematically rigorous sign conventions are not essential in any of the above three practices.

A.2.4.2 Sign Convention as a Mathematical Necessity

As now-a-days the mathematical methods of analysis are used, a consistent sign convention becomes unavoidable, even for the methods suitable for hand calculation. Association of positive/negative signs before the numbers representing opposite physical effects, such as directions of opposite forces or compressive/tensile strains etc. becomes necessary. One of the most commonly used sign convention, which has been used by Timoshenko in his work on theory of elasticity is shown in **Fig.A.3**.

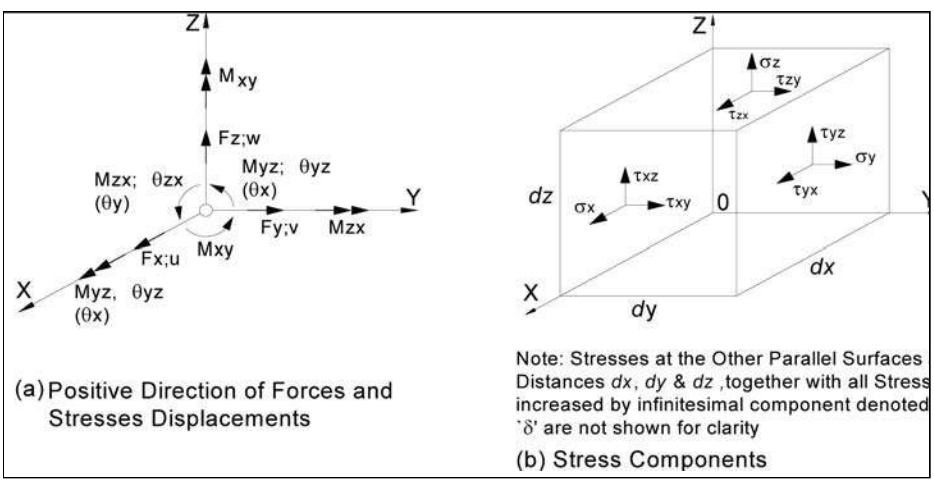


Fig. A.3 3-Dimensional Orthogonal Co-ordinate System following Right Hand Rule

A.2.4.3 Conventions used in two Dimensional and three Dimensional Plates and Shells

The Plates and Shells are three dimensional structures. In the theory of elasticity, some of the commonly met shapes like domes (surfaces of revolution) have exact solutions available for some simple loading conditions. Many other shapes, like cylindrical shells, have approximate solutions which can be accepted and safe structures can be designed using engineering judgement and experience. For applications where such structures are required (e.g. fish belly type cross-section for super-structure) the Code has suggested a general method in the Informative **Annexure B1**: Concrete Shell Structures. For such applications this or other specialised text books or literature may be referred to. Strict adherence to the sign convention is vitally important in such analysis.

A.2.4.4 Use of Varying Sign Conventions and Precautions

Mathematical methods require use of internally consistent sign conventions within the analysis, but these conventions are not necessarily the same in different methods of analysis. The designer must be wary of what these signs represent in the analysis in relation to the signs being followed in his design process. The best precaution to avoid blunders is to try and understand the overall deformations of the structure and the deflected shapes of the members obtained in the analysis and compare the same with individual’s own perceptions.

A.2.4.5 Further Developments in Methods of Analysis

Classical methods of static and dynamic analysis have developed over large number of years. The exact closed form solutions of the differential equations of the theory of elasticity are available only for a limited number of simplified structural elements and loading patterns. Therefore, many simplified, but sufficiently accurate methods for engineering applications have been developed. A review of these methods is outside the limited scope of this Chapter. Numerical techniques such as finite differences have general applicability in such cases, but these are fairly cumbersome to use.

Many other solution techniques suitable for hand analysis having special applications have been developed. ‘Moment Distribution’ method of Hardy Cross and its further generalisations using step-by-step relaxation method are suitable for indeterminate frame structures with and without sway. A number of ‘Strain energy’ methods were developed. Method of Three Moments, Conjugate Beam Method and Column Analogy are some of the other normally used methods. The powerful ‘Slope Deflection’ method developed in this era could be fully exploited only after advent of computers with application of ‘Matrix Algebra’ to deal with the large number of unknowns and equations.

A.2.4.6 Simplifications Introduced in methods of Analysis

Simplification of mathematical models is of great help to designers. Some of the simplifications commonly used in the analysis of bridges are mentioned bellow:

A.2.4.6.1 Two Dimensional representation in longitudinal and transverse directions

Analysis of structures idealised as if they are lying in one plane simplifies the understanding of behaviour, the sign conventions, as well as the mathematics of analysis itself. However, the real life structures have three dimensional geometry, and the effects in the out-of-plane direction (such as buckling) should not be lost sight of while using the two dimensional approximations.

A.2.4.6.2 Simplified Analysis of Plates for deck-slabs, webs of deep beams and box sections

Plates, which are two dimensional elements but are subjected to three dimensional loading, are very commonly used in bridge engineering. Many simplified methods of analysis involving acceptable loss of accuracy have been developed.

A.2.4.6.3 Simplification of loading

Most of the real life loads are complex in nature. The codes use simplified loads which produce more or less equivalent effects on the structure to that of complex loads. The use of uniformly distributed load in combination of single point load to represent actual effects of vehicular live loads was used earlier by the British Codes. In comparison, the currently used hypothetical train of axle loads are more difficult to use. The train of loads specified in Appendix of IRC:6 were developed by NATO countries for the military use. The present live loads used by Eurocodes are developed based on the number surveys carried out in Europe using statistical methods of stochastic analysis.

A.2.4.6.4 Simplification of dynamic effects

Use of Impact Factor to increase the static value of traffic load to produce equivalent dynamic loading of vehicles travelling at high speed is the well known example. Also use of static wind pressure steadily applied on the structure in place of real life dynamically applied time-varying wind pressures for bridges which are not dynamically sensitive to wind loading, is also common.

A.2.4.6.5 Seismic Effects

The Indian Standards are using simplified static inertial loads to calculate dynamic effects of Design Basis Earthquake, (DBE). Static 'Equivalent Forces' based on ground acceleration and natural period of vibration of the structure are used together with static linear elastic analysis in place of dynamic non linear analysis by reducing inertial forces by response reduction factor.

A.3 Modern Methods of Analysis

A.3.1 General

It may be understood that the theory of elasticity does not properly predict the behaviour of reinforced concrete structures near their ultimate state of strength. This is due to the prominent non linear behaviour of concrete in compression, effects of cracking and plasticity of steel.

New methods are developed in the last 60 years or so, based on the knowledge about the plastic behaviour of materials and resulting ductility of structural elements. These methods require more complex mathematical methods of analysis. However, their application in practice is simplified by tools like design charts and computerised algorithms.

A.3.2 Classification of Methods of Analysis

The different types of analysis are classified as per the constitutive laws of materials used in the analysis and whether the equilibrium of the structure is evaluated based on the original geometry of the structure (first order analysis) or based on the deformed geometry (second order analysis). Variously used idealised constitutive laws for stress-strain and moment curvature relations for uniaxial relationships are shown in **Fig A.4.** This usage has led to the following self-descriptive definitions:

(1) **First order linear-elastic analysis without redistribution**

Elastic structural analysis based on linear stress/strain or moment/curvature laws and performed on the initial geometry of the structure.

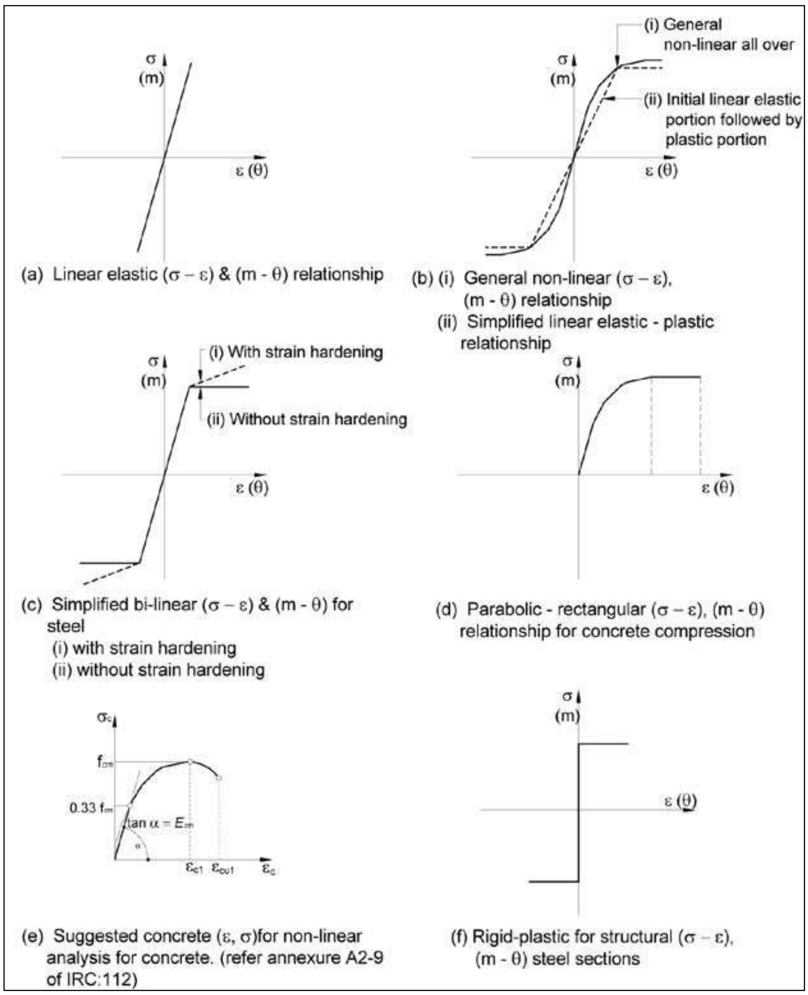


Fig. A.4 Various Constitutive Relationships

(2) **First order linear-elastic analysis with redistribution**

Linear elastic analysis, in which the internal moments and forces and external reactions are modified, but which remain consistent with the given external actions, if this is done within limits (for structural design), more explicit calculation of the rotation capacity, required for validity of the re-distribution, is not carried out.

(3) **Second order linear-elastic analysis**

Elastic structural analysis, using linear stress/strain laws, applied to the geometry of the deformed structure.

(4) **First order non-linear analysis**

Structural analysis, performed on the initial geometry, that takes account of the non-linear deformation properties of materials

This type includes different types of non-linearity such as a continuously varying stress-strain relation, bi-linear stress-strain relation with elastic branch followed by second linear branch with strain hardening or having perfect plasticity (e.g. reinforcing and prestressing steels). These are shown in **Fig. A.4**.

(5) **Second order non-linear analysis**

Structural analysis, performed on the geometry of the deformed structure that takes account of the non-linear deformation properties of materials as listed in type (4) above.

(6) **Rigid, fully plastic analysis, first or second order**

In the ultimate state of some of the materials (e.g. steel) the contribution of elastic phase to the total deformations is small and can be neglected for simplifying the calculated response in the analysis as shown in **Fig. A.4**. This method is not applicable for concrete structures.

A.3.3 Applicability of modern non-linear and plastic methods in bridges

The present trend in bridge design is to use different types of analyses for calculating different types of responses for different design situations (load combinations), even for the same bridge elements. For example, while overall analysis of an element is elastic, the sectional design is based on elasto-plastic methods. The use of various methods is described in **Table A.1** Design Situations and Types of Analysis.

Table A.1 Design Situation and Type of Analysis

No.	Type of Analysis	Design situation
1	First order linear-elastic analysis without redistribution	Global analysis for stability of bridge structure and its components.
2	First order linear-elastic analysis with redistribution	Calculation of load effects to be considered in ULS and SLS analysis of bridge as a whole for integral bridges and for super-structure only for continuous bridges, with limitation of 10 percent on redistribution.
3	Second order linear-elastic analysis	Analysis for imposed deformations (e.g. buckling) to verify satisfaction of the limit of 10 percent on second order deformation.
4	First order non-linear analysis	- Design of sections under ULS using bi-linear stress-strain relationship with sloping upper arm. - Design of sections under ULS using bi-linear stress-strain relationship with simplified (horizontal) upper arm. - Shear and torsion design by truss analogy.
5	Second order non-linear analysis	Analysis for imposed deformations (e.g. buckling) for design of slender elements.
6	Rigid, fully plastic analysis, first or second order	Not used for concrete bridges

A.3.4 Applicability of theory of plasticity to concrete structures

Theory of plasticity was developed for analysis of steel structures since steel exhibits large plastic deformations before failure, both in tension and compression. This theory can be applied to reinforced concrete structures provided that the plastic strains on compression side are large enough to permit formation of plastic hinges/yield lines, which allows redistribution of internal resisting forces while satisfying the new static and/or kinematical equilibrium conditions. The concrete section also needs to be 'not-over-reinforced' to the extent of not allowing steel to enter the plastic zone on tension side.

A.3.5 Strut and Tie Models

The code permits use of strut and tie models for local analyses. However, in any situations of loadings the possible number of solutions being very large, it is difficult to predict the most likely solution that will evolve in practice. It is advisable to develop model which is a logical development of the elastic phase into plastic stage as load increases. It should also be kept in mind that the idealised elasto-plastic diagram is not a true representative of the real behaviour. In reality the diagram exhibits a declining arm after reaching peak, and this behaviour puts practical limits on exploiting the plasticity fully, if the redistribution calls for mobilising strains beyond this peak.

It is relevant to mention here that in case of concrete corbel, highly non-linear strain distribution exists at re-entrant corners where effect of the declining arm may seriously reduce the capacity of plastic solutions. Such corners should be avoided in practice, thus minimising the stress concentrations, unless more rigorous analysis or testing of the experimental corbels is carried out.

A.4 Combination of Global and Local Effects

A.4.1 Global Analysis

This method is used for the overall analysis of the bridge structure. It consists of determining, in a structure, a consistent set of either internal forces and moments or stresses that are in equilibrium with a particular defined set of actions on the structure. The internal sets of forces depend on geometrical, structural and material properties. One of the commonly used applications is box girder super-structure, where the box cross-section may be treated as a line member.

A.4.2 Local analysis

Results of global analysis are valid in all parts of structure, except in local zones such as points of application of concentrated loads including support reactions, areas of discontinuity like openings in structural members and joints/connections of members at locations other than geometrically smooth transitions where sharp changes in the flow of stresses are involved. The Saint-Venant's Principle described earlier assures that the effects of these disturbances in the structure are strictly local and limited in their extent. The stresses at some distance away from these disturbances get back to the original state without vitiating the global analysis.

This principle allows considering equilibrium of a small part of the structure considering the equilibrium of a free body containing the disturbance, but having its boundaries sufficiently away at locations where the original stress distribution is as by the global analysis. In hand analysis this distance is taken as the depth of the element. A generalised method of analysis for such cases is to use the computerised analysis. Alternatively, other types of local analysis based on non-elastic methods or those using theorems of plastic analysis like strut and tie method are used where the normal elastic methods become unusable. One of the examples of such local analysis is idealisation of box girder super-structure into a transverse slice of unit width for load effects in the transverse direction.

A.4.3 Combined global and local analysis

In global FEM analysis the local disturbances can be included as a part of global geometry with refined finite element mesh surrounding the disturbance to yield distribution of rapidly changing local stresses as a part of global analysis. Similarly, in the case of box girder super-structure results of global analysis and local (transverse direction) analysis are algebraically added to get the total design forces.

A.5 Computerised Analysis and Computerisation of Design Process

A.5.1 Development of digital computers and use of numerical solutions, Matrix Algebra and applications to Dynamics

The availability of large capacity computers, small enough to be put on table tops in hands of designers, has revolutionised the computing capacity in design offices.

The slope-deflection method was developed by classical theorist. It involved setting up a number of linear equations connecting deformations of members at its end with the set of forces acting at its end by the members' stiffness or flexibility. The full rigidity (or full flexibility) of connections of such members enforced common deformations on members (or the individually separate deformation of fully flexible members). In case of partial fixity, the members can be connected by springs with required spring constants. Solution of such set of linear equations, which numbered the same as the number of unknowns, provides a unique solution to the structural deformations and forces. This method had limited application in practice due to the fact that the number of unknown increase very rapidly as the number of members and connections increase and the solution for each loading case has to be separately obtained. This was inspite of the availability of the mathematical tool of Matrix Algebra.

However, with the advent of digital computers the situation changed dramatically.

It became, not only possible but even simpler to solve the large sized frames, once the computer programmes for use of Matrix methods were developed. Many other types of indeterminate structures such as members of varying dimensions, two and three dimensional elements such as plates and shells etc. could be solved using various numerical techniques. Most significant development of finite elements took place based on the ability to solve the mathematical equations.

Even the non linearity of the material properties, and geometric non-linearity of structural response (i.e. second order effects) could be analysed using numerical techniques.

The otherwise un-solvable equations involved in the field of dynamics could be solved using step-by-step static solutions by small time steps and integration of the same to represent dynamic process. The meaningful earthquake analysis of structural response became possible.

However, this development has increased the need for designers to be more knowledgeable about the basic mathematical methods and to become proficient in their application, while using commercially available softwares. It is even more important to be aware of their strengths and limitations.

A.5.2 General observations

The large size of memories of present day computers are capable of dealing with very large number of unknowns arising from the large number of compatibility requirements, thus increasing the ability to find mathematical solutions for arbitrarily defined shapes, support conditions and loadings. However no solution can be more accurate than the accuracy of the data and the extent to which the assumptions made in the solution techniques are valid. It is, therefore, important to understand the basic theory and assumptions underlying the methods used by the computer programme.

Further more, computers are also being used to make design choices in addition to analysing the structure. However, indiscriminate use of automated procedures, both for analysis and design has a few major flaws, including amongst others;

(1) Creation of false sense of accuracy. Since the computer can calculate with equal accuracy solutions based on correct models as well as those based on the unrealistic models, the accuracy of solution depends on the use of correct model.

(2) It is not possible to improve the characteristics and behaviour of basic element (liner or multi dimensional) to take into account more refined properties of the same than the constitutive relations built-in in the programme.

(3) The mechanical use of software to obtain black- box solutions is not conducive for imbibing in the designers a physical feel of the behaviour of structures and structural materials, unless special efforts are taken to understand the same.

(4) In the computerised analysis large amount of data is required to be prepared as input data and the mistakes are difficult to detect.

(5) The process of carrying analysis is totally programme-driven and mechanical. The results are arrived at without the computer having any intelligent understanding of what it is doing and why. Whether the results look reliable or appear to be not- so- right in an engineering sense is to be judged by the user.

All this is known, but inspite of the difficulties and risks, computerised analysis is a very useful tool, if properly deployed and properly interpreted. The most important care to be taken in the analysis is the totally correct use of the signs in input data and the correct interpretation of the output. Since these sign conventions are programme - specific it is essential for the user to become fully familiar with the same before starting their use. This is specially required for using programmes originally developed for primary use in mechanical engineering applications for the civil engineering applications. Usually these are very advanced programmes and through familiarity with the methods, limitations as well as sign conventions is a pre-requisite.

A.5.3 Use of Finite Element Methods

The above discussion is especially true for using techniques of 'Finite Element Methods', (FEM). The assembly of finite elements should be examined visually or plotted as a hard copy for all structures to insure correct assembly of the elements. However, it should be remembered that the apparently identical looking twin- image shown by the assembly of Finite Elements does not correctly represent all properties and all behaviours of the structure of whose mathematical model it represents. The validity and accuracy of any output is decided by the properties of the finite element (i.e. shape functions) used and their suitability to represent the primary and secondary structural behaviour. For example, a 3-D solid element using compatibility of translational (u, v, w) deformations only at nodes cannot represent, at the element level, the effects of angular rotations and bending moments. For full understanding of the capabilities and limitations of various elements used in a specific programme reference shall be made to the specialist literature.

A.5.4 Computer Aided Designs

While using commercially available software as design aid it is necessary to be aware of the basic requirements of the code for which the programmes supplying design aids are written. If the same are different from the IRC codes being used (either old versions or the current version), these design aids cannot be used for the purpose of verification (i.e. to demonstrate the acceptability of the design).

A.6 Theories of Small Deflection, Large Deflection and their Applicability for Bridge Structures

A.6.1 Theories of small and large deflection

For many types of structures only limited deflections are acceptable in order to meet the functional and serviceability requirements. In other words, the complete structure and its structural elements should be sufficiently stiff i.e. non-flexible. It is found that for such structural elements the equilibrium of external forces and internal resisting forces need not be calculated on the basis of the deflected shape after loading by 2nd order analytical methods, but can be established with acceptable level of accuracy based on its geometry before loading (i.e. first order analysis). On the other hand, some of the structures may meet the requirement of stiffness of the structure as a whole, but some of its elements/members are more flexible and their stabilised deflected shape (i.e. their deformed shape) under action of load has to be taken in to account for calculating the overall equilibrium of the structure and load shared by the flexible elements/members.

The theory in which sufficiently accurate equilibrium between the load and resistance is established, based on the original unloaded geometry is termed as a theory of small deflections. This definition appears subjective and not mathematically precise but is enough to make a judgement about applicability and use of one or other type of analysis.

A.6.2 Applicability for Bridge Structures

The theory of small deflections provides the basis for most of the analytical work needed in bridge engineering with notable exceptions of second order analysis needed for analysis and design of slender members, which may be required in normal types of bridges having tall piers. This analysis is covered in the Code in **Section 11**: ‘Ultimate Limit State of Induced deformations’.

The analysis based on the theory of large deflections is needed for special types of bridges. The suspension and cable stayed bridges and large span arch bridges belong to this type. Suspension cable of the suspension bridges can only be analysed based on its deflected shape. The stiffening girders carrying load may be designed on the basis of the theory of small deflections, but the overall stability under localised live loads also has to be based on the large deflections.

The cables of cable-stayed bridges and the cables of extra-dosed bridges belong to an intermediate category depending on their geometry and length. However, at least for the construction stage analysis they have to be treated as highly flexible elements.

The IRC:112 on its own does not cover all design requirements of such elements. However, the Code can be used for such designs in combination with the help from specialised literature and/or international codes.

A.7 Prestressed Members and Structures

IRC:112 has covered prestressing in six sections. The global effects of prestressing on structures including its time dependent variations are covered in **Section: 7** ‘Analysis’; the design properties are covered in **Section: 6** ‘Material Properties and their Design Values’; the local effects, such as spalling and bursting behind anchorages and the technological aspects are covered in **Section: 13** ‘Prestressing Systems’; **Sections: 15 and 16** cover detailing and **Section: 18** covers ‘Materials, Quality Controls and Workmanship’.

Although various methods of considering prestressing in the analysis and design have been used in the past, the Code requires in **Clause 7.9.2** that:

“General

(1) Prestressing is considered as an action and its effect should be included in the forces/moments and applied to the structure.

(2) Prestressing force is time-dependent. Its magnitude also varies from the intended value due to technological reasons. Both the effects should be considered in selection of design prestressing force.

(3) The contribution of prestressing tendons to the resistance developed by the member shall be limited to the additional forces mobilised by their further deformations, consistent with the ultimate deformation of the member."

In view of the above, discussion in this Chapter is restricted to the evaluation of 'Prestressing Action' as a load due to prestressing. The appropriate methods of analysis are to be used for obtaining the action effects.

A.7.1 Concept of Prestressing as a Load

Historically, prestressing was developed in era of the 'working load/allowable stress' philosophy of design. Its target was to increase the capacity of concrete members to carry tensile loads without cracking, allowing the optimum use of materials in which the stiffness from cracked concrete need not be neglected. The prestressing of a structural member in this way may be defined as the creation of an initial stress of opposite sign to the stress produced by the working load, in order to increase the working load without increasing the actual stress in the member. The most logical way to achieve this is to apply opposite force as a pre-load, which is created by prestressing. For prestressing to be most advantageous, it is therefore necessary that the working load should act mainly in one direction and the creation of initial stress of opposite sign is achieved by prestressing. In theory it is not essential that pre-loading is achieved by stressed steel tendons anchored to the structural member although it turns out to be the most convenient method, especially after steels of high strength having adequate residual force after accounting for relaxation loss were developed.

However, it should be remembered that achievement of enhanced working load capacity does not automatically ensure sufficient safety margin under Ultimate Limit State. For Instance, in case of unbonded tendons, which do not undergo large increase of force under factored ultimate load, the required margin of safety cannot be provided by prestressing tendons and it needs to be achieved by other means.

The force in the tendons is transferred to the structure at all points of contact between the two. At all locations the force on the structure is equal and opposite of the force acting on the tendon. At location of anchorages force equal to locking force acts along the direction of the tendon. A curved tendon pressing against the structure transfers pressure equal to T/R where T is the local tensile force in tendon and R is the radius of curvature at that location. For a circular profile it represents a constant radial pressure along the tendon profile. For a parabolic profile it is equivalent to uniformly distributed load, since 1/R = constant for the parabola. For moderately flat profiles this is easily calculated by dividing the change in vertical component between two points of the tendon by the length of the horizontal portion in between.